GEOMETRY OF SPHAERICS.

(A pedagogical experiment in stereographic projection with Maximiliano Londono.)

by Pierre Beaudry
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1. THE SUCCESSFUL FAILURE OF OUR GEOMETRIC MODEL.

There are many wrong underlying assumptions about man's investigation of the heavens, but I am going to deal with only one of them because of its pedagogical importance. The most significant wrong assumption one makes about the heavens is when one thinks that the motion of the starry canopy is really taking place inside of an immense sphere. That is a {*sense-deception*}, as Lyn put it. And, such a {*sense-deception*} should be looked at as an essential feature of the Noosphere and should be represented as a necessary companion to human reason. In other words, {*sense-deception*} is a divine irony, a principle of difficulty that God has imposed on all human beings in order to teach them humility. From that vantage point, {sense-deception} is truly different from animal perception. No more than the biological body of man, should perception be considered as an animal part of man. There is no animal part of any human being. Furthermore, this astrophysical {sense-deception} was also probably an experience that led Plato to take up the metaphor of the Cave. Remember that in Greek, the word cave is {*koilos*} meaning cavity and concavity, which is where the French word {*ciel*} comes from.

However, the sense-impression of the existence of a heavenly sphere is a very real human impression, even though it does not correspond to the reality of the universe. And therefore, the stars only appear to be rotating around a fixed point called the Celestial North Pole. This poses a fundamental problem, perhaps the most important problem concerning truth in all of its forms, and that is: how do you determine the role of sense perception with respect to real knowledge of the universe? It is only after having successfully failed on a number of observations that one realizes how the celestial North Pole is nothing but the subjective projection of our perception of the earth's north-south axis of rotation. The ancients, as far back as to Pythagoras himself, knew this. In other words, such an experiment in {*sense-deception*} should immediately reveal to us two things: the first is that our sense perception does not represent true knowledge and the second is that this lie, when educated by reason, tells us that the starry canopy's rotation is merely the result of the rotating motion of the earth which appears not to be moving at all. Thus, an educated lie becomes useful to find the truth.

Therefore, thanks to that fallacy, it was from that deception that the ancient science of {*Sphaerics*} became the means by which man was able to cognitively have a first scientific grasp of the universe and began to infect the planet with that cognitive power. It was from the vantage point of that central paradox of {*Sphaerics*} that the universe could be considered as being both finite and without external boundary; that is to say, that we are living in a universe outside of which the effects of universal physical principles cannot exist. But, this also means that if the effects of universal physical principles could exceed apparent formal boundary conditions of the universe, then, it would be lawful that we could go beyond such formal limits of the universe. I will now submit to you and examine a case of this paradox, and I will show you, from that vantage point, how the successful failure of our geometric model will be able to lead us to the truth about the universal characteristic of boundary conditions.

Furthermore, since what you actually see with your eyes is not the real universe, there is only one way to restore your confidence in a truthful knowledge of the world, and that is, by recognizing that what you are experimenting is like seeing, as if through a glass darkly, the shadowy distortion of universal physical principles that your mind is attempting to discover by means of normalizing a process of change. Thus, what your mind sees is that the sphere provides you with a means of normalizing what you see with your eyes in congruence with what you see with your mind. Then, the question becomes: " how do you know with certainty that this is truthful? Where and how did you acquire that knowledge? Is it because someone said so on television and showed you the earth turning in empty space, like a Google Earth, or did you discover the truth of the matter in a different way, by means of some cognitive principle?" Before answering these questions, imagine yourself living in Egypt 5,000 years ago and consider that you have come to the very same conclusion, but without a television set. How would you then know with scientific certainty, that this was true?

Here is the answer that Archimedes gave in his {*Sand-reckoner*} to this last question with respect to the greatest Greek Astronomer, Aristarchus (c. 310-230 BC).

"You [King Gelon] are aware that 'universe' is the name given by most astronomers to the sphere the center of which is the center of the earth, while its radius is equal to the straight line between the center of the sun and the center of the earth. This is the common account, as you have heard from astronomers. But Aristarchus brought out a book consisting of certain hypotheses, wherein it appears, as a consequence of the assumptions made, that the universe is many times greater than the 'universe' just mentioned. His hypotheses are that the fixed stars and the sun remain unmoved, that the earth revolves about the sun in the circumference of a circle, the sun lying in the middle of the orbit, and that the sphere of the fixed stars, situated about the same center as the sun, is so great that the circle in which he supposes the earth to revolve bears such a proportion to the distance of the fixed stars as the center of the sphere bears to its surface." (Thomas Heath, {*A History of Greek Mathematics*} Vol. II, p. 3.)

Here, Archimedes reports on the masterful Aristarchus hypotheses in the form of a great irony. The irony is that Aristarchus was hypothesizing that a true boundary condition of the universe as a whole must be grounded on the epistemological power of the human mind to grasp the infinite within the finite. This is what Lyn had identified as a universe which is both finite and without external boundary. Thus, Archimedes confirmed that it was Aristarchus that brought closure to the hypothesis of heliocentrism and that his ontological proof of it laid in the discovery of the universal physical principle of incommensurable proportionality between man and the universe as a whole. In a sense, Aristarchus was also responding to the Pythagorean doctrine of harmonic ordering and of Plato's principle of proportionality, which stated, "…God created and bestowed vision upon us so that we, contemplating the orbits of intelligence in the heavens, might put them to use by applying them to the orbits of our reason, which are related to them…." (Plato, {*The Timaeus*}, 47.b.)

Thus, the ancient boundary conditions of the universe were set, epistemologically, in harmonic proportionality with human vision together with human hearing. And, as Lyn keeps emphasizing, both human vision and human hearing are ontologically coordinated through the Noosphere and, therefore, are not to be considered as functions of the Biosphere. In other words, there is nothing in man that pertains to the animal domain. In the same way that the chemicals found in living beings are nowhere to be found in non-living beings; similarly human vision and hearing are ontologically unique to man and cannot be found in the animal domain. From that standpoint, it should be clear that the Biosphere and the Noosphere are each ontologically unique.

2. THE IDEAS OF LIMIT AND BOUNDARY CONDITIONS IN SPHAERICS.

I want to go through two considerations with respect to what Lyn has been saying on the subject of boundary conditions and how this relates to the ancient discovery of the astrolabe, to stereographic projection, and to solving the paradox of projecting the sphere onto a plane. Though there may be doubts about the attribution of the Astrolabe to Hipparchus, since none of his writings on astronomy have reached us except through the sophistry of Ptolemy's {*Almagest*}, there is no doubt, as we shall see below, that the method Hipparchus used for determining the position of stars on a celestial sphere was the necessary step to the discovery of the astrolabe.

First, I want to identify that this report is in response to two questions that Max Londono and Pedro Rubio have asked me about the ideas of limit and of force in the universe. Those two questions appear to have the same underlying assumption, and that is: {*Outsidedness*}. In that form, this pertains to the Euclidean a priori boundary condition of the universe that Lyn has been criticizing and which has to be replaced by universal physical principles in accordance with the anti-Euclidean approach of both Riemann and Vernadsky. See EIR, April 13, 2007. {*Is the U.S. Congress Dying? The Baby-Doomers*.} In other words, if the idea of external formal limit is replaced by the idea of self-developing boundary condition of the universe and the idea of external force is replaced by the appropriate idea of power, then you automatically will perceive cognitively how the fallacy of mechanics must be replaced by dynamics.

As Lyn emphasized, the question of limits and of harmonic relationships in the universe has to be viewed from the vantage point of breaking with the Euclidean domain of apriori axioms, postulates, and definitions and of compartmentalized knowledge, such as music separated from physics, or life separated from thinking, and should be looked at from the standpoint of interdependency of different principles affecting all of these different areas of knowledge. I think that a good way to rethink the whole thing is to use the sphere as a metaphor of Lyn's paradoxical idea that the universe is {*finite but without external boundaries*}. Think of boundary conditions as being subjective and internal to a self-developing process as opposed to being imposed formally from the outside.

For example, when I discovered the integral 10-circle Egyptian sphere, which integrated the twelve interval well-tempered musical system, the five Platonic Solids, the Great Pyramid of Egypt, and the Doubling of the Cube, I realized that what the boundary condition of the sphere was not determined by the outer surface of the sphere as such, but by the internal dynamic of a self-bounding principle of harmonic ordering, which came from inside of the sphere, and which defined its harmonic proportionality only internally, and without any external limitation. This helped me realize that the twelve stars that appeared, as shadows, on the surface of that sphere were merely an expression of what was in the dynamic center of the sphere and that, therefore, everything that was integral to the sphere was everywhere a reflection of a universal physical principle of proportionality that was present in the infinitesimally small as well as in the extremely large. It was from that vantage point that the dynamic of the ten-circle sphere began to appear as the boundary condition of the five Platonic solids; just a Plato had expressed the function of his nurse {*Chora*}, his phase space of transformation, in {*The Timaeus*}. Such a {*Chora*} is also expressed in the Kepler notion of the sphere as he developed it in his {*Paralipomenes a Vitellion*} for the determination of harmonic relationships between the curved and the straight. That is the key.

"First, it is fitting that the nature of all things be in the image of God the Creator, and to the extent possible in accord with the creation of each thing's own proper essence. For when the Most Wise Creator strove to perfect all things in the most beautiful and excellent way possible, He found nothing more beautiful and more perfect, nothing more excellent, than Himself. For that reason, when He took the physical world into consideration, He settled on a form for it as like as possible to Himself. Hence were generated the entire gender of quantities, and within it, the distinction between the curved and the straight, and the most excellent figure of all, the spherical surface. For in forming it, the Most-Wise Creator played with it to form the image of His Reverend Trinity. Hence the central point is therefore like the source of the entire spherical solid, the surface is the image of the intimate point, and one may conceive that each pathway leading to it is generated by an infinite emanation (in all directions) coming out of the point itself and extending itself in an equality of all emanations, the point communicating itself into this extension and becoming equal to the surface, in accordance with the variation of the ratio of density. Hence, between the point and the surface there is everywhere an utterly absolute equality, a most compact union, a most beautiful convenience, connection, relation, proportion, and symmetry. And since these are clearly three – the Center, the Surface, and the Interval, - they are nonetheless one, inasmuch as

not one of them, even in thought, can be separated from the others without destroying the whole." (Kepler, {Paralipomenes a Vitellion}, p. 107.)

This is the way to eliminate from your mind the wrong sense of Euclidean outer limit or external force that your sense perception tends to attribute to an externally imposed boundary.

Secondly, the question arises also as to how this can be related to Hipparchus and to his discovery of stereographic projection as applied to the solution of the paradox of projecting a sphere onto a plane, which resulted in his invention of the astrolabe. This requires some geometric constructions that will lead us to understand some aspects of Riemannean hypergeometry.

The initial astrophysical observations of Hipparchus at the observatory of Rhodes during the second century BC, involved the mapping of stars onto a sphere and the mapping of the starry sphere onto a plane disc. There is a French tradition of popular astronomy on that question that Francois Arago kept alive after the Napoleonic demise of the Ecole Polytechnique, and which Charles Davies initiated also at the United States Military Academy of West Point, where Monge's descriptive geometry was taught for a period of at least fifty years. This West Point program should be revived within our membership as well, especially emphasizing the role of Hipparchus as Arago insisted. Just to give you a sense of how he constructed his model of the celestial sphere, consider the following few steps.

3. HOW HIPPARCHUS MAPPED 1,028 STARS ONTO A SPHERE.

What Hipparchus did, from his observatory in Rhodes, at about 200 A.D., was to create a large sphere representing a miniature copy of the celestial sphere on which he mapped the "fixed stars" of the heavenly sphere with total precision by determining their angular distances. The key was to discover a method of *{measuring and projecting angular distances between stars}.* For that purpose, Hipparchus invented the method of measuring the circle in 360-degree angles, 60 minutes, and 60 seconds, which is still in use today.

However, the reader should note a double difficulty, here. First, the study of Hipparchus is made most difficult because only one of his books has survived from his early period, his {*Commentary on the Phenomena of Eudoxus and Aratus*}. Secondly, people have lost the passion for seeking the truth beyond the existence of original documents and fall easily prey to second hand documented evidence that may not be reliable. As a result of this lack of documentation and unreliability of secondary sources, you may hear slanders against Hipparchus; especially that he was a "geocentric Babylonian magician." Pay no attention to this kind of concierge gossiping, and think for yourself. Though there may be lacking direct evidence for his discoveries of the astrolabe, of spherical trigonometry, and of the precession of the equinoxes, the fact that Claudius Ptolemy was one of the last people on record to have made use all of the known astronomical works of Hipparchus, in order to construct his own geocentric system of epicycles, is not sufficient evidence to qualify Hipparchus as a Babylonian sophist. That would be fallacy of historical composition. Why the books of Hipparchus no longer exist after Ptolemy had access to them is not known. It may well turn out that Ptolemy subverted the {*Sphaerics*} of Hipparchus precisely in the same fashion that Euclid subverted the {*Sphaerics*} of the Pythagoreans. So, my references to Hipparchus must rely more on trusted secondary sources than on primary evidence, and especially on the epistemological methodology required to achieve the discoveries that are attributed to him. As an example of method, witness how, according to Francois Arago, Hipparchus plotted stars onto a sphere. The method itself is quite telling about the cognitive thinking process of its inventor.

Hipparchus started his celestial sphere mapping with Sirius, the brightest star in the night sky. Sirius was both his starting point, his reference point to determine position for the rest of the stars, and the last star he returned to at the end of his process of discovery. He first plotted Sirius arbitrarily anywhere on his sphere. Then, he took a second star whose angular distance from Sirius he measured with some sort of portable sighting device that Arago called "pinnules." .

The second star he spotted was located at the angular distance of 20 degrees from Sirius. Hipparchus opened his compass by 20 degrees and traced a circle of 20 degrees around Sirius as the center. He knew the second star would be somewhere on that circumference, but he did not know where. So, he decided on an arbitrary position of the second star somewhere on that 20-degree circumference, but not as arbitrary as the choice of position of Sirius had been. The second star had to be somewhere on that circle, and once he chose its arbitrary position, it then became precisely determined with respect to Sirius. From that point on, everything had to be precisely established into a sort of spherical locked-in position, so to speak.

His next move was to locate a third star, which was at 22 degrees from Sirius, and 30 degrees from the second star. That appeared to be a little more complicated, but not really. Hipparchus then took his compass and drew two spherical circles. The one opened by 22 degrees was again centered on Sirius, and the other; open by 30 degrees was centered on the second star. Of the two intersecting points, between the two intersecting circles, there was only one point that had the property of being locked into the right position at 22 degrees from Sirius, and at 30 degrees from the second star. Thus, the position of the third star was entirely determined by this geometry of position. Then, Hipparchus obtained similar results when he chose a fourth star, and so on, subsequently.

Thus, by measuring the precise angular distances of 1,026 stars, and establishing the dynamics of boundary conditions on a physical sphere, Hipparchus had invented what Leibniz later called *{analysis situs}* or geometry of situation without axioms, postulates and definitions. That is what Carnot also called "geometry of position," and which I prefer to call "geometry of situation," because the notion of situation has the advantage of being descriptive of both historical as well as physical space-time. You should also know that the very first lesson that Gaspard Monge gave on January 20, 1795, at the Ecole Polytechnique, was entitled "{*How to Determine a Point in Space*}" and he had used this very same Hipparchus method for his construction. Charles Davies did the same thing in his classes at West Point.

Once Hipparchus had mapped all of the 1,026 stars on his sphere, the most difficult part of his investigation began. How was he going to map these stars onto a circular plane disc? This problem immediately raised the question of the astrolabe. Indeed, the discovery of the astrolabe requires the mapping of the celestial sphere onto the equatorial circle of that sphere! In other words, as I will develop below, whoever discovered the principle of stereographic projection, required for the astrolabe, also required to know how to map all of the stars onto a sphere! Why is that?

Well, the discovery of the astrolabe implies that the same angular projection of the sphere had to be replicated onto a flat surface, thus, realizing that the same compass conic function was common to both the plane and the spherical domains! It is the non-linear conical function that solves the paradox between the plane and the sphere. The circles were not of the same size, but were translated in the same proportion. Low and behold, both angular compositions of the sphere and the plane were different and similar at the same time. They were stereographically maintaining the same proportionality of angles between the azimuth circles (longitude) and between the almucantar circles (latitude). I am giving you the Arabic names of these circles because the oldest existing astrolabe was built by Muhammad Al-Fazar (796-806) and the astronomical tradition has kept this language.

Furthermore, in order to completely construct the proof of this astrolabe discovery, you will have to walk through several paradoxes with respect to the apparent motion of the sun. I will now attempt to reconstruct what appears to be some of the likely steps that were taken to establish the calendar year of the sun. Moreover, I will also show that whoever discovered the astrolabe also had to be highly aware of {*sense deception*} of heliocentrism as opposed to geocentrism.

4. THE DISCOVERY OF THE SUN'S PATHWAY DURING THE YEAR.

The discovery of how to locate the position of the sun in the heavens with respect to each day of the year is probably one of the most difficult questions of ancient astrophysics. In fact the answer to that question requires the projection of the entire sphere of the heavens onto an astrolabe. However, what kind of representation can an observer on earth have of the position of the sun at any time during the year? Remember the first approximation that I gave you with the different zenith angular positions of the Sun's shadows on the site of the Great Pyramid of Giza in Egypt? What were the key features of that representation? Two things had to be emphasized: the zenith angular distance of the shadows of the noonday sun, the two limits of the winter and summer solstices, and the half of the entire angular span, which determined the equinoxes.

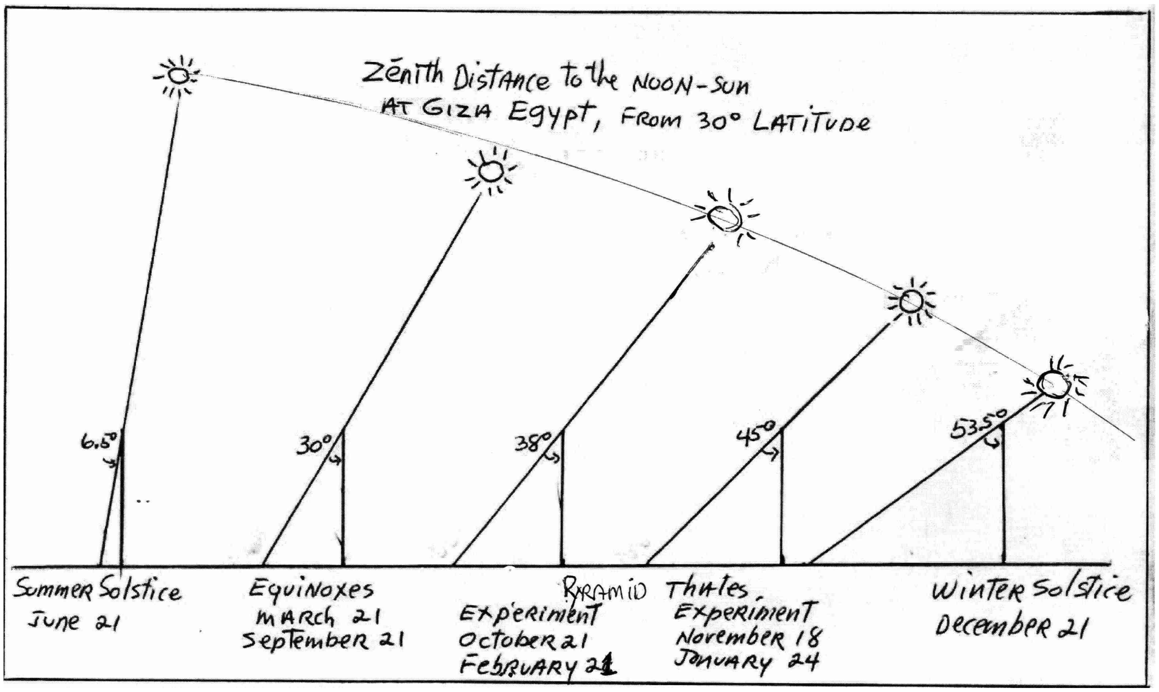

Figure 1. [Noonday shadows from Giza Egypt.]

Now, how can this shadow reckoning idea of Figure 1 help us discover a complete circular mapping of the sun for the duration of an entire year? How can I project this from an apparent sphere of the heavens onto a plane? What needs to be established are the fixed position of the solstices and the equinoxes, but they keep on moving all the time. The timing between them may change somewhat, from year to year, but the axis of those four cardinal points must be established in some form of stable framework. It is that universal workable framework that had to be discovered with respect to the fallacy of our {*sense-deception*). As Lyn keeps emphasizing, it is the epistemological limitation of our sense-perception experience, which has to be understood properly with respect to the physical universe. "{*Perhaps the most controversial, and most important issue which permeates every corner of efforts to understand physical science and art, alike, is the question whether or not, and in what manner and degree, our sense-perceptual experience as such corresponds to the universe in which we actually exist.*}" (Lyn LaRouche, {*Euclid's Essential Fraud*}, MORNING BRIEFING, April 27, 2007.)

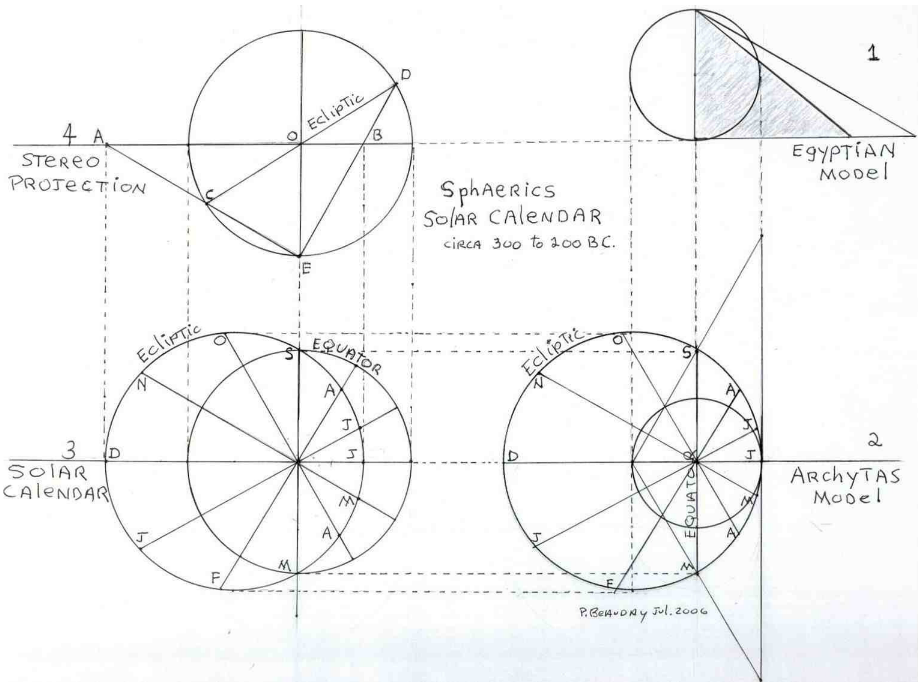

Figure 2. [Transformation of the Archytas model into a solar calendar].

Even though there are no direct ancient documented sources showing the construction of the following solar calendar, during the post dark age period of 700 BC to about 200 AD, there are sufficient traces of ancient Egyptian and Greek knowledge to assume its constructability. This constructability also involves, as the Archytas demonstrates explicitly, an original understanding of not only the domains of rational power and of irrational power, but also the domain of transcendental power. Thus, from the Egyptian and Archytas models of the doubling of the cube alone, it is readily feasible to trace back what the actual thinking process of an ancient solar calendar must have involved based on their astrophysical knowledge of {*Sphaerics*}. A summary investigation of the Archytas model for the doubling of the cube, for example, will reveal the essential components of such a calendar and their connections to the discovery of principle of Hipparchus.

For example, considering that the Archytas doubling of the cube reflected an early form of the complex domain in which one could relate to incommensurable magnitudes between rational, irrational, and transcendental powers, ask yourself: "how can the projection of the Archytas model of a Cylinder, a Torus, and a Cone be the discrete manifold shadow of a Sphaerics continuous manifold representing the twelve month solar calendar of an astrolabe?" Also ask yourself: "how can such a calendar reflect the universal common boundary condition of the dodecahedral sphere generating the Five Platonic Solids, the Great Pyramid of Egypt, the Doubling of the Cube, and the Twelve Intervals of the Well-Tempered System?" In other words, the Pythagorean idea of the Egyptian calendar of 360 days/angles is to establish a division of a year into twelve equal months, which also corresponds to the partitioning of a sphere into twelve equal angular divisions. How do you construct that?

If you use the Archytas Model 2 for the doubling of the cube, as projected from the Egyptian Model 1, you can transform the plane area covered by the Torus into a projection of the twelve unequal divisions of an Ecliptic circle connected to the twelve equal divisions of the Cylinder base as established by the Scalene Conical function cutting the two points of the equinoxes. Thus, each of the twelve divisions of the Ecliptic is made to correspond to an equal 30 days/degrees, which correspond to the ancient Egyptian calendar of 360days/angles.

This is truly perplexing because we all the time experience the effects of such yearly events that cannot be seen with our physical eyes, and yet we know such effect to exist with certainty and we can prove their actual truthfulness without ever requiring a sense perception proof of its existence. For example, the yearly pathway of the sun, which corresponds to the orbit of the earth around the sun during a year, is such a non-visible experimental phenomenon. You can never have a perception of its pathway during an entire year, because it is a construction of {*sense-deception*}, and yet, you can have a comprehensive knowledge of its deception and, therefore, you can reproduce the ironic cycle that corresponds to it. Let me show you what I mean.

In a stereographic projection, the solar cycle of the Ecliptic is a fascinating irony because the sun apparently spends half of the year traveling outside of the universe! Now, that should raise eyebrows about the limits of the universe, shouldn't it? Indeed, you may have noted that, in sections 3 and 4 of **Figure 2**, the stereo-projection of the Tropic of Cancer segment of the Ecliptic Circle is projected at 30 degrees inside of the Equatorial plane of the celestial sphere, while the Tropic of Capricorn segment of the same Ecliptic Circle, which is located in the southern hemisphere of the universal sphere, is entirely projected outside of the universe from the south pole. This must definitely pose some questions about the boundary conditions of the universe, especially if you think that there must be a formal external limit to this universe. Is there any problem with that?

Well, there should not be any problems, really, because, since there is no external boundary of the universe, the irony of the matter is that whatever appears to be the inside or the outside of the universe must be determined by a universal physical principle of proportionality and not by some formal geometrical limit. This may be confirmed by the following proportional limitation between the two manifolds (3) and (4).

AB : CD :: DJ : MS because **AE : AC :: DE : DB**

Thus, the Ecliptic Circle and the Equatorial Circle in the sphere are proportional to corresponding circles in the plane because the stereo-conic projection is what connects proportionately the two domains together, providing assistance to both the curved and the straight. The truth of the matter is that the universal physical and harmonic proportionality of this process is in conformity with the Noosphere of reason. This is the reason why whatever is located north of the equator is considered to be inside of the sphere of the universe and whatever is located south of the equator must appear to be projected outside of that same universe. This is what the anomaly of the stereographic projection is all about. This is also what I mean by the successful failure of our geometric model. There should be no difficulty with this if you have abandoned the idea of an external limit to the universe for the subjective idea of internal self-development.

5. THE SINGULARITY OF THE STEREOGRAPHIC ZENITH FUNCTION.

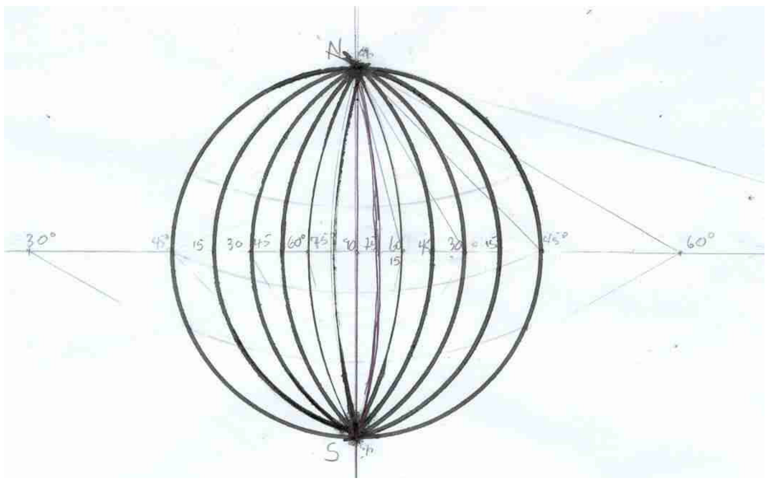

Figure 3. The twelve azimuth partitions of the stereographic sphere

For the purpose of construction, all of the twelve azimuth circles of the sphere are determined by the angular conical projections of only two conical functions, the 45/90-degree cone and the scalene 30/60/90-degree cone.

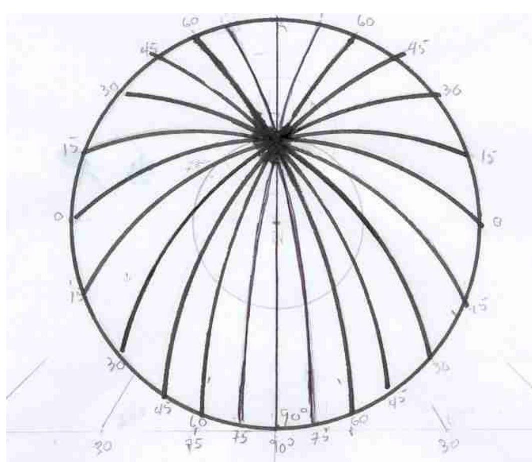

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Figure 4. The twelve azimuth circles projected at the Zenith of 23.5 degrees on the stereographic sphere.

Note that all of the twelve azimuth circles are generated by the same six angular measurements of the conical-compass projections of 15, 30, 45, 60, 75, and 90 degrees. The angular projections are all different, but their projected results are all the same! These degrees all reflect the degrees of right ascension of stars taken from the horizon of an observer on earth. It is by this conical function that the stereographic projection maintains the proportionality of angles in the transformation between the Solid Locus and the Plane Locus. It is also in this manner that the stereographic projection maintains the proportionality of azimuth circles in the transformation from the sphere to the plane. The crucial singularity underlying azimuth circles resides in the discovery of the Zenith function from which they can be mapped onto a plane.

A Zenith function is the unique subjective projection of a personal spherical point onto the heavenly sphere taken from the fixed position of an observer on earth. Its purpose is to act as a {*point de capiton*} that holds together the fabric of the starry dress of night like a well-tailored arachnidan web of azimuth and almucantar circles. It is from that Zenith function that the angular position of stars is determined, as they appear to be dancing overhead in the night sky during the year. Therefore, the zenith point corresponds to any latitude position on earth and it constantly changes its angular web with respect to the fixed Celestial North Pole and the fixed position of a chosen observatory on earth. Consequently, the position of stars rotating on the imaginary sphere of the heavens will appear differently depending on the position of the observer, but will be made to cross the circles of his web at precisely identifiable moments of the year. The whole process is, therefore, timed by the precise position of the sun on the ecliptic, because it is the sun who is the director of this great symphony!

6. HIPPARCHUS, APOLLONIUS, AND FERMAT ON THE SOLID LOCUS.

As Lyn has often suggested on the question of astrophysics, otherwise known as {*Sphaerics*} for the Ancients, the epistemological crux of the matter is reflected in the method of constructive geometry that was designed as an anti-Euclidean method of geometry by Apollonius of Perga (262? – 197? BC). In this regard, the work of Apollonius on conic sections greatly influenced Kepler, Fermat, Roberval, Pascal, and Leibniz not only on the geometrical approach to quadratures, rectification of curves, maxima and minima, and other problems of tangents, but also most significantly on the epistemological question of establishing an axiomatic difference, in mathematical physics, between the geometry of situation in the plane domain and the geometry of situation in the solid domain. In a lost manuscript on {*Plane Loci*}, Apollonius had originally established a crucial difference between the straight line and the surface as belonging to the plane domain, on the one hand, and the ellipse, parabola, and hyperbola belonging to the solid domain, on the other hand. Apollonius made this distinction in direct response to the sophistry of Euclid.

The point to be made, here, is that the reason why the ellipse the parabola and the hyperbola were called Solid Loci was not only because they were sections of a solid cone, but because Apollonius considered these figures to be generated from a higher dimensionality than those of the plane figures that Euclid generated. These figures were treated as transcendental because they each related to the infinite. And, consequently, Euclid was not able to deal with them properly. [<fn2> In his General preface of Book I on the {*Conics*}, that he had sent to his friend Eudemus, Apollonius identified the shortcomings of Euclid with respect to the subject of conics. He wrote: "{*The third book contains many remarkable theorems useful for the synthesis of solid loci and for {diorismi} (determination); the most and prettiest of these theorems are new, and it was their discovery which made me aware that Euclid did not work out the synthesis of the locus with respect to three and four lines, but only a chance portion of it, and that not successfully; for it was not possible for the said synthesis to be completed without the aid of the additional theorems discovered by me.*}" Thomas Heath, {*A History of Greek Mathematics*}, Volume II, Dover publication, 1981, p. 129.)]

In his reconstruction of the lost text of Apollonius, Pierre de Fermat also emphasized this question with respect to identifying a uniquely determined curve considered as the locus of an indeterminate equation of first, second, or higher degrees. In his treatise on Apollonius, Fermat stated: "{*There is little doubt that the Ancients wrote many works on loci; witness Pappus who, in the beginning of the seventh book (of the Collection) asserts that Apollonius wrote on the Plane Loci and Aristaeus on the Solid Loci. But, unless we are mistaken, their investigation did not satisfy them sufficiently. This we gather from the fact that they did not express many loci in sufficient generality, as we will show below.*}" (Pierre de Fermat, {*Ad Locos et Solidos isagage*}, (Introduction to Plane and Solid Loci), quoted by Michael Sean Mahoney, {*Pierre de Fermat 1601-1665*}, Princeton Paperback, Second Edition, 1994, p. 91) [<fn1>Fermat also added the following: "{*Whenever the local endpoint of the unknown quantity describes a straight line of a circle, a plane locus results; and when it describes a parabola, hyperbola, or ellipse, a solid locus results. If (it describes) other curves, the locus is called linear. We will say nothing concerning the latter, because a knowledge of a linear locus is very easily derived, by means of reduction, from the investigation of plane and solid loci.*}"]

Moreover, in his {*Paralipomenes a Vitellion*}, Chapter IV, Kepler also referred to Apollonius topic and identified not three, but five conic sections. In a manner reminiscent of Cusa, Kepler emphasized the singularity of passage at infinity between each and all of the conics. In this way, Kepler established a bridge between Cusa and Leibniz by implying the existence of a {*principle of continuity*} between all conic sections and through which "{*there is an ordering dependent on their properties*}." As he said: "{*There exists different cones: the rectangular cones, the acutangular cones, and the obtusangular cones; similarly there are right cones, or regular and scalene cones, or irregular, or again flattened: on that account, see Apollonius and the commentaries of Eutocius. For all of these cones, without exception, there are five species of sections. In fact, the line formed at the surface of a cone by its section is either a straight line, either a circle, either a parabola, either a hyperbola, either an ellipse. Between those lines – and speaking more from analogy than from geometry, there is an ordering dependent on* *their properties, which is the passage from the straight line to the parabola by the intermediary of an infinity of hyperbolas, then from there to the circle by the intermediary of an infinity of ellipses. In reality, of all of the hyperbolas, the most open one is the straight line, while the most acute one is the parabola, and similarly, of all of the ellipses, the most acute one is the parabola and the most open one is the circle.*}" (Kepler, Op. Cit., p. 220-22.)

In a way, those two different methods reflect the same principle, which implies the existence of a Riemannean axiomatic change between dimensionalities, the existence of an epistemological gap. It was also by means of making the same axiomatic difference in curvature between a Plane Locus and a Solid Locus that Fermat was able to disprove Galileo on the experimental question of a cannon ball falling from a tower. Contrary to Galileo who initially thought the cannonball fell in a straight line, as sense perception would have it, Fermat demonstrated that the locus of the curve must be that of a spiral caused by the additional dimensions of two other motions, the earth's orbit and rotation. Thus, the apparent straight line of a falling cannon ball is in reality the result of the higher power of a conical function.

Similarly with Hipparchus, we must treat the stereographic projection of the sphere onto an equatorial circle not as a simple Plane Locus, but as the expression of the Solid Locus of a conical function that is reflected by the action of gravitation and therefore requires the introduction of a higher universal physical principle of proportionality than plane geometry requires, a double mean proportionality, as Plato stressed. As in the case of the principle of continuity, the discontinuity of stereographic projection between the sphere and the plane is everywhere effective within the dynamics of conical spiral action.

Thus, from the vantage point of principle, the very idea of stereography accredited to Hipparchus is quite different from the Euclidean-Cartesian approach that was designed by the conformal projection of Poincare's model of a spherical projection onto a hyperbolic plane. The projection of Hipparchus, understood both from the standpoints of epistemology and of practical astrophysics, takes fully into account the Apollonius distinction between a Plane Locus and a Solid Locus. This may be hinted at by the fact that there cannot be a direct orthographic bi-univocal projection of an Azimuth circle from a sphere onto a plane. The only two spherical points that can be projected orthographically are the zenith and the nadir points. Otherwise, there is an epistemological leap between the two dimensionalities, an incommensurable gap, which is filled by an angular conic projection relating the two incommensurable domains proportionately. As a result, the angular projection of equal spacing by 15 degrees in the Solid Locus cannot be projected orthographically, but can only find its proportionate spherical equivalent stereographically in the Plane Locus by means of a compass conic function! Compare the projection of both azimuth and almucantar circles.

For example, take the following case of the zenith function projected from the latitude of 39 degrees in Leesburg Virginia. **Figure 6** is indubitably the stereographic projection of **Figure 5**, yet only the two extremes of azimuth and almucantar circles can be connected orthographically one on one by two different conical projections!

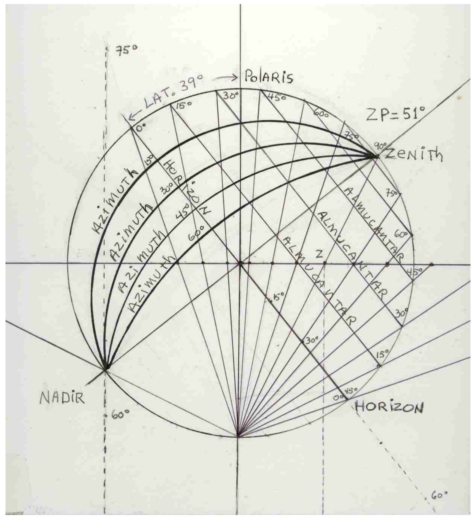

Figure 5. Stereographic Azimuths and Almucantars in the sphere.

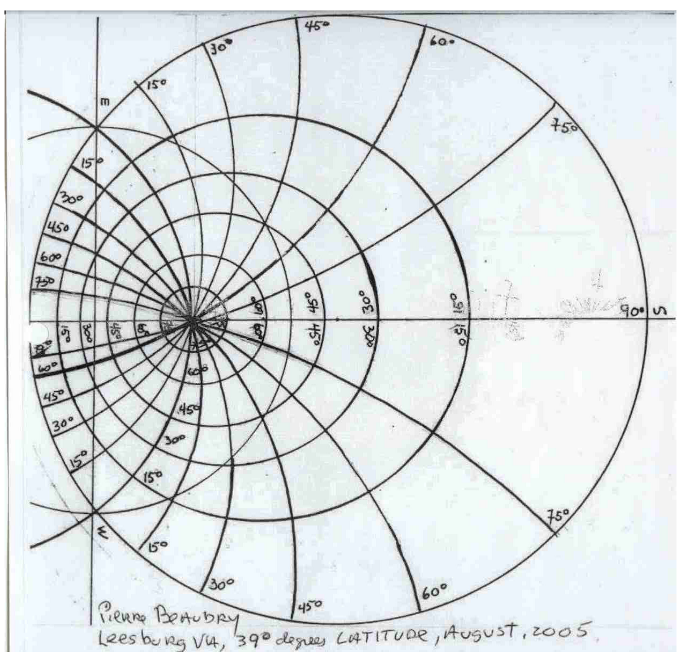

Figure 6. Stereographic Azimuths and Almucantars projected for Leesburg VA.

Such an incommensurable projection between the two domains of a Solid Loci and a Plane Loci may appear to be purely geometrical, yet the arachnidan web it projects reflects an axiomatic change of the physical reality between the sphere and the plane, as precisely as any astronavigator would require, in ancient times, to travel the oceans of the globe and find his direction to any location from any point on the planet. Both almucantar and azimuth circles exhibit different and well-proportioned transformations. Let's construct the almucantar first and the azimuth afterwards. Project, for example, the first almucantar from a zenith located on the equator, let's say from The City of Quito in Ecuador.

As shown in **Figure 7**, the key to the almucantar construction at this latitude is the transformation of the diameter of the spherical circle into a chord of a different plane circle. Note how the four points **ABCD** coincide with two different circles: **AB** is the diameter of the spherical circle and **CD** is the diameter of the plane circle. The two are the same circle, but they are not of the same size! This fascinating irony demonstrates the extraordinary harmonic proportionality between the sphere, the cone, and the plane!

Furthermore, all of the almucantars of that latitude projection will reflect the same anomaly when projected from the equator; and from nowhere else on earth. This is a very interesting singularity that reflects the internal boundary limit of the system. The significance of this Cusan anomaly lies in the fact that this is the only place on earth where the equatorial circle and the zenith-nadir circle coincide; because this is the only location where the projection of the horizon circle and the north-south axis of the terrestrial g lobe are represented by a same infinite straight line!

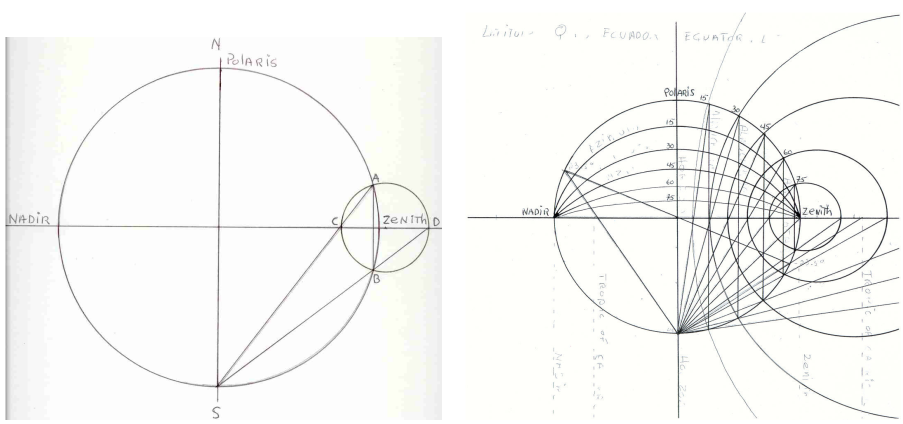

Figure 7. The anomaly of spherical almucantars transformed into plane almucantars at the equator of the celestial sphere.

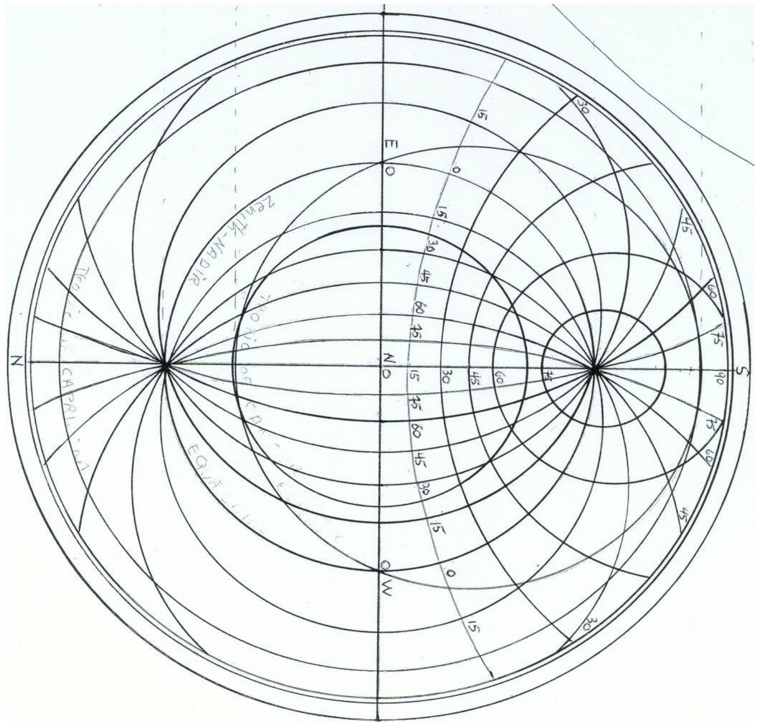

Figure 8. Projection of spherical azimuths and almucantars at Quito Ecuador.

7. THE {*PUNCTUM SALIENS*} OF THE ZENITH FUNCTION: A PEDAGOGICAL EXERCISE FOR YOUR MENTAL EYE.

There are two points to be stressed here with respect to Fermat and Apollonius. The first point is the greater generalization that Fermat brought to mathematics from the work of Apollonius and more specifically to an entire family of algebraic equations that were without any geometric determinations; that is, the indeterminate equations in two unknowns. What Fermat did was to determine the mathematical specificity for the Plane Loci and Solid Loci respectively. In other words, he showed how a geometric construction was the necessary basis for mathematical equations of the first and second degree. The second point to be stressed, which Fermat touched on but to my knowledge, did not develop, is the function of the {*punctum saliens*} in the projection between the two axiomatically different domains of the plane and the solid.

Apollonius implied that the optical focus of an ellipse is such a function when its region is treated as a caustic of light, that is, as the region of an {*envelope of evolute inversion*} reflecting the image of a burning point in your soul as in the image of God. As Lyn put it in his {*My May* 8^{th} *Declaration*}: "{*If man is in the image of the Creator, he is also in the image of the process of development within the universe as a whole*.}" It is the nature of this second point that I wish to emphasize here because it provides us with a useful insight into what happens at the limit of an axiomatic change between two dimensionalities, between two powers. As Schiller demonstrated historically and socially, a {*punctum saliens*} is a point of limit at which, when a system must continue its progress, it must either change or break down. As Lyn stressed, if the change is made, then the discovery of a new universal physical principle has been applied. The Zenith Point in the stereographic projection of Hipparchus represents such a principle of continuity function. It is a {*point de capiton*} holding the entire spherical fabric harmonically together through the axiomatic transformation between two dimensionalities.

Thus, to go straight to the {*punctum saliens*} in question, let us now examine the constantly proportional rotation of a tangent conic envelope around the conic section of an ellipsoid as designed heuristically by Apollonius in Book III of his {*Conics*}. Think of this process as a precursor to the inversion of tangent method of Leibniz.

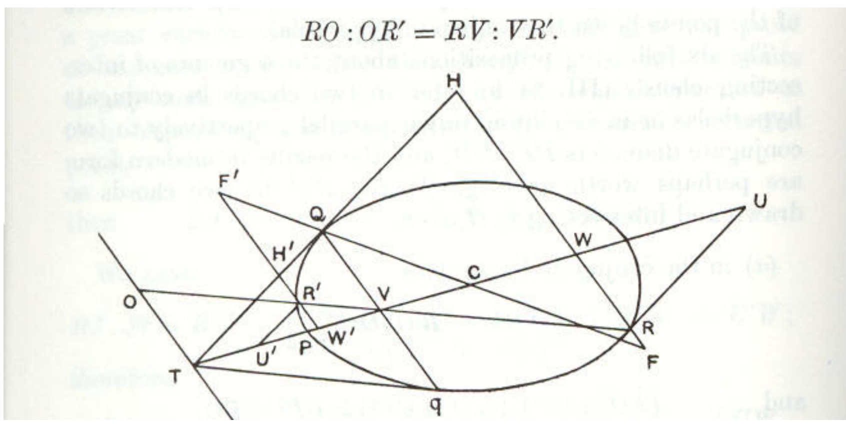

Figure 8. Apollonius harmonic properties of a conic envelope tangent to the conic section of an ellipsoid. (From Thomas Heath, {*A History of Greek Mathematics*}, Volume II, Dover Publication, 1981, p. 154.)

Consider **QR’PqR** to be any elliptic conic section and **OT** a straight line generated in the same plane and on which point **T** represents the apex of an enveloping cone that is tangent to an ellipsoid at points **Q** and **q**. If you move point **T** along the straight line **OT**, line **Qq,** changing its position proportionately, will always go through point **V** and the plane surface sections of the two solids will always pass through the same straight line **OT**. As a result, the straight line **OR’VR** will generate a harmonic range such that **RO : OR’ :: RV : VR’**. The proof of it can be made simply by conceiving a conic projection from point **O** enveloping the same ellipsoid. The two new tangents **T’Q’** and **T‘q’** shall touch the ellipse at two new points of tangency **Q’ and q’.** The new straight line **Q’q’** shall also pass through point **V**.

Gaspard Monge developed a similar theorem during his first classes at the Ecole Polytechnique, and in which the ellipse was a shadow conic section of the higher domain of a Solid Locus generated by the intersection of a cone and of an ellipsoid. Indeed, given the intersection of two solids in which a ellipsoid surface is enveloped by a conic surface circumscribing it, if the summit **T** of the conic surface moves anywhere on the perpendicular plane where the straight line **OT** lies, the rotating surface of the cone will generate a caustic {*envelope of evolute inversion*} in the region of **V**.

Lastly, consider the focus **V** of the Apollonius ellipsoid projection **(Figure 8.)** and the zenith function **Z** of the Hipparchus spherical projection (**Figure 6**.) as being two different but comparable species of {*punctum saliens*} determining an axiomatic change between a Solid Locus and a Plane Locus as per the Keplerian-Leibnizian principle of continuity. This alone might have caused a little revolution, in ancient times, if the political forces of the flat-earth oligarchy had not supported Euclid instead of Apollonius and Hipparchus.

As you may have concluded, it is useful to compare the two cases of Apollonius and Hipparchus, because the two ideas were created in opposition to Euclidean geometry at approximately the same historical period of time during the third and second centuries BC; that is, both of them rejected apriori axioms, postulates, and definitions in their geometrical constructions of {*Sphaerics*}. Furthermore, both of them represented non-visible mental regions of singularities, which organized a harmonic congruence between what is visible to your physical eyes and what is visible only to your mental eye. Both required subjective internal boundary conditions that are truly scientific expressions of discontinuities of universal physical principles through which everything changes and reflects the existence of a finite universe without external boundary.

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