

## EGYPTIAN SPHAERICS AND THE ARCHYTAS DOUBLING OF THE CUBE.

[12. Bogotá LYM class: SPHAERICS. January 24, 2007.]

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### 1. EGYPTIAN SPHAERICS AND THE DODECAHEDRAL CALENDAR.

No matter how you cut it in Washington these days it is LaRouche who has the moral high ground, because he has the future. Whoever has the youth in the United States today, has control of tomorrow's politics; and the Congress knows that LaRouche has the youth vote and has the youth thinking and singing. So, take it from there. LaRouche represents the New America and the Congress will follow his New Politics and New Economics.

The reason I am saying this is because one of the very first passionate aspects of creative mental developments in the physical science of economics by ancient man was the Promethean astronavigator attempt at mastering the universe by establishing a calendar, an actual {*dodecahedral calendar*,} that is to say, by establishing a twelve month astrophysical pathway of the sun across the stars expressing the ordering universal physical principle underlying the recurring cycles of the Sun, from year to year, for the purpose of establishing transoceanic travel and also for expanding civilization throughout the planet. That is the same spirit of the LYM today. That is why we have the future.

This Promethean knowledge of ancient times was reflected in the first astrophysical science of shadow reckoning developed by astronavigators going back thousands of years before Christ, as reported, for example, by the origami writings in ancient caves discovered in Kentucky USA by students of Barry Fell, as also expressed by the Neolithic ball carvings of Scotland dated at the earliest 3,200 BC, and as demonstrated by the construction of the Great Pyramid of Egypt as an astronomical observatory.

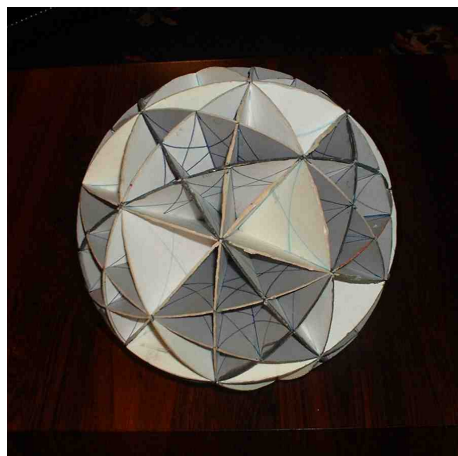
So, in this perspective, I would now like to show you how to construct a three step portable organizing model for the Archytas doubling of the cube, which comes out of that ancient Promethean knowledge and indicate to you how it was not only derived from the knowledge of such ancient Egyptian Sphaerics, but how it may also have been related to an early form of solar calendar leading to the discovery of the astrolabe by Hipparchus.

First, however, I would like to give you a brief historical account of my discovery of Egyptian Sphaerics and of the different steps that I went through, following the footsteps of Lyndon LaRouche. The idea of a {*dodecahedral calendar*} comes originally from Plato's {*Timaeus*} and relates both to the astronomy of the solar system, to Classical Artistic composition, and to music. I came upon this complex discovery when Lyn wrote his extraordinary paper {*On the Subject of METAPHOR*}, in the fall 1992 issue of Fidelio. I highly recommend that every single one of you master that paper in

which Lyn had provoked us to discover the single integral sphere that would generate all of the Five Platonic Solids.

The idea fascinated me and I was quite aware of the limitation of the principle of equal partitioning, that is, of 3 great circles partitioning each other into 4 equal parts, of 4 great circles partitioning each other into 6 equal parts, and of 6 great circles partitioning each other into 10 equal parts. Lyn had shown how these different forms of equal partitioning had established the boundary conditions for the Five Platonic Solids. So, I knew that this axiomatic boundary condition had to be changed if a single integral sphere was to be discovered, and that equal partitioning had to be replaced by some superior division. That was when I got the idea that Sphaerics divine proportionality might be the way to discover that higher domain. I was right in that hypothesis.

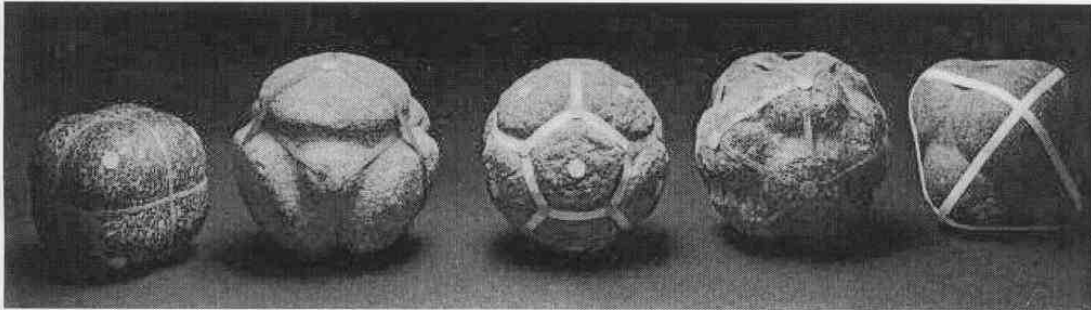
So, I started to study Sphaerics, first from the standpoint of constructive geometry at first, and I discovered that the unique sphere I was looking for had 16 great circles and each one was proportioning itself and the others by the spherical golden section. I sent a note to Lyn, in jail, and he replied that the 16-circle sphere I had constructed “{*pertained to 256.*}

” Those were his exact words, and I was totally excited. And so, he encouraged me to work some more on it, especially with respect to what he had termed the {*zenith function*}. So, I did and I dove into the question of proportionality and wrote an article on {*The Metaphor of Perspective*} for the Summer 1995, Fidelio.

**Figure 1. [16-Circle Sphere.]**

During the summer of 2001, I gave a 6-week class in Leesburg to our children aged 9 to 12, which gave me the idea to write a Geometry book for children that I called {*LANTERLAND, a Rabelaisian World of Platonic Discoveries*}. It was never published but it circulated among some of the first members of the LYM, and it was especially useful for the further development of angular construction of Sphaerics. That was the time when I discovered the existence of 5,000-year-old Scottish Neolithic Spheres which represented the earliest know attempts at treating the Five Platonic Solids from Sphaerics.

## NEOLITHIC PLATONIC SOLIDS



Archeologists have uncovered in Aberdeen shire, Orkney, Skye, and in other sites of Scotland, hundreds of carved stone balls dating back to the Neolithic period; that is, between 3200 to 2500 BC. Most of the carved rocks are generally the same size, each less than three inches in diameter, and can be easily carried by hand.

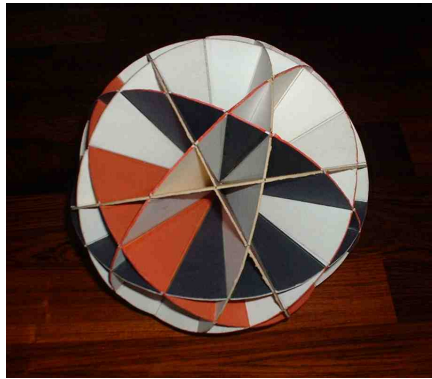
Note that some of the regular features of the stones reflect early forms of spherical Platonic solids.

1. The ball on the extreme left is divided into 8 equal sections forming a rounded CUBE.
2. The second ball from the left is divided into 4 knobs suggesting the shape of a spherical TETRAHEDRON.
3. The ball in the middle has 12 knobs, defining the oldest known DODECAHEDRON.
4. The second ball from the right has 14 knobs forming an incomplete ICOSAHEDRON.
5. The ball on the extreme right divides the spherical space into 8 equal parts, close packing 6 knobs forming an OCTAHEDRON.

**Figure 2. [Neolithic Spheres.]**

It was not until 2004 that I discovered the astrophysical function of that 16-circle sphere, that is, when I discovered that not only the sphere had been a Pythagorean sphere, but that it also contained the angular measurement of the Great Pyramid of Egypt. That was when I wrote my article *{Pythagorean Sphaerics: The Missing Link Between Egypt and Greece}*, Summer 2004. 21<sup>st</sup> Century. That was the crucial turning point.

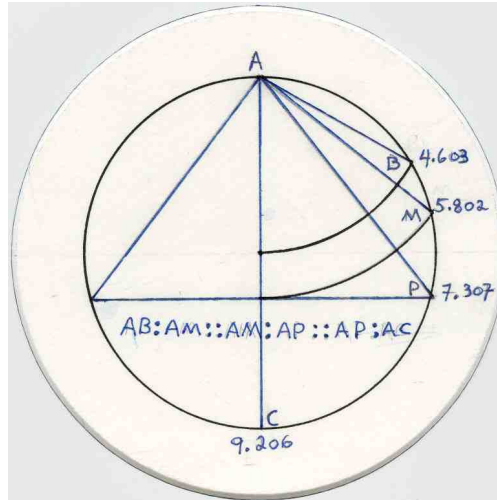
So, these were the main steps that made me discover that ancient Egyptian Sphaerics generated the Solar System intervals, the Bel Canto musical intervals, the Great Pyramid of Egypt, and the Five Platonic Solids. Those were a lot of things to pull together all out of one universal hat, and it all had to be proven by angular construction alone, without axioms, postulates and definitions; that is, ironically without methamagics. So I went to work and pulled all that together as a reflexion of Egyptian Sphaerics. Now, when I started looking at the Archytas doubling of the cube in 2004, I hypothesized that there had to be a similar connection between the Egyptian 10-circle sphere and the Archytas construction. Everything else was coming out of it. So, I said: “why not also the doubling of the cube.”



**Figure 3. [Egyptian 10-circle sphere.]**

However, I realized that it was impossible to discover this purely by intuition or by computer modeling. It had to be done by hand, by construction, and with hard work. The discovery had to be physically constructed in accordance with LaRouche’s precept that said: “{*Believe nothing that for which you cannot give, yourself, a constructive proof.*}

” So, here we are, I now have for you a constructive proof from Egyptian Sphaerics by means of which you can construct the famous Archytas problem of doubling the cube directly from the Great Pyramid. So, if you are ready, we shall begin the construction. Take the medallion of the Great Pyramid in **Figure 4.**



**Figure 4. [Medallion of the Great Pyramid for Doubling the Cube.]**

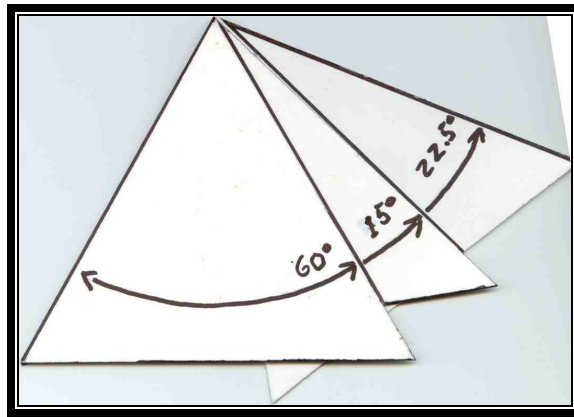
First, imagine the Great Pyramid of Egypt inscribed into a cylinder and project its shadow onto the circular base **AC** of that cylinder. Secondly, put your compass at the summit of the Pyramid at **A** and rotate the radius of the circle to become the secant **AB** of the cylinder circular base. Thirdly, keep your compass at **A** and rotate the height of the Pyramid to the position of a second secant **AM**. The third secant **AP** represents the apothem of the Pyramid. Behold, the two secants **AM** and **AP** are two mean proportionals between two extremes **AB** and **AC**, which are in a ratio of **2/1**. The proportion is such that **AB** is to **AM** as **AM** is to **AP** in the same proportion that **AP** is to **AC**, or:

$$AB : AM :: AM : AP :: AP : AC$$

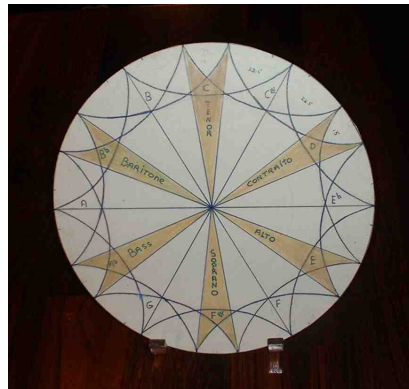
In other words, { *The height of the Great Pyramid is to its apothem as two mean proportionals are for the doubling of the cube.* } In other words, secant **AP** corresponds to the side of a cube whose area is double the area of the cube whose side is **AM**. This must be viewed not as a way to generate a solid from a surface, or the surface from a line, but rather a way to determine surfaces and lines from the higher domain of the Sphaerics solid. This is why you must first understand that the angular shadow of the Great Pyramid is originally derived from Sphaerics.

Now, for the construction principle of the Great Pyramid itself, you should go to the WLYM website to see my pedagogical on how it is derived from a 10-circle Dodecahedral Sphere. I will not have time to go through that with you in this series of classes, but I encourage you to do a physical construction of that sphere, on your own, for your Pedagogical Museum. If you wish to conceptualize the angular relationship between the Great Pyramid and this dodecahedral sphere simply compare **Figure 4** with **Figure 5**. The angular difference between the Pyramid Triangle and Equilateral Triangle is  $75 - 60 = 15$  degrees.

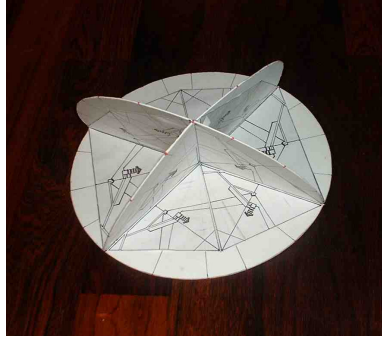
To summarize briefly the construction of that sphere, what you need to do is to generate 10 great circles, each of which must be partitioned by overlapping a six-part Great Pyramid triangle partitioning (75-degree angles) with a six-part equilateral triangle partitioning (60-degree angles); and thus, create an excess of 6 register shifts of 15 degrees each to establish the location of the twelve stars of the Egyptian Dodecahedral Sphere. The true angular measurements of the Great Pyramid Meridian Triangle are 75 degrees at the summit and 52.5 degrees at the base. Correspondingly, each great circle of the Egyptian Dodecahedral Sphere has to be partitioned precisely into six twin angles of 22.5 degrees separated by six single angles of 15 degrees each. See **Figures 5, 6, and 7.**



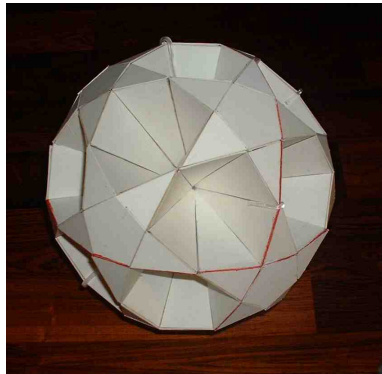
**Figure 4.5 [Egyptian Angle Ruler.]**



**Figure 5. [Great Circle Golden Section Partitions.]**



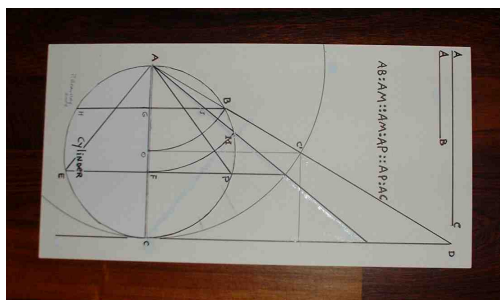
**Figure 6. [Great Pyramid Partitions.]**



**Figure 7. [Egyptian Starred Dodecahedron.]**

**2. HOW TO CONSTRUCT THE ARCHYTAS DOUBