A GEOMETRICAL METHOD TO INVESTIGATE THE FUTURE

(On the anti-Euclidean Geometry of Apollonius of Perga, Cusa, Kepler, Leibniz, Fermat, and Steiner.)

by Pierre Beaudry, March 11, 2013

"Don't nurse your discovery, go to the next one!"

Lyndon LaRouche

Figure 1 Snapshot of the Russian meteorite of February 15, 2013, by Marat Akhmetaleyev. A warning sign of the times: an asteroid exploding over Russia following the same principle that Kepler identified for the planet that exploded between Mars and Jupiter.

FOREWORD

This method of constructive geometry is based on the method of *inversion of tangents* that Leibniz used in his discovery of the catenary principle: *given the property of an anomaly that is known, discover the unknown principle that lies behind it.* This method of constructive geometry is teaching you how to discover a new principle by increasing the energy-flux density of your own mind. The report has five sections:

1. APOLLONIUS OF PERGA, CUSA, KEPLER, FERMAT, AND LEIBNIZ
2. A STEINER CONSTRUCTION BASED ON THE LEIBNIZ METHOD OF INVERSION
3. THE EPISTEMOLOGICAL SIGNIFICANCE OF CONICAL SPIRAL ACTION
4. THE CONSTRUCTION OF THE LOGARITHMIC SPIRAL MUSICAL SYSTEM
5- RABELAIS AND THE SINGULARITY OF THE PYTHAGOREAN TETRAD

INTRODUCTION

This paper may as well have been called ***The Panurge Axiom Busting Drama of the Pythagorean Tetrad***, as Rabelais demonstrated during Panurge's Lanternland visit, because the present state of the world breakdown crisis has reached a similar point of historical axiomatic change where reality turns into ***pure irony that which leads humanity to its proper end***; that is, in a condition where fiction becomes more real than what seems to be reality, when reality turns out to be totally fake. That's where we're at, as Lyn indicated with the case of Shakespeare dramas. A true drama is always a projection of the future and not of the past. But, the truthfulness of reality is taking place off-stage, because history is forcing the truth to be located exclusively in the imagination of the audience, and this is also where geometry meets its most needed challenge, when the question becomes: Is the intention of mind an expression of a principle that is to be experienced as a legitimate part of the universe? If so, how do you reach out to the real world by properly integrating your sense perception within your creative imagination? Lyn put the point as follows:

> "What are to be distinguished, as by merely raw popular opinion as what may be identified as the respectively separate subject-matters so-called fact and fancy, are now united under metaphor: two, nominally distinguished domains, are being as if a fusion of two domains of the imagination, the sensed versus the imagined, are now fused into the combined reality which is the actual experience of the combined powers of the human mind. Neither medium truly exists as a proper experience without the concurrence and conjunction of the other." (Lyndon LaRouche, A DOCTRINE CONCERNING MAN, LaRouchePAC, February 22, 2013)

In view of the necessity to reestablish the future mission of NASA and the need to develop new forms of technologies for the ***Defense of the Earth***, a constructive form of anti-Euclidean geometry is required if we are to understand the pathways that asteroids travel through from as far as the orbit of Neptune to the orbit of Mercury. We may not know what technological advances will be required to solve such complex problems, but we, at least, have been given the harmonic property of how and where to look for such a solution. This harmonic property is found in J. S. Bach's well-tempered musical system and in the projective property of constructive geometry developed by Jacob Steiner. I will use both methods, with the help of Leibniz, in order to construct the kind of constructive geometry that Lyn had in mind when he wrote our Music Manual more than twenty years ago. As Lyn said then:

> "The bel canto register shift is a physical event of fundamental importance, and not merely a technical question for the voice. In physical terms, the register shift constitutes a singularity, a non-linear phase change comparable to the transformation from ice to water or water to steam. An even better comparison is to the biological process of cell division (mitosis). In every case, we see that in C=256 tuning, the region of this singularity coincides with the principle *geometrical division* of conical spiral action. (Here we take the soprano voice, for musical and developmental reasons, as the fundamental reference for the human voice in general.)

> "Our Solar System also makes a 'register shift.' It has long been noted that the inner planets (Mercury, Venus, Earth, and Mars) all share such common features as relatively small size, solid silico-metallic surface, few moons, and no rings. The outer planets (Jupiter, Saturn,

Uranus, and Neptune) share a second, contrasting set of characteristics: large size, gaseous composition, many moons, and rings. The dividing-point between these two sharply contrasting "registers" is the asteroid belt, a ring-like system of tens of thousands of fragmentary bodies believed to have arisen from an exploded planet.

"It is easy to verify that the Solar System register shift falls exactly in the same geometric-mean position, as the shift of the soprano voice in the proper C=256 tuning. If we begin at the outer layer of the Sun, and construct a self-similar (logarithmic) spiral making exactly one rotation in passing from that layer to the orbit of the innermost planet, Mercury, then the continuation of that spiral will make exactly one full cycle in passing from Mercury to the region defined by the overlapping orbits of Neptune and Pluto…" (Lyndon LaRouche, A MANUAL ON THE RUDIMENTS OF TUNING AND REGISTRATION, Book I, Schiller Institute, 1992, p. 10)

We know that the unknown to be discovered inside of the Solar System has to have the property of the astronomical harmonic ordering of Kepler and the musical principle of artistic composition established by J. S. Bach, and that such harmonics must be replicated in a form of anti-Euclidean geometry of conics that goes at least as far back as Apollonius of Perga, Nicholas of Cusa, Johannes Kepler, Pierre de Fermat, Gottfried Leibniz, and Jacob Steiner. We know that the modern form of such constructive geometry had been established as a curriculum of study by Gaspar Monge, Lazare Carnot, and Jean Victor Poncelet of the French Ecole Polytechnique, whose pedagogical principles were later transmitted to Gottingen University in Germany, where it was further enriched by Freidrich Gauss and Bernhard Riemann. But, the more ancient forms can also be found in Pythagoras, Plato, and Archytas following in the footsteps of the ancient Egyptian builders of the Great Pyramid of Giza. Such is the great human tradition of constructive geometry that is required to be revived today.

For that stated purpose, I propose a crucial thought-experiment that will institute an anti-Euclidean principle based on a physical ***constructive measure of change*** that is essentially oriented, by virtue of its constructive geometrical characteristic, to investigate the future. Steiner's method is the best anti-Euclidean form of geometry in the sense that it is not deductive and does not depend on any ***a priori*** knowledge, because it is not based on sense perception, but on the collaboration between sense perception and imagination. In essence, Steiner's method is based on task orientation. (For all intents and purposes, the best elementary constructions of Jacob Steiner can be found in ***Geometrical Construction with a Ruler***, Scripta Mathematica, Yeshiva University, New York, 1950.)

Such a method displays, therefore, two fundamental epistemological conditions that Lyn has been emphasizing as necessary for any discovery of principle: one, it must be ***"lodged within the ability to forecast important aspects of an increasingly energy-dense pre-shaping of mankind's actual future,"*** which is exhibited by the nature of the human mind itself; and two, it must perform a unique form of human power, which is "***the ability to employ what is a distinctly human power of insight into an actual foreseeing, and thus the creating of the actual future***." (Lyndon H. LaRouche Jr., A GOOD OLD THOUGHT REVIVED, LaRouchePAC, February 3, 2013)

The problem that most people have is that they don't know where they are going, so they believe that randomness is the basis for their choice of action. Consider that projecting is foreseeing, but don't

forget that probability is not true projecting. Statistical probability is merely the way that oligarchies hold chaos together before they unleash it on the people when they can no longer handle it. And probability is made socially acceptable only because people have been made stupid and government leaders have abandoned their responsibility vis-à-vis the people.

As Lyn keeps stressing, human society is not random; it has direction. Just by observing how people react and by understanding what their intentions are, you can tell which way society is going, and you can see which direction it needs to be steered in order to make it progress. Steiner's constructive geometry functions in the same way. Simply by understanding the property of the projective means you are given, you can discover which direction to take and which problem to solve next, because that property has intentionality. So, what you need to master about the future is to discover the right orientation. You are looking for a definite direction to take, a definite task orientation to adopt, and a lot of patience to carry you through. Again, there is no probability involved, only a precise task orientated pathway in which the step you willfully take will give you the indications of where you must go next. But, first, let's look at how the precursors of Steiner used this method.

1. APOLLONIUS OF PERGA, CUSA, KEPLER, FERMAT, AND LEIBNIZ

As Apollonius and Fermat both explicitly noted, one typical problem to solve in constructive geometry is to correct the flatness of the past by projecting the solid characteristics of the future. In other words, if you know, for instance, the time-distance it takes for an asteroid to go from the orbit of Jupiter to the orbit of the Earth, you can also discover harmonic points between those two extremes wherefrom you can identify locations where asteroids would be located into mean positions, ahead of time, along that stereographic pathway, and therefore, forecast the harmonic positions of any object traveling along that complex least action pathway. This means that harmonics of conical spiral action can help you identify where things will be located within the changing process of a Solid Locus function. For an introductory approach to such an epistemologically unresolved question, see my report on PONCELET PARADOX OF THE VANISHING POINT.

As much as the understanding of the motive for the precession of the equinoxes requires the discovery of a higher hypothesis located above the heavenly sky, so does the motive for investigation of elliptic functions must come from the higher hypothesis of the ancients. As Lyn often suggested on the question of ancient astrophysics, otherwise known as *Sphaerics*, the epistemological crux of the matter is reflected in a form of solid geometry that has been explicitly designed in opposition to the sense perception platitudes of Euclid (c. 323-283 B.C.).

Johannes Kepler made a similar point in his *Paralipomenes à Vitellion*, in which he identified the necessity to establish a higher principle of continuity between the discontinuities of conics, as Leibniz later derived from his *principle of continuity*. What is to be considered, as it were, is not some a priori given object, but a preestablished ordering principle which is discoverable through the property of some discontinuous singularity which expresses causality from the future. As Kepler said in the spirit of Cusa, when you project your mind into the infinite unknown of the future, there is always a continuous series of

discontinuities that emerges and pushes the human mind into such least action pathways that increase their creative powers of discovery:

> "There exist different cones: the rectangular cones, the acutangular cones, and the obtusangular cones; similarly there are right cones, or regular and scalene cones, or irregular, or again flattened: on that account, see Apollonius and the commentaries of Eutocius. For all of these cones, without exception, there are five species of sections. In fact, the line formed at the surface of a cone by its section is either a straight line, either a circle, either a parabola, either a hyperbola, either an ellipse. Between those lines – and speaking more from analogy than from geometry, there is an ordering dependent on their properties, which is the passage from the straight line to the parabola by the intermediary of an infinite number of hyperbolas, then from there to the circle by the intermediary of an infinite number of ellipses. In reality, of all of the hyperbolas, the most open one is the straight line, while the most acute one is the parabola, and similarly, of all of the ellipses, the most acute one is the parabola and the most open one is the circle." (Kepler, *Les fondements de l'optique moderne : Paralipomènes à Vitellion (1604)*, traduction Catherine Chevalley, Paris, Vrin, 1980, Chapitre IV, p. 220-22.)

What Kepler proposes, here, is not what it seems. He is not giving us an itemization of things to look for in the study of conics, but rather, a method of how to look for their non-apparent intentions. This is probably the reason why he used the term "*Paralipomènes,*" which means "discarded things." What Kepler draws our attention to is to not forget the causal principle behind the conics, which is something that can only be discovered by turning the mind to certain limits, boundary conditions, and singularities of discarded things that Euclidean geometers never pay attention to, as a matter of course. Following in Kepler's footsteps, Leibniz did the same thing when, in his MEMORANDUM OF 1671, he advocated the search for expressions of the "Divine Proportion" that reveals how man is created in *Imago Dei*:

> "9. From this it follows inexorably that charity, the love of God above all, and true contrition, on which the assurance of blessedness depends, is nothing other than that love of the public good and of universal harmony; or rather, on that account, the glory of God and to understand are the same, and how great it is in itself to make greater, for there is no more distinction between universal harmony and the glory of God, than between body and shadow, person and picture, between a direct and a reflected ray of light, since the one is what is in fact, the other what is in the soul of him who knows it. For God creates rational creatures for no other reason but that they should serve as a mirror, in which His infinite harmony would be infinitely multiplied in some respects. From which must arise in due course the completed knowledge and love of God, in the beatific vision or the incomprehensible joy which the mirroring, and to a certain degree the concentrating of the infinite beauty in a small point in our souls, must bring with it. And thus, a burning mirror or burning glass is the natural image here." (Gottfried Wilhelm Leibniz, OUTLINE OF A MEMORANDUM: ON THE ESTABLISHMENT OF A SOCIETY IN GERMANY FOR THE PROMOTION OF THE ARTS AND THE SCIENCES (1671), in THE REAL POLITICAL ECONOMY OF THE AMERICAN REVOLUTION, edited by Nancy Spannaus, Second Edition, Washington D.C., 1996, EIR, p. 217.)

Once *Imago Dei* is understood as the divine proportion, as opposed to a golden mean section of visual straight lines, then the principle you are seeking is at hand. As for Apollonius of Perga (c. 262 –

190 BC), he was one of the first to launch a similar fight against Euclid, in Alexandria, by locating similar physical singularities inside of conics. In this regard, the work of Apollonius on conic sections greatly influenced Kepler, Fermat, Pascal, Leibniz, and Steiner not only on the geometrical approach to quadratures, rectification of curves, maxima and minima, and other singularities of tangents and chords, but also, most significantly, on the epistemological question of establishing an axiomatic difference between the *loci* of the plane domain and the *loci* of the solid domain.

The reason why Apollonius emphasized the distinction between "plane" and "solid" was not only because these geometrical figures were generated by a so-called "three dimensional cone," but most significantly because he related those figures to a higher dimensionality of the mind that required such an inversion of the past into the future, rather than persisting with the bankrupt platitudes of plane figures that Euclid related to from the past. Think of it in this way: equate sense perception with flatness and the imagination with the solid. The gestalt that unites the two is true knowledge as opposed to fictitious constructs. The question is to discover how to integrate the two into a secured manifold. If you wish to exercise your mind with a simple example, take six sticks of equal length and arrange them in a manner such that you can generate four self-similar equilateral triangles. That's the gestalt you wish to discover.

It was Apollonius who first raised this epistemological issue for solving constructive geometric problems in a now-lost manuscript on *Plane Loci*. In this report, Apollonius reportedly established a crucial difference between, on the one hand, straight lines and surfaces as belonging to the plane domain and as reflexive shadows of a higher solid domain. Similar distinctions had to be adduced from ellipses, parabolas, and hyperbolas as reflecting a higher generative solid domain discoverable through motion. That difference might first appear to be a very banal distinction for people who live in the Flatland of the past, but it is of the utmost significance for those who wish to live in the higher dimensionality of the future. The difference requires that one understands the Fermat principle of the least action pathway in all physical processes, understood from the top down as opposed to flatness. In his treatise on Apollonius, Fermat stated:

> "There is little doubt that the Ancients wrote many works on loci; witness Pappus who, in the beginning of the seventh book (of the Collection) asserts that Apollonius wrote on the Plane Loci and Aristaeus on the Solid Loci. But, unless we are mistaken, their investigation did not satisfy them sufficiently. This we gather from the fact that they did not express many loci in sufficient generality, as we will show below. […]

> "Whenever the local endpoint of the unknown quantity describes a straight line of a circle, a Plane Locus results; and when it describes a parabola, hyperbola, or ellipse, a Solid Locus results. If (it describes) other curves, the locus is called linear. We will say nothing concerning the latter, because knowledge of a linear locus is very easily derived, by means of reduction, from the investigation of plane and solid loci." (Pierre de Fermat, *Ad Locos et Solidos isagage*, (Introduction to Plane and Solid Loci), quoted by Michael Sean Mahoney**, *Pierre de Fermat 1601-1665*, Princeton Paperback, Second Edition, 1994, p. 91-92)

The point to be made, here, is not to simply identify the difference between a plane and a solid domain, but how you pass from one to the other. The point is not to construct ellipses, parabolas, hyperbolas and see their generative source in your mind, but to discover how these figures are generated

through singularities of a higher order from the mind. In his General preface of Book I on the *Conics*, Apollonius identified the shortcomings of Euclid with respect to this epistemological subject matter:

> "The third book contains many remarkable theorems useful for the synthesis of solid loci and for *diorismi* (determination); the most and prettiest of these theorems are new, and it was their discovery which made me aware that Euclid did not work out the synthesis of the locus with respect to three and four lines, but only a chance portion of it, and that not successfully; for it was not possible for the said synthesis to be completed without the aid of the additional theorems discovered by me." (Quoted by Thomas Heath, *A History of Greek Mathematics*, Volume II, Dover publication, 1981, p. 129.)

It was by means of making this same polemical-epistemological difference of curvature between a Plane Locus and a Solid Locus that Fermat was also able to disprove Galileo on the experimental question of a cannon ball falling from a tower. Contrary to Galileo, who thought the cannonball fell in a circular path because of gravity, Fermat demonstrated that the locus of the curve was a spiral caused by the doubly-connected motion of the earth; that is, of both its orbit and rotation. Similarly with Hipparchus, we must treat the stereographic projection of the sphere onto an equatorial circle not as a Plane Locus, but as the expression of a Solid Locus, that is, by adding a new and higher dimensionality that must also include the process of a principle of change coming from mind.

The point I wish to stress, here, is how to determine an epistemological Solid Locus, specifically. In other words, the method must involve specific functions of mind that also demonstrate the shortcomings of sense perception. Thus, the mind must see and account for what escapes sense perception. The case that Apollonius chose represents a constantly proportional rotation of a tangent conic envelope around a conic section, as he developed in Book III of his *Conics*. See with your mind's eye how Apollonius projected the shadows of that rotating Solid Locus in **Figure 2**.

Consider **QR'PqR** to be any conic section shadow of a rotating cone, and **OT** a straight line generated in the same plane as the figure, and on which point **T** represents the apex of an enveloping cone tangent to the ellipse at points **Q** and **q**. If you move point **T** along the straight line **OT**, all of the lines **Qq,** changing position proportionately, will go through the fixed point **V** and the plane of the two surfaces will always pass through the same straight line **OT**. As a result, the straight line **OR'VR** will generate a harmonic range such that **RO : OR' :: RV : VR'**. The proof of it can be made by drawing from point **O** two tangent lines to the ellipse. The straight line joining the two new points of tangency of the ellipse shall also pass through point **V**. Lastly, consider point **V** as the zenith point of a sphere. Similarly, all circles crossing through that point will be harmonically ordered in position, and in a fashion congruent with this harmonic conic envelope.
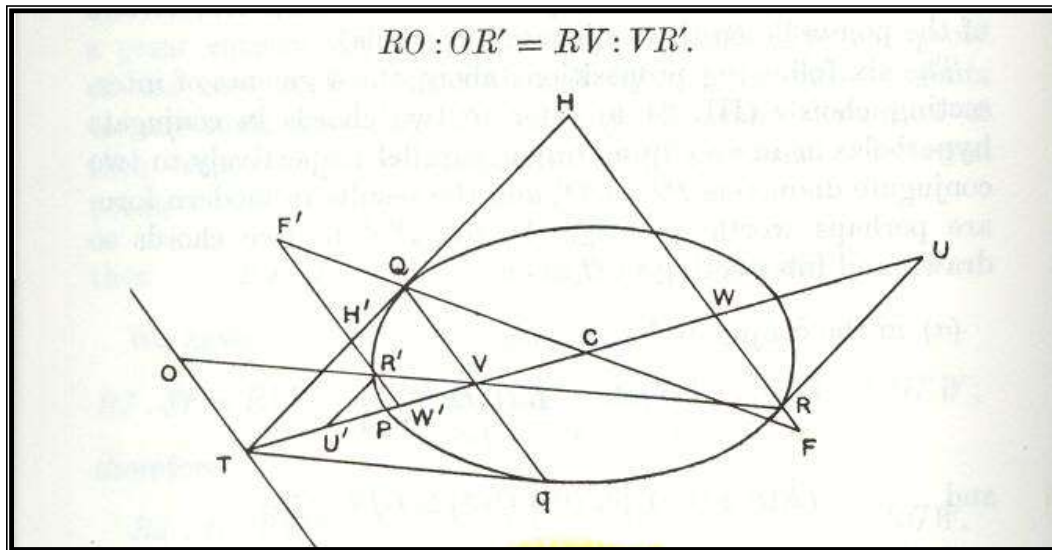
Figure 2 Apollonius harmonic properties of a conic envelope tangent to a conic section projected onto a plane. (Illustration from Thomas Heath, *A History of Greek Mathematics*, Volume II, Dover Publication, 1981, p. 154.)

Gaspard Monge had developed a similar theorem during his first series of classes at the Ecole Polytechnique. The theorem is only a particular case of a more general one that is generated from the Solid Locus of an ellipsoid. See the Monge construction in my report on THE GEOMETRY OF THE ONE AND THE MANY: THE METAPHOR OF PERSPECTIVE.

The problem that Apollonius solved, here, is similar to the one that Alexander Hamilton solved with the American Revolutionary War debt. *The solution lies in turning the past into the future.* That is how Hamilton turned debt into credit. That's the "General Welfare" principle of the American Constitution: you issue credit in a way that discards the past debt, which no longer worked or was illegitimate, and you bring in the future based on the anticipated work that will pay-off later. That's the gestalt. As Michael Kirsch demonstrated in the introduction of his Draft Legislation to Restore the Original Bank of the United States, the gestalt of the Hamiltonian idea of credit is in the increasing power of wealth for the future.

> "Under a strict system of law, where the payment of debts is rigorously enforced, credit, in its simple and primitive form, is preferable to money. The man who enjoys the one, has nearly an equal facility with him, who commands the other, in the purchase of merchandise, or materials for trade, or manufacture. But the stimulus to industry, or exertion, is very different in the two cases. The mechanic, who has a hundred dollars, can live without work so long as it lasts. He may spend the whole, or part, in his pleasures, or for his sustenance, and may work proportionally less. But the mechanic, who can command credit to the amount of a hundred dollars, has nearly the same capacity to earn money, as the other; but his privilege will not sustain him in idleness, or

dissipation. It can only be of use to him, through his skill and industry; unless he be so wicked as to become a swindler, and is permitted by the law to swindle with impunity.

[…] "Prone in common with all substantial and hereditary wealth, to subside into channels rather ample than numerous, the precious metals flow through a country in large streams, which carry out as much as they bring in, and contribute more to partial magnificence, than to general fertility: while credit, springing up in innumerable self-created rills, diffuses a fertilizing influence throughout every region." (Robert Hare*, **Proofs that Credit As Money In a Truly Free Country is to a Great Extent Preferable to Coin,** abstraction from a pamphlet written in 1810, Printed by John C. Clark, Philadelphia, 1834.p. 5 and 12)

2. A STEINER CONSTRUCTION BASED ON THE LEIBNIZ METHOD OF INVERSION

The implications of reviving Jacob Steiner's geometry work, in the spirit of the Apollonius and Fermat approach of a constructive form of solid geometry, is of the utmost importance if we are going to get rid of the impotent reductionist approaches of Euclid, Descartes, and Newton that have dominated science until today. The most effective method for accessing the future is based on the Leibniz principle of inversion which can be simply stated as follows: ***Given the property of something that is known, find the unknown***. Leibniz's art of inversion is based on a very simple hypothesis: ***the unknown of the future can always be discovered by means of the property of a constructive type of geometry.*** So, the question is: how can you construct from the future?

Leibniz's method of constructive geometry challenges you to discover the future by inversion, that is, by giving you something that is known, but only to make you discover that the unknown does not proceed directly from what is already known. You may object that you can only access the unknown from what you already know, but that is never true. There is nothing in the past which can generate the future, nothing in the known that can generate the unknown. In order to discover the unknown, you must find some inversion that will connect your mind to the principle of what needs to be discovered in the future, a sort of in-betweenness that is not an ***a priori***, but an actual bridge-principle. Let me give you an example taken from Charles Stevens report entitled, ***Toward the Determination of the Non-numeric Value of the Golden Section.***

I want to stress that one of the unknowns that Chuck was looking for was that of a Steiner construction for determining the center of the circle. What he did with his research on the Gauss ***Pentagramma Mirificum*** was not only to revive the constructive geometrical approach of Jacob Steiner, but he also, implicitly, revived the work of Apollonius and of Fermat on conics. I want to reemphasize, here, the point that he made with respect to the constructive method of Steiner, which is that the specific pedagogical case study of the construction of the quadrilateral is a typical constructive method for investigating the future. The method was adduced from the Leibniz hypothesis that would say: ***given the property of four points on the circumference of a circle, find the center of that circle.*** (**Figure 3**)
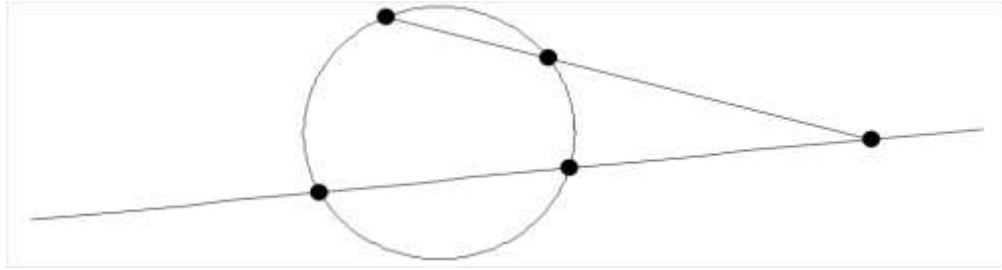
Figure 3. The construction of a quadrilateral from the circle without a center. (Illustration by Charles B. Stevens)

Although Chuck developed the construction of this centerless fixed circle much more extensively in his paper, I only want to point out, here, that this is a good example of how to discover the future from only a few shadows projected onto the plane from a Solid Locus. The point, here, is not to find the center of the circle with a straight edge alone, but rather, to discover from the future the center of the sphere to which this circular form of quadrilateral construction is centered.

First of all, there are two essential considerations to be adduced from this type of construction. One is that you must use a higher domain to solve a problem located in a lower domain. Two, you must seek the solution from the future and not from the past. In other words, since all problems relating to plane figures must be solved from the higher domain of solid figures, how do you proceed to discover the center of a sphere whose shadow coincides with the center of this circle? The answer can be found simply considering this circle as the equatorial circle of a sphere and fold the two sets of two points onto each other, two by two. Their invisible folds will form two mental diameters crossing each other at their mid-intersecting-point of the sphere. That is where the center of the circle lies.

As I stated above, a solution like this one cannot proceed from deductive investigation of those known points on a plane surface, and it doesn't need any mathematics. All you need is a creative imagination in congruence with sense perception. The center point can only be created by imagining a form of spherical action such as a folding action does with two chords of a plane circle. On the other hand, you can stare at those four circular points until you become blue in the face; they will never yield that solution from the plane, because the plane domain doesn't have, in itself, the power to solve this problem. What is known about these points is useless for what you are seeking to discover from the top down. On the other hand, what you have to discover is the relevant unknown geometrical property of what could bring those four points together into a single unity of effect; that is, into a gestalt of truth. Therefore, what you have to think about is something like what Leibniz called the ***property of the tangent***, or the ***property of the chord***. What is that property? The given property is to be at right angle to the center of curvature. That is the flank.

As in the case of military strategy, all higher manifold discoveries are deployed like a flanking maneuver at right angle, as Lyn exemplified with General Douglas Mac Arthur's Inchon landing in Korea. That is the in-between step, the blind spot of the enemy, so to speak, which is required to be adduced beyond the shadows of sense perception. When dealing with conics, both Kepler and Leibniz understood this strategy as ***the ordering which is dependent on the property of some discontinuous singularity pulling you from the future?*** So, you see, what you are looking for is not really the center of

the circle, as such, but the principle of the property of a function by means of which the unity of effect of a new principle can be discovered, as if from between the cracks, and through which the center of a sphere can be found. Don't focus on the center; focus on the principle of centering a flank.

Therefore, the geometric principle you are looking for is not something that you discover by some mechanical deduction, as from a Euclidean theorem. It is not as the reaction of something like the animalistic response to some provocation, either. It pertains to the creative imagination as opposed to what common sense would look for. It is not obvious, yet, it is not farfetched to the point that no one in his right mind would ever think of it. It is exactly the opposite. You are looking for something that is barely noticeable and that everyone takes for granted and never pays any attention to, because it is considered insignificant. And, that is precisely what is significant about such an inversion.

Let me put it in a different way. The formalist student will tend to grab the result of this geometric discovery as if it were the center of a circle that had to be found and nothing else. That is wrong. The inquisitive student, instead, will look for something else, because, during this whole process of seeking a solution, he will have realized that the question was not so much how to get a result, but, rather, how to discover the pathway of the intention behind the inversion of the problem; that is, the reason why the problem was posed in the first place.

So, while the former will become noisily elated by his stumbling onto things he did not know before, the latter will continue silently to seek the principle of the process that keeps calling his passion ahead from the future and pulls him toward the next step. That is the difference between looking for the result and looking for the intention behind the result. While the former will be tempted to look at things in the sunlight, the latter will continue to forage for the truth in the domain of shadows until the moment when the principle establishes itself in an inescapable definite manner. I don't know where, and I don't known when, but this is how you will discover the crucial significance of the Asteroid Belt.

3. THE EPISTEMOLOGICAL SIGNIFICANCE OF CONICAL SPIRAL ACTION

The Asteroid Belt is the greatest region of dissonance of the whole solar system, and it has to be treated, as such, not like a scientific object, but like in a classical musical composition. This is where classical artistic composition and astrophysics come together in a higher unified conception, because they proceed from the same intention, even though they may produce some very different effects. Poetry and physics don't clash, they work in the same way and they can be defined as belonging to the same domain of reality, when they are looked at from the future.

As Kepler adduced from the harmonics of the solar system, the Asteroid Belt is the region of an exploded planet through a complex arrangement that went beyond the Solid Locus, but he had no way of knowing what new ordering lay beyond that axiomatic passage of the first half of the solar system. Kepler did not know that he was passing from a lower set of small and hard surfaced inner planets to an outer set of the gaseous giants. However, Gauss did know that, when he discovered that Ceres was based on the Kepler axiomatic hypothesis. He knew that you could not go beyond this axiomatic region of the asteroid belt without making a complete axiomatic transformation of what you though you knew before.

The same process takes place in a bel canto voice register shift moving from the lower chest register to the higher head register. As this matter of mind was only partially demonstrated geometrically

in our Music Manual, I propose to illustrate the matter in a more completed manner below. I restate the point that Lyn made at the time:

"The harmonic orderings of the well-tempered system, centered upon the well-tempered values for the minor third, major third, fourth, arithmetic-geometric mean, geometric mean, and fifth, are identical to the correct values for astronomy, and are congruent with the Golden Section. Kepler was correct as far as he had progressed in detail; the modern Gauss-Riemann physics of the complex domain permits us to provide the corrections in method and values in a rigorous and conclusive way. (See *Figure C*.)

[…] "By these combined efforts we have mapped completely each of the thirteen half-tones, from C to C' in the well tempered system of twenty-four major and minor keys. The values so determined are those congruent with harmonic orderings based upon the Golden Section. Hence, this is the only musical arrangement which is coherent with the principle of life, and thus the only musical arrangement in which natural beauty is possible." (Lyndon LaRouche, A MANUAL ON THE RUDIMENTS OF TUNING AND REGISTRATION, Book I, Schiller Institute, 1992, p. xxi)
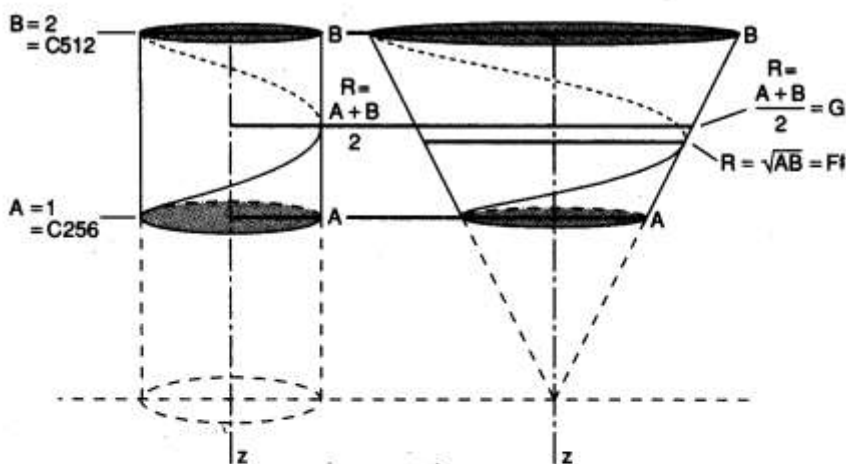

Figure C. Simple spiral action in the complex domain (left) is cylindrical in form; at one-half rotation (R), the distance moved along the axis z is one-half of the distance moved along z by a full "octave" rotation from A to B. One-half rotation therefore corresponds to the arithmetic mean (A+B)/2, or G. In self-similar spiral action (right), which characterizes the Gauss-Riemann complex domain, the distance moved by one-half rotation is the distinct geometric mean ($\sqrt{AB}$) of the total distance, corresponding to the movement from C to F-sharp.

The point here, however, is that the conical illustration of *Figure c* is missing a crucial point. It does not reflect the required unity of effect between the object of sense perception and the object of the imagination as Lyn warrants it. It demonstrates the difference between a cylinder and a cone, but it does not express the axiomatic difference between a lower and a higher mental domain, as does the relationship between sense perception and imagination. A change in the geometrical shape of an object of perception is not sufficient to prove anything. As a result, the conical spiral action in the cone in *Figure C* cannot project the spiral action properly onto the plane. The point to be stressed, geometrically speaking, is that the passage from the solid to the plane is missing. That axiomatic passage, as the dissonant passage of the human voice or between Mars and Jupiter should be a passage that marks, performatively, the fundamental difference between plane geometry and solid geometry, between the Plane Locus and the Solid Locus. That is the connection that is missing between the cylinder and the cone in *Figure C*.

As shown in the next **Figure 4,** for instance, the observer can see and hear with his mind the axiomatic difference the register shift makes by imagining the interaction of the three soprano voices without hearing them as sense perceptions, but the connection between perception and imagination is made. The boy on the right is singing in the lower chest register, the one in the center is moving his voice through the register shift range, and the one on the left is singing in the higher head register. The connecting passage between the lower domain and the higher domain is made.

As for the Solar System, the implication is that the region above the asteroid belt is the locus of a higher energy-flux density that we are not familiar with under the asteroid belt. That is the domain of the unknown. The discovery of what makes this higher form of existence of the Solar System, within the outer half of the "Gas Giants" is, therefore, a completely new outward challenge for mankind, but similar to the register shift of the human voice.

From the vantage point of what has just been said, the purpose of this report is to study how constructive geometry can become a tool of forecasting through anomalies and paradoxes of astronomy as

Kepler studied, mastered, and resolved in his own time. This implies that astronomy cannot be simply understood through approximate geometrical models, but through the critical anomalies that such least inadequate models represent for the benefit of our own approximate representation of reality. From that vantage point, astronomy is not a deductive science founded on geometry, because geometers will tend to force reality to correspond to their models and to their mathematics. That is how geometry can easily become a trap and a nasty source of sophistry.

Figure 4 The Register Shift Choir Stall of the Cathedral of Florence, by Luca della Robbia. (Illustration from the Schiller Institute, *Manual on Tuning and Registration*, 1992.)

For example, the three models of Copernicus, Ptolemy, and Brahe that Kepler used to discover the principle of gravitation were fallacies of composition. The only model that was not a fallacy was Kepler's own "elliptical model." Why was that the case? What made Kepler's geometry superior? It was not because the ellipse looked more like the real thing, and the circle did not. If that is what you think,

then, you have fallen into a sense perception trap. What makes you think that reality is like what you "see"? You cannot even see the pathway of a planet; so why would you assume that it is elliptical?

There has to be another reason, and it is because Kepler's model called for a higher accommodation between the sensed and the imagined. It called for the existence of a universal physical principle which fuses both into a unique experience that which can only be captured fugitively through the apertures of astronomical anomalies. That's what Lyn called Kepler's "vicarious" hypothesis. That hypothesis cannot be demonstrated by sense perception or by the imagination separately. As Lyn stated above: "Neither medium truly exists without the concurrence and conjunction of the other." (Lyndon LaRouche, A DOCTRINE CONCERNING MAN,) So, what unites sense perception and imagination is the anomalies of metaphor.

Therefore, if you don't include such anomalies of metaphor into the process, you have missed the boat. Kepler's model is superior precisely for the reason that it admits that it is the least inadequate constructive form of geometry that takes into account the ***anomaly of metaphor as an expression of the failure of sense perception***. Thus, Kepler showed that the most important part of the ***proof of fallacy,*** which is inherent to geometric models, lies in the fact that geometry tends to make you believe, falsely, that you have succeeded in explaining the reality of the physical phenomenon, as it is in itself, when, instead, it should show you that you have failed, and by how much. That's where the Leibniz method of inversion by means of a calculus comes in.

So, the question is not how do I succeed in making a better and more "look-alike" geometric model, but what causes the process of reality to go to higher levels of existence from a lower one? What sort of singularities is the universe throwing at us with the intention of generating such progress? These questions cannot be answered without going through the axiomatic discontinuities that a process of change must execute in its upward progression. This is what the following pages intend to demonstrate by geometrical construction.
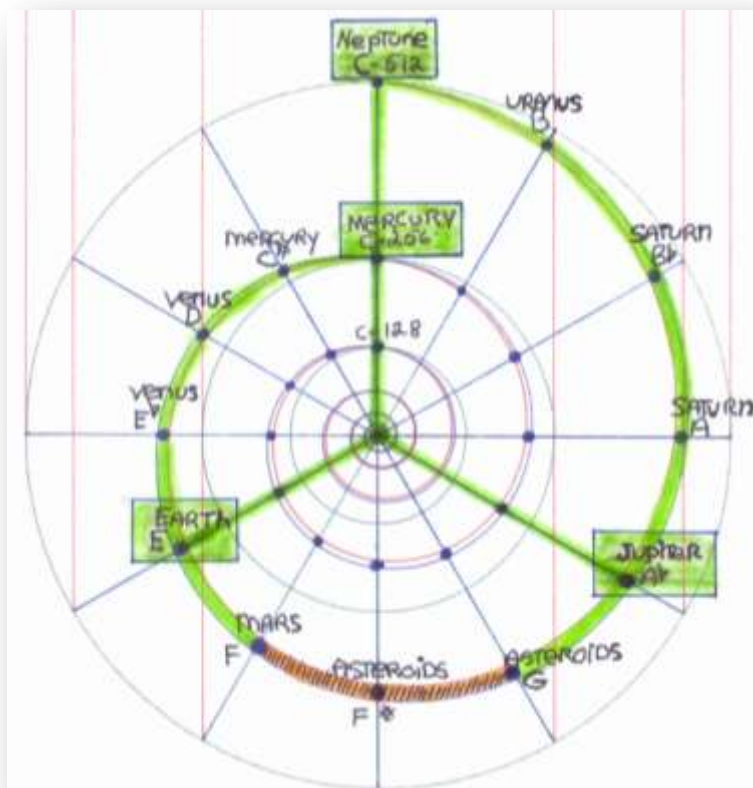

Figure 5 Plane projection of a conical spiral action of the Solar System and the Equal-Tempered System. (Illustration by Mark Fairchild and Pierre Beaudry, 1985.)

Take, for example, the logarithmic spiral action that Lyn called for in the introductory part of his Music Manual. (See **Figure 5**) How can this be constructed in a least inadequate form of geometry that reflects the Solar System? As the Earth and other planets rotate and orbit in a counter-clockwise manner around the Sun, as viewed from above the North Pole, so is the conical projection of this solar and musical ordering moving counterclockwise, as it proceeds from the Sun-apex outward, from C - 256 (Mercury) to C – 512 (Neptune).

THE PLANETARY ORBITS AND THE EQUAL-TEMPERED MUSICAL SYSTEM
by WILLIAM BOHDAN

PLANETS	ASTRO. UNITS	Log. 10x	ADDED CONSTANT	MULTIPLE CONSTANT	CYCLE EQUIVALENT	MUSICAL CYCLES	PLANETS
MERCURY	(P) 0.310	0.5086	+2.496	x 128.8	255.97	C = 256	MERCURY
MERCURY	(A) 0.470	0.3279	" "	" "	279.25	C#=271.22	MERCURY
VENUS	(P) 0.715	0.1457	" "	" "	302.72	D = 287.35	VENUS
VENUS	(A) 0.725	0.1397	" "	" "	303.49	Eb =304.44	VENUS
EARTH	(P) 0.983	0.0074	" "	" "	320.52		EARTH
EARTH	(A) 1.017	0.0073	" "	" "	322.42	E = 322.54	EARTH
MARS	(P) 1.379	0.1396	" "	" "	339.46	F = 341.72	MARS
MARS	(A) 1.661	0.2204	" "	" "	349.86		MARS
ASTEROIDS	(P) 2.2	0.3424	" "	" "	363.32	F#=362.04	ASTEROIDS
ASTEROIDS	(A) 3.6	0.5563	" "	" "	393.13	G = 383.57	ASTEROIDS
JUPITER	(P) 4.95	0.6946	" "	" "	410.95	Ab=406.37	JUPITER
JUPITER	(A) 5.45	0.7364	" "	" "	416.33		JUPITER
SATURN	(P) 9.006	0.9545	" "	" "	444.43	A = 430.54	SATURN
SATURN	(A) 10.074	1.0032	" "	" "	450.69	Bb=456.14	SATURN
URANUS	(P) 18.288	1.2622	" "	" "	484.05	B = 483.26	URANUS
URANUS	(A) 20.092	1.3030	" "	" "	489.31		URANUS
NEPTUNE	(P) 29.799	1.4742	" "	" "	511.36		NEPTUNE
NEPTUNE	(A) 30.341	1.4820	" "	" "	512.37	C = 512	NEPTUNE

Figure 6 Lyn's hypothesis of the Mercury-Neptune octave (green) and the sub-dominant-dominant register-shift range from F to G (orange). The logarithmic conical spiral action applied to both the solar system and the musical system is represented by a logarithmic correlation between astronomical units and the cycles of musical intervals. As the reader can easily recognize, the location of the planetary orbits in correlation with the musical intervals of an equal-tempered logarithmic spiral are close approximations of planetary cycle-equivalence. (Illustration by William Bohdan.)

Note that the angular interval between Jupiter and Earth reflects the partitioning of the double conical spiral action into a register shift range spanning from F (Mars), F# (Asteroids), to G (Asteroids). The partitioning of the whole system into three equal spiral intervals reflects the ARCHYTAS CONICAL FUNCTION for the doubling of the cube. In fact, the two Archytas proportional means for doubling the cube are E-Earth and A^b- Jupiter, which means that the register shift region from F to G corresponds to the Solid Plane interval of doubling the cube. Thus, the Archytas doubling of the cube represents the outer boundary conditions for the register shift range and should be recognized, as such, as defining the general pathway to a higher energy-flux density. This is how to define the axiomatic Solid Locus as being geometrically located within the arithmetic, geometric, and harmonic range of conical spiral action. (The

chart of the cycle-equivalence between the planetary system and the musical system was provided to me by courtesy of William Bohdan of Calgary Canada.)

It is, therefore, this least inappropriate type of geometrical construction which prompts the Kepler question: why is there an exploded planet between Mars and Jupiter? What is the underlying generative principle that causes the dispersed ordering of asteroids and where is that principle located? What is the nature of the intensity that causes a planet to not be solid enough to be like the lower inner planets and yet not gaseous enough to be part of the larger outer planets? As Lyn asked: "How does water go from liquid to vapor?"

Obviously, the first place to look for answers seems to be in the Sun, but, ironically, this is not where they are to be discovered. It should be obvious by now that the answers are to be found through the increase in energy-flux density of the human mind, because this is the best experimental place they can be found with certainty. If you look anywhere else, you will commit the dramatic error of omitting the function of the human mind, and you will fall flat on your face. So, it is in that sense that your best experimental laboratory is your own mind, by means of which the least inadequate geometry will demonstrate how it fails or succeeds by design, and not by mistake. That is the point that Kepler made with respect to Ptolemy, Copernicus, and Brahe: their models failed by mistake, not by design.

This is the reason why I have proposed the following constructive geometry; not as a representation of how the solar system works objectively, but how it works subjectively and by design, like the mind works when it is investigating singularities and anomalies in the least inadequate logarithmic form of spiral action and elliptic functions. The Kepler model is right for the same two reasons: 1) it is a correction of the previous models by showing their fallacies, and 2) it generates an increase of energy-flux density that the previous models could not by looking at the mind of the Sun.
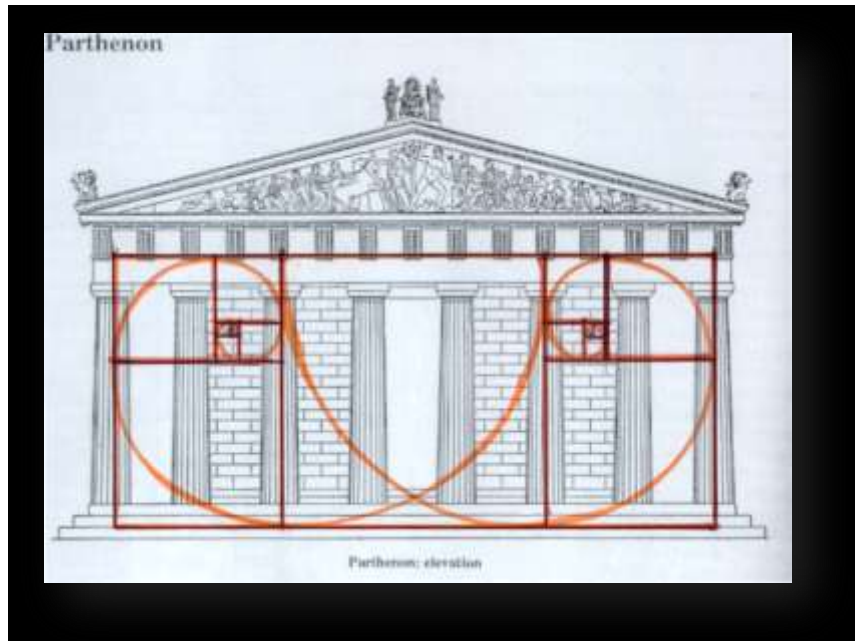

Figure 7 Parthenon demonstrating the chirality of the left and right handed nautilus half shells. (Dover Illustrated Dictionary of Historic Architecture, by Cyril M. Harris, 1977.)

4. THE CONSTRUCTION OF THE LOGARITHMIC SPIRAL MUSICAL SYSTEM

As Lyn demonstrated in A MANUAL ON THE RUDIMENTS OF TUNING AND REGISTRATION, the best experimental proof demonstrating the existence of increase in energy-flux density, as an expression of the principle of progress in the universe, is located in your own mind. And, the most effective demonstration of this performative discovery of principle is to actually experiment the bel canto register shift as a physical event that speaks to the mental increase in power of the human mind.

This is also an experiment which defines a complex conical spiral action based on the singularity of an axiomatic change in the middle of the Solar System. However, there exists no geometry that can express the reality of such an event, but only a less inadequate form of geometry which can point to it by construction. The point I wish to make, here, is that the form of complex spiral action has a deliberate outward and upward directionality which increases in energy-flux density as it progresses. The experiment is made as if something had been missing or mistaken in the human singing voice, before the register shift, and that a higher form of projection was needed to correct it, by passing through it. The process is such that a definite future is anticipated and moral conditions are set for the creation of a new principle that did not exist before, but which comes at the exclusion of the principle that existed before, because it is no longer valid.

Think of the range of change as being defined musically by means of what Lyn located as the axiomatic turbulences of the soprano singing voice range between the sub-dominant and the dominant above the key of Middle-C = 256. As Lyn put it:

"The well-tempered system of twenty-four combined major and minor keys is certified by the interaction of two sets of considerations. The first consideration is the defining of the scale itself; this is done from the standpoint of physics as such. The second consideration, is the fact that the properly trained singing voice, is moving upward from Middle-C must change singing voice register, in passing through F# from the sub-dominant interval of F, to the dominant interval of G.

"Since the natural points of passing from one register to another are essentially fixed in terms of the absolute values of the well-tempered scale, the human activity of music differs in two fundamental respects from an instrumental music not subordinated to human considerations.

"Most simply grasped, of these two distinctions, is the fact that every key has a distinctive "color," differing from that of each of the other keys.

"Take the well-tempered key of C, major or minor, from the G below middle C, through F above middle-C, we have a register we shall name as "B"-register. In other words, from the Lower G to the F above C, we have a constant register. On the F#, the seventh step of C-major, the voice registration becomes the relatively higher register, denoted here by "C"- register. This is true only for the case that Middle-C is equal to 256 cycles, or very nearly so. Thus the natural singing value of the F#, at the point of passage from F to G, defines a required value for Middle-C, as 256."

"The value of the F, as the subdominant interval, and G as the dominant, for the key of (Middle-C = 256), has a very precise musical significance, best understood from the standpoint of constructive geometry. For the moment, it is sufficient to indicate, that a key which divides the octave at subdominant to dominant, in this way, is the most natural of keys from the standpoint of physics and principles of classical composition. The congruence of the division of the octave by singing-voice register, with the passage from the sub-dominant to dominant is crucial for understanding the interconnectedness of singing with definitions of well-tempered scale. This interconnectedness is the ground-principle which distinguishes human music from the abstract music of such dead objects as musical instruments. This is what defines human music, the only real music, as situated within a doubly-connected manifold." (Lyndon LaRouche, TRUTH IS BEAUTY AND BEAUTY IS TRUTH: UNDERSTANDING THE SCIENCE OF MUSIC, 9/9/1986, unpublished report, p. 33-34)

The point that Lyn is making, here, is that you can demonstrate the truth of this axiomatic principle in two ways, one is by appropriate Bel Canto singing, in accordance with the well-tempered requirements as he just indicated, and two, by developing a least inadequate constructive geometry for the conical logarithmic spiral. Therefore, the process of this construction is like walking through the truth of a discovery of principle of going from the future to the past, as if from the solid dimensional domain to the plane dimensional domain.

The construction from the solid dimensional domain requires that you go through all of the steps from the sub-dominant to the dominant in order to complete the circle of fourths. As in a Bach fugue, this will require that you develop a precise number of cross voice interactions and that you learn how to adjust them within the whole of a complete conical function as it progresses. The whole composition cannot be constructed in any other way but by following the same process as that of a Bach fugue. And, it is only after you have completed this apparently impossible fugal task that you will realize how what had to be
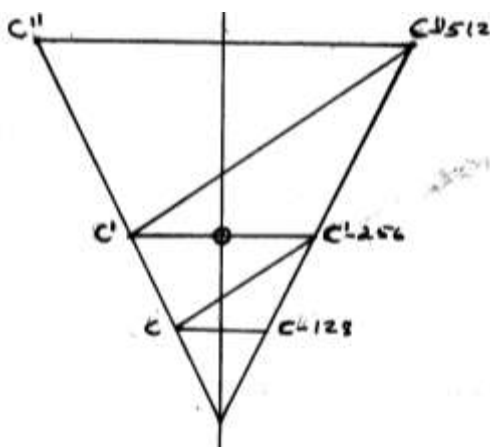

done was to complete a circle of composition that transformed you into a new person that is completely different than you were before you started. That's the method of composition of J.S. Bach, that's the method of the Shakespearian drama, that's the rigor that you have to master in order to be truthful in order to become both a scientist and an artistic human being. The following elementary steps will show you how to proceed in locating the intervals of the logarithmic spiral action partitioning of a cone with an elliptic function in accordance with the precise equal-tempered musical divisions of two octaves.

Figure 8.1

(**Figure 8.1**) The twelve half-tone intervals of the logarithmic equal-tempered octave can be constructed through a projection of the circle of fourths and a progression of Lydian divisions by means of a straight edge only. First, draw a cone whose height is equal to the width of its base and turn it upside down, on its apex. The motion up the scale is from the apex up the cone. Second, determine the range of two octaves

to be in proportion of 2/1, and draw two elliptic cuts C-C' and C'-C", joining the three circular cuts of the two octaves of [C-128- C-256], and [C-256- C-512].

　　　(**Figure 8.2**) Thirdly, draw two more circular cuts F and F' at the intersection points of the two initial elliptical cuts with the axis of the cone. Draw a third elliptical cut F-F' and generate a new circular cut B^b which intersects the axis of the cone. Fourthly, at this point, you have to discover something new that did not exist before. So far, I have shown you how to discover the fourth elliptical cut of C which is F. The question is: how do you find the fourth elliptical cut of F which is B^b?
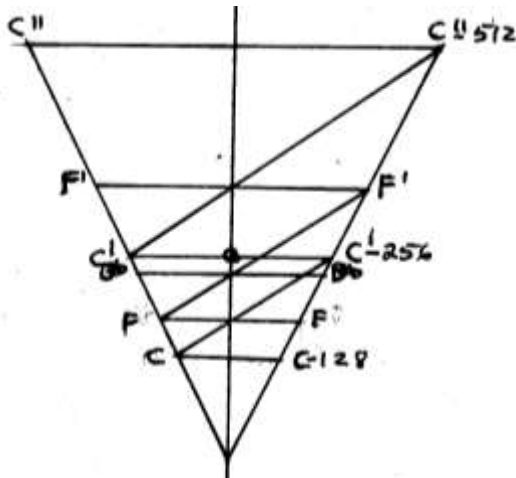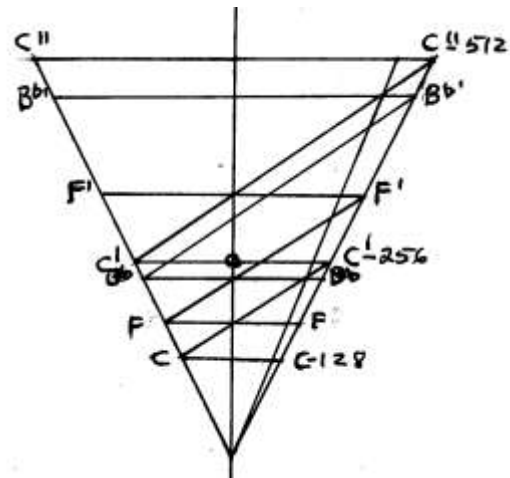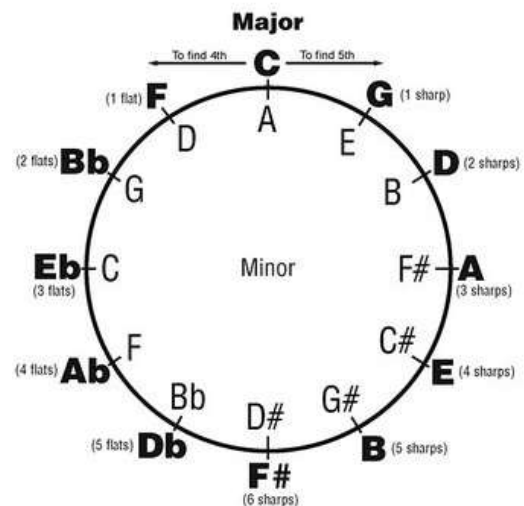

Figure 8.2

Figure 8.3

(**Figure 8.3**) Project a line from the apex through the intersection of the B^b circle and the elliptical cut C-C'. All the circular cuts of one octave will make you discover the circular cuts of the other octave. This is a new form of cross-voicing of which there are only four in the entire cycle of construction. By generating all of the circular and elliptical cuts between CC' and C'C", by means of those four conical radii, you will have generated the complete set of two octaves, from C-128 to C-512.

Figure 8.4

The Circle of 4ths and 5ths

(**Figure 8.4**) At the end, you will also have discovered that the three sets of Lydian divisions are also in their proper right angle cross positions inside of the circle: (C D# F# A), (C# E G B^b), (D F G# B). Thus, this construction also generates the circle of Lydians. The exercise requires a lot of patience and total military discipline.
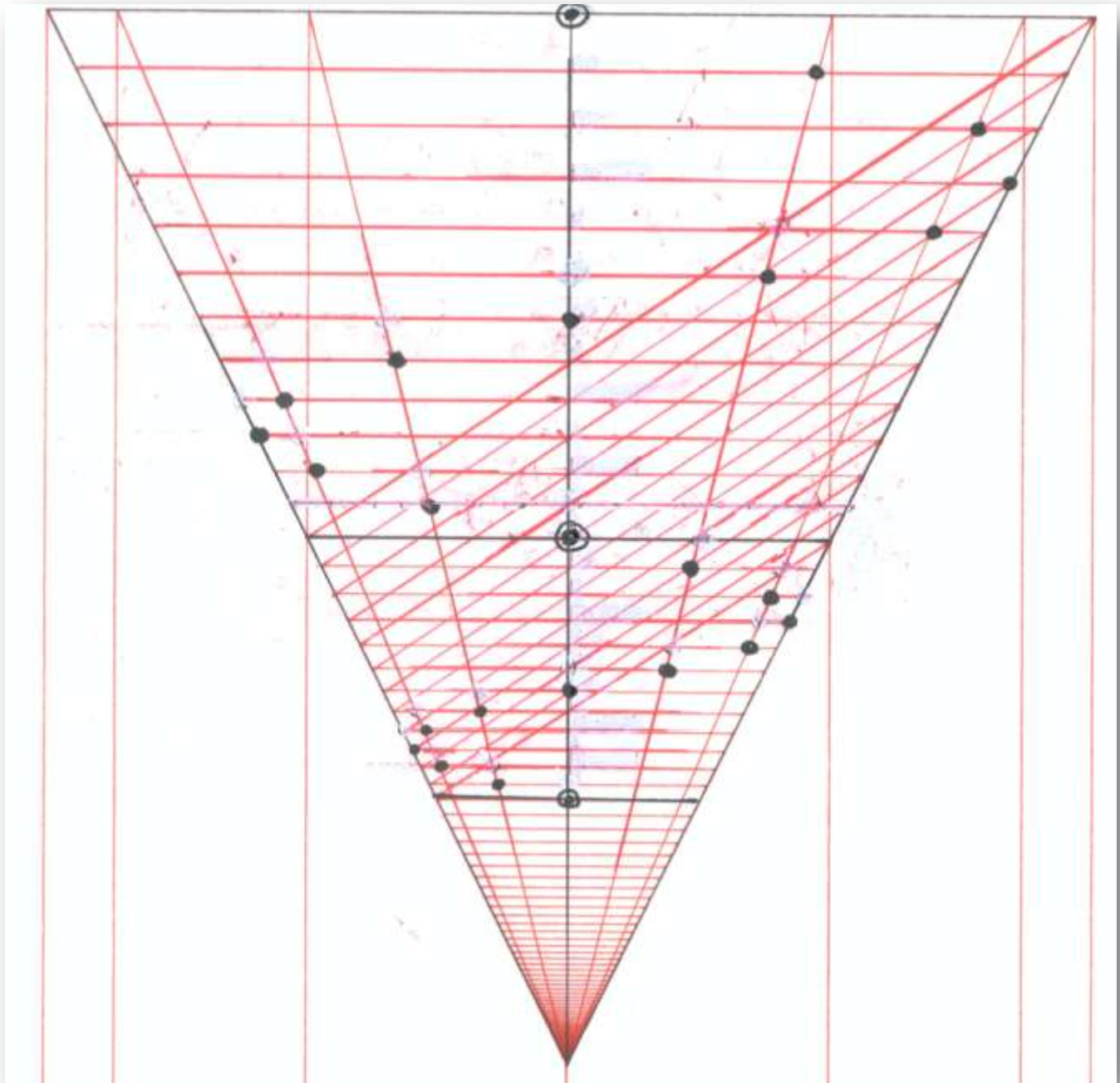
Figure 8.5 The complete logarithmic divisions of the equal-tempered cone. The black-dot intersections of all of the circular cuts with the apex-radii locate the position of the twelve half-notes of two octaves, thus marking the shadow location of the invisible logarithmic least-action pathway of the spiral. (Illustration by Mark Fairchild and Pierre Beaudry, 1985.)

Diatessaron ratio $4/3 = c/F$
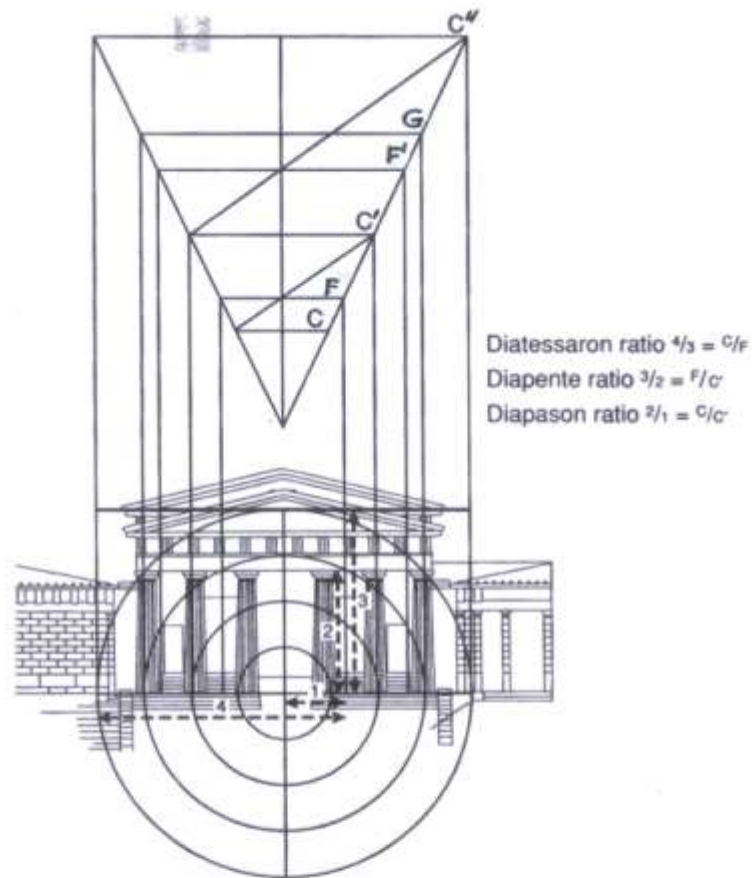Diapente ratio $3/2 = F/c'$
Diapason ratio $2/1 = c/c'$

Figure B.   The works of Iktinos and Mnesikles, the two leading architects of
Athens during the fifth century B.C., show mastery of a method of constructive
geometry based on a conical projection of a Golden Section ratio and self-
similar spiral action. This constructive method formed the basis for the design
of the Propylaia, the Parthenon, and the Erechtheion on Mount Acropolis. For
example, the west elevation of the Propylaia is a composition of mixing the
ratios of 2:1 (the octave), 3:2 (the fifth), 4:3 (the fourth), and the Golden
Section.

Figure 9 As Lyn demonstrated in the Music Manual, the sectioning of circular motion between F and G,
that is, between the dominant and the subdominant of C-256 of the well-tempered musical system, is the
location of the register shift for the soprano and tenor voices. As he put it: "The Athenians recognized that
beauty of form is associated with certain harmonically ordered constructions based upon the sectioning of
circular motion. In Plato's dialogues, it is emphasized that all beauty of form, including that of music, is
congruent with harmonic orderings cohering with the Golden Section of circular motion." (Lyndon
LaRouche, Manual on the Rudiments of Tuning and Registration, Volume I, Introduction and Human
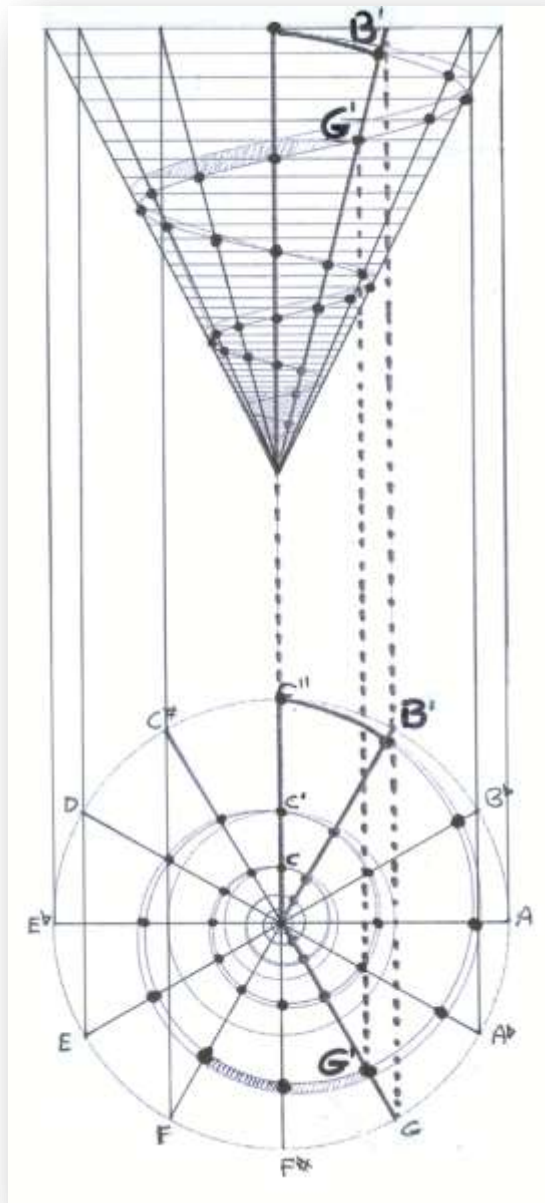Singing Voice, Schiller Institute, 1992, p. xx. Illustration by Pierre Beaudry.)

Figure 10 You are no able to map, orthographically, the shadow events from the higher Solid Locus to a lower Plane Locus in congruence with the least action pathway of the arithmetic-logarithmic spiral. The reason for the existence of these two spirals resides in the fact that all bel canto voice register-shifts, such as the axiomatic discontinuity of the Asteroid Belt between Mars and Jupiter, are defined by the singularity of an arithmetic-geometric mean. Note how all of the points of the spiral in the plane below can only find their "***raison d'être***" from the conical higher level. Those points have been entirely determined from the higher dimensionality of the conical spiral action. In other words, they were generated from a domain of higher energy-flux density. (Illustration by Mark Fairchild and Pierre Beaudry, 1885.)
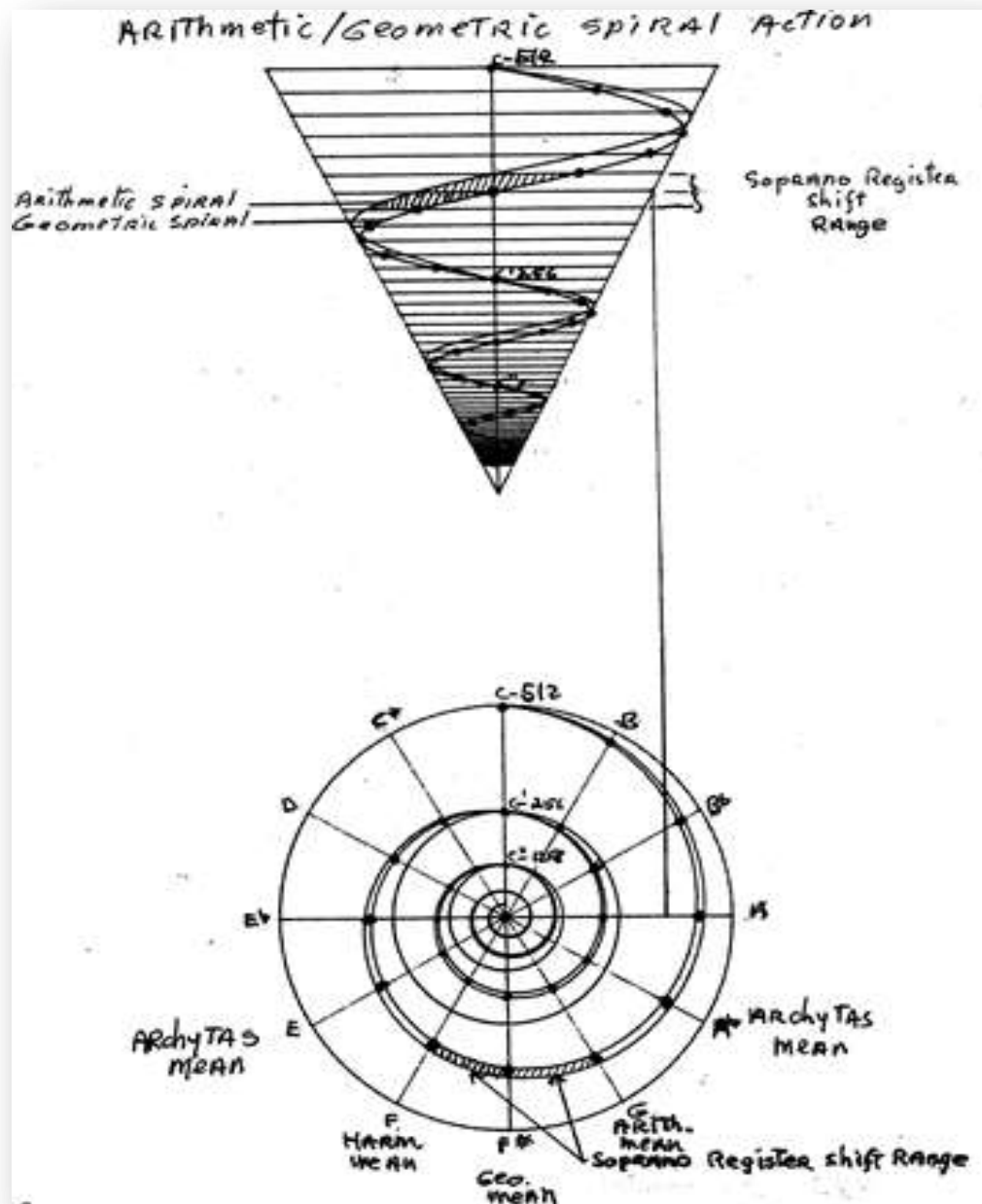
Figure 11 The complete range of two octaves of the equal-tempered system from C-128 to C-512, following a construction by the circle of fourths progression: C, F, Bᵇ, Eᵇ, Aᵇ, Dᵇ, F#, B, E, A, D, G. The circle of fourths, or of fifths, is a derivative of the arithmetic-geometric register shift which is the primary elliptic function of the singing human voice and of the Lydian division of the well-tempered system. The idea of the double arithmetical-logarithmic spiral was conceived as a reverberation of the voice register-shift affecting the entire range of an octave. In order to better understand the nature of the arithmetic-geometric mean as an irony of artistic composition that goes beyond mathematics, I reproduced, below, my 2007 report on **RABELAIS AND THE SINGULARITY OF THE PYTHAGOREAN TETRAD**. (Illustration by Mark Fairchild and Pierre Beaudry, 1985.)

5- RABELAIS AND THE SINGULARITY OF THE PYTHAGOREAN TETRAD, 4/17/2010
(Reproduced from THE AXIOMATIC SIGNIFICANCE OF MATHEMATICAL SINGULARITIES.)

       One of the best examples of how a truthful irony overflows mathematics from all sides is found in Francois Rabelais' story of Panurge conducting an experiment with the Arithmetic-Geometric Mean discontinuity of the Pythagorean Tetradic Steps, during his visit to the Temple of the Bottle in Lanternland. To establish the context of this story, let me start with the prophecy that Rabelais made to all visitors of Lanternland. He used the statement of the Greek Stoic philosopher, Cleanthes, who said: **"Destiny leads the willing, but the unwilling drags."** (*Les destinées meuvent celui qui consent, tirent celui qui refuse.*)

       This principle of the future is found in **Book Five,** the last book that Rabelais wrote about the last adventures of Pantagruel, Panurge, and Friar John, who had traveled to the Island of Lanternland, on their last expedition before returning home. When they arrived at their destination, the visitors were greeted by Midnight-Oilers who immediately started having philosophical discussions with them, especially on the subject of final causality; as they wished to know from their visitors if they knew that the nature of the future was to make **"all things move to their ends."** The habit of Midnight-Oilers is to stay up all night and feast on ideas of the future generated exclusively from their guide-lanterns which are modern forms of **Pythagorean Sphaerics.**

       I refer, most emphatically, to **Chapters 32 to 48** of that last book, because, of all of Rabelais' writings, it is in that last section of the **Fifth Book** that the axiom-busting method of Rabelais is best displayed. However, for our purpose, here, I will only refer you to **Chapter 36: Our Descent of the Tetradic Steps; and Panurge's fright.** This section is a pure delight of **Pythagorean Sphaerics.** Here, Rabelais brings the reader to make a fundamental discovery of universal physical principle by using Leibniz's **principle of continuity.** Consider that this discovery is of such significance and importance that it may one day save your life.

       In **Chapter 36,** Rabelais playfully restored to civilization the Pythagorean Tetradic principle of progress by means of an experiment that he conducted with a conical spiral action. He wrote the story with such tongue in cheek gusto that the reader cannot help but be provoked to investigate the seriousness and truthfulness of his **numberironies**. This is a joke that is not a joke. It is a fiction which is in fact more truthful that what appears to be reality. It is a funny story which has a deadly serious twist to it. So, as Lyn once put it: **"Believe nothing that for which you cannot give yourself a constructive proof."** Go work it out for yourself, and see what the Rabelaisian construction adds up to for you. Count the numbers and draw your own conclusion. You don't need a mathematical degree to do it. In fact, if you had such a degree, I know you would be incompetent to do it. Here is the relevant part of Panurge's crucial experiment.

       The idea that Rabelais is reconstructing, here, reflects the metric of change in the domain of ancient Egyptian and Greek **Sphaerics**. In the **Pythagorean Tetrad** construction, the point is 1, the line is 2, the surface is 3, and the solid is 4. These represent the four-dimensional world of Pythagoras, and the passing from one level to the next requires a non-linear leap, an epistemological jump, as in Leibniz's principle of continuity or as in Riemannian manifolds. This **Pythagorean Tetrad** represents the ancient

Greek Cosmos as a four degree expanding and axiomatically changing continuous manifold. Einstein would call it a finite and unbounded universe.

The numbers that Rabelais used may appear to be pure numerology, but don't be fooled, they are not. Those numbers are merely shadows that Rabelais is playing with, as a means of getting at the truth of his crucial thought-experiment which is that of an ontological axiomatic change. Here, again, you have to make the appropriate distinction between what is fact and what is fiction, because, here, the fiction is more real than you think. The ordering of these numbers is a metaphor expressing the different levels, or different changes in powers that the human mind is capable of discovering by getting to know the universe. Just walk through the Rabelais construction with me, and you will see what his intention was. Just pay attention to the intention and you won't get lost.

"Book Five, Chapter 36: *Our Descent of the Tetradic Steps; and Panurge's fright*.

Then we descended an underground marble staircase, and came to a landing. Turning to the left, we went down two other flights, and came to a similar landing. Then there were three more to the right, ending in a similar landing, and four to the left again.

'How many flights have you counted?' asked our splendid Lantern.
'One, two, three, and four' answered Pantagruel.
'How many is that?' she asked.
'Ten' answered Pantagruel. **[That is, 1+2+3+4 = 10, PB.]**
'Multiply this result by the same Pythagoreal Tetrad,' said she.
'That's ten, twenty, thirty, forty,' answered Pantagruel.
'How many does that all make?' she asked.
'A hundred, answered Pantagruel.
'Add the first cube,' she said, 'which is eight. **[That is, 10+20+30+40 = 100 + 8 = 108, PB.]** *At the end of that foreordained number of steps we shall find the Temple door. And note most carefully that this is the true psychogony of Plato, which was so highly praised by the Academicians, but so little understood. The half of it is made up of unity, of the first two plane numbers, two squares, and two cubes.* **[That is, 1+2+3+4+9+8+27 = 54, PB.]**

In descending these numbered stairs, underground we had good service from, firstly, our legs, for without them we could only have rolled down like barrels into a cellar; secondly, our illustrious Lantern, for we saw no other light as we descended, any more than we should have done in St. Patrick's hole in Ireland, or in the cavern of Trophonius in Boëtia. When we had gone down seventy-eight **[78, PB.]** *stairs, Panurge cried out to our most luminous Lantern:*

'Most wonderous lady, I beg of you with a contrite heart, let us turn back. For by God's truth, I am dying from sheer fright. I agree never to marry. You have taken great pains and trouble for me, and God will reward you for it in his great rewarding-place. I shan't be ungrateful either, when I get out of this Troglodyte's cave. Let's turn back, if you please. I'm very much afraid that this is Taenarus, which is the way down to hell. I think I can hear Cerberus barking. Listen, that's he, or I have a signing in my ear. I've no liking for him at all, for there's no toothache so bad as when a dog has got you by the leg. And if this is only Trophonius cave, the ghosts and goblins will eat us alive, as they

once devoured one of Demetrius's bodyguards, for lack of scraps. Are you there Friar John? I beg of you, old paunch; keep close to me, I'm dying of fear.' " (François Rabelais, *Gargantua and Pantagruel*, Penguin Books, 1955, p. 686.)

After having gone through this astonishing psycho-epistemological drama, investigate the different constructions for the three numbers that Rabelais generated. Why are they 108, 54, and 78? What is the significance of those numbers? Project their shadows on the wall of Plato's cave. Have a look at the Gauss Arithmetic-Geometric Mean function and see how it works. What is the significance of those three numbers with respect to the Gauss Arithmetic-Geometric mean? How do they relate to the crisis that Panurge has gone through? What is the significance of the geometric relationship to the epistemological crisis of Panurge as he is going down the spiral staircase? How is that related to music, or to the solar system? This experiment is very similar to the one that Benjamin Banneker had made when he related his mathematical puzzle of proportionality to the issue of slavery in a famous letter to the master of Monticello.

Figure 12 Why did Panurge freak-out as he went over the 78th step of the Pythagorean Tetrad? (Illustration by Pierre Beaudry, 2001.)

If you take the total number of steps in the spiral Tetradic staircase, the conical spiral function as a whole has 108 steps forming a musical octave starting from step 54 to 108. Then, there is the complex halfway step between them. The numberirony, here, is that Step 78 is both an arithmetic mean and a geometric mean step which represents the singularity of a threat that Panurge perceived as deadly when he was about to put his foot on it. Why? What is the threat? What danger does number 78 represent? I

have never seen a number threaten anyone before. Is this merely an imagined fear or is it a real existential fear?

This axiomatic crisis, here, is the excruciating moment of a high density of singularities that a political leader experiences at a crucial historical moment of decision in his life. This experiment is also, quite literally, what the arithmetic-geometric mean function represents at the asymmetrical halfway mark of a double spiral progression in a musical octave as well as in the solar system. This is where the voice breaks or goes to a higher register; this is where a planet actually exploded between Jupiter and Mars. It could not make up its mind about whether becoming a small and rocky planet like Mars or a large and gaseous planet like Jupiter. The point is that this is the locus of change, the locus of increase in energy-flux density that is required for the progress of the solar system. This is also the point of the greatest tension that opposes the arithmetical and the geometrical. The same principle applies to the human mind and to the economy, which Lyn also represented by a conical function in his *So You Wish to Learn all About Economics*. If you do the calculation yourself, you will find the Arithmetic-Geometric mean of that octave as being more precisely, 78.666. Rabelais did not include the 0.666 part for reasons that should be obvious. So, how do you calculate the Arithmetic-Geometric mean of the octave that Rabelais gave us between 54 and 108?

1) First, take the arithmetic mean of those two original values, which is:
$\frac{54 + 108}{2} = 81$. Then take the geometric mean of the same two values, which is the
square root of 54 x 108 = 76.3675...

2) Second, take the arithmetic mean of the last two values, which is:
$\frac{81 + 76.3675}{2} = 78.6837...$ Then take the geometric mean, which is the square root of
81 x 76.3675 = 78.6496...

3) Third and lastly, take the arithmetic mean again of the last two results: which are:
$\frac{78.6837 + 78.6496}{2} = 78.666...$ Then take the geometric mean, which is the square
root of 78.6837 x 78.6496 = 78.666... the Arithmetic-Geometric mean of that octave.
Simple isn't it!

Thus, you have arrived at an apparent limit of 78.666... after three iterations, which generate the delta volume of the Leibniz calculus, the singularity of the quantum of action of the Arithmetic-Geometric mean which, during the Renaissance, had been associated with the fearful *devil's interval of F#* during the Renaissance. This is why Panurge is freaking-out. This infinitesimal interval was used to scare the hell out of people during the Renaissance and made them politically impotent for fear of being burnt as witches at the stake for telling the truth. Now, what is interesting here with Rabelais is that he

used this as the creative singularity of an axiomatic change. He used the crisis as an opportunity, like the Chinese people do.

Here, Rabelais described how a creative moment is always fearful, because, at the point where one has to make a decisive step that changes one's entire life, one becomes totally perplexed and wants to run back to a ***comfort zone***, for fear of not being able to break through to the next higher degree of responsibility and freedom that history puts on one's shoulders. ***Then, appears the fear of fear itself.*** Therefore, the decision to undergo such a crucial experiment has universal implications and carries with it a heavy load of consequences.

That is the secret of Panurge passing the test of his moral commitment to change history. The passage is based on a single existential question: "Am I going to hammer my personality in order to become a world historical figure?" Such a decision is based purely on the will power of an individual who is capable of understanding the balance between reason and power in the process of changing the world for the better. Whatever happens when faced with such an unknown, the individual will emerge victorious from the ordeal, if there is balance between his capacity to understand and his capacity to undertake appropriate action in accordance with his talents. However, if the balance between understanding and action were to be off kilter, the individual may become, as Leibniz put it, in his MEMORANDUM OF 1671, either a tyrant or a slave. This is why this passage singularity represents the most important decision in someone's life.

However, such a decision is based solely on the willpower to change and to understand the consequences of one's action. The risk that is taken is that the discovery of the truth can only come after the deed is done and the fear of the unknown has been carried through the apparent discontinuity of the process, such as when an ellipse is transformed into a parabola. The question is always: are you willing to take that risk? Do you have the required morality to do it? As in the dynamics of Leibniz, **"*as the data are ordered, so the unknowns are ordered also*."** And that is how **"*all things move to their ends*."** This is what Panurge went through when he defied death and passed over the critical step of the F# register shift of his mind.

Now, listen to Panurge claiming his sublime victory over mortality, stating that he is ***willing*** to go on to the next battle as a new man: "***Let's go on, then***," said Panurge, "***and charge ahead foremost through all the devils. We can but perish, and that is soon done. I have always been preserving my life for some battle. Let's move, let's get moving, and let's press onward. I have enough courage and more. It's true that my heart is pounding. But that is from the chill and staleness of this cave. It's not fear, oh no, it's fever. Let's move on, let's pass on, push on, and piss on. My name is William the Fearless.***" (Francois Rabelais, ***Gargantua and Pantagruel***, Penguin Books, 1955, p. 686.)

The limit Rabelais described also reflects the singularity of the dissonance that is associated with the register shift interval in bel canto singing, that is to say, the passing "***wolf tone***" from the chest register to the head register of each and all six human voices. Such a limit is also the best pedagogical representation for Kepler's explanation as to why a planet must have exploded and disintegrated in the complex middle region of the solar system, between Mars and Jupiter, and whose debris resulted in the Asteroid Belt.

The question is: what do the Archytas means (E and A♭), the harmonic (F), geometric (F#), and arithmetic (G) means have to do with scientific progress and increases in energy-flux density? Ever since Plato and his *Timaeus* discovery of how the different means relate to classical artistic composition, the idea of truth and beauty has been understood as being at the center of the development of the human mind. These, and their combinations, are the most elementary functions of universal progress of the human mind. They are the most truthful and most universal means as Lyn demonstrated in his paper on the double-connectedness of the classical musical manifold quoted above. In other words, it is the singularity which defines the process of change as an axiomatic change of the process.

CONCLUSION

In conclusion, this beautiful *Pythagorean Tetrad* problem that Pythagoras had posed as an axiom buster to his students was also replicated by the 20-year-old Freidrich Gauss, more than 2,000 years later, when he discovered the pathway of the asteroid Ceres, the first proof of scientific evidence that Kepler's exploded planet was real and appropriately located. A similar idea can also be experimented by an inversion of the Arithmetic-Geometric Mean process. In fact, if you inverse the process of the A-G Mean, you can simulate the Cusa idea of transforming a circle into an infinite straight line. Be careful, however, because you will have created the surprising effect of a shock-wave explosion where, after only three iterations, the original ellipse becomes flattened into an infinite straight line. **Figure 13**.
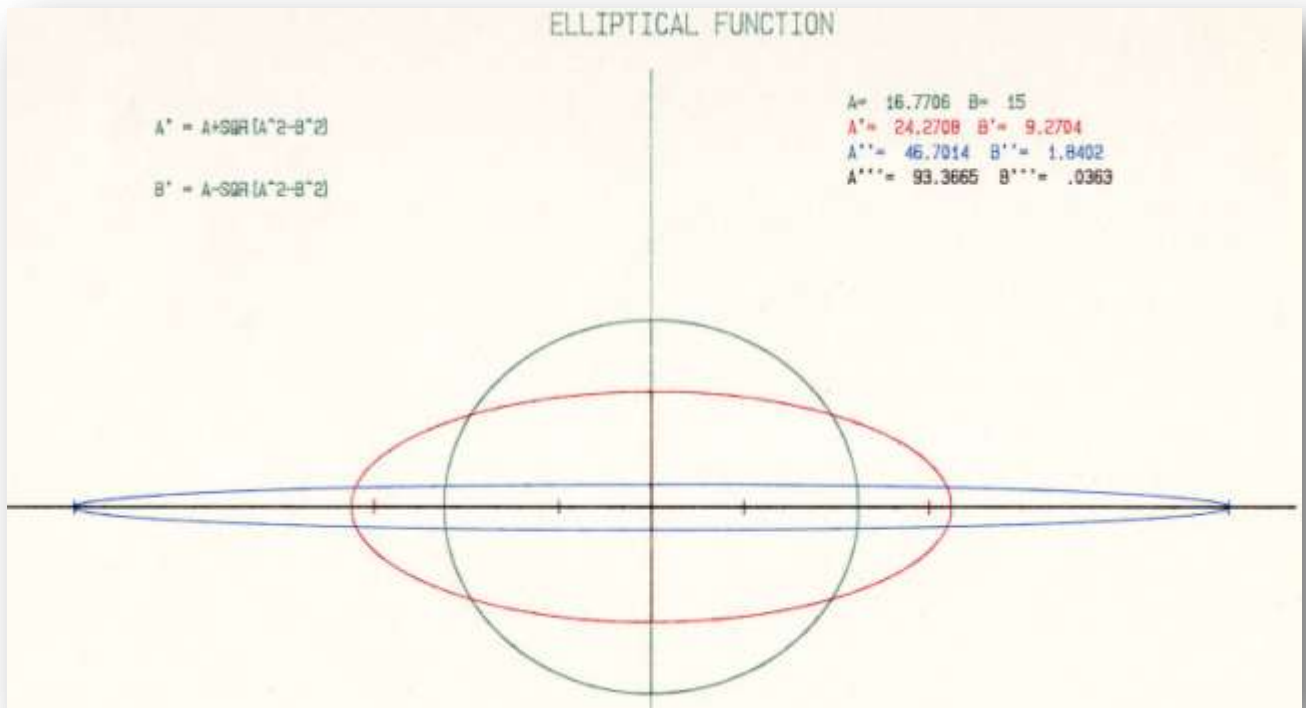

Figure 13 How an ellipse explodes into an infinite straight line by going through the singularity of an Arithmetic-Geometric Mean Inversion. (Illustration by Mark Fairchild and Pierre Beaudry, 1985.)

This singularity is also the less inadequate geometric representation of the "awful" responsibility that citizens have to assume during the present Constitutional crisis of our nation, because the United States has the responsibility for the rest of the world, and its citizens must also act accordingly. The boundary conditions of the general breakdown crisis are such that if the present Constitutional crisis is not resolved by a minority of willful individuals very soon, the system will break down completely, and at an accelerated rate, like an asteroid exploding into pieces. In fact, isn't it obvious that the U.S. House of Congress has already reached the state of an exploded asteroid?

Isn't that fact the most important reality that should spur you into action to save the Constitution of the United States in spite of the exploding Congress? The dramatic state of the world breakdown crisis, at this time, is entirely dependent on what the United States citizens will do, or not do, to shape the future of mankind. The good news is that we have a great advantage over our political enemy, because the beauty of this method of constructive geometry is such that if you are evil, or if you have a perverse nature, you will not understand it, and you will not be able to make the implied discovery of principle. This is the reason why the British Oligarchy can't win this war of nerves we are engaged in. On the other hand, if we were to let them win from a lack of our own willful creativity, then, everybody would lose. The question is: are you prepared for what the future Defense of the Earth is holding for you? Since it is your duty to avoid such a breakdown of human civilization in the near future, are you willing to help your fellow-citizens bust their axioms through the *Pythagorean Tetrad* as Panurge did for the betterment of mankind?

FIN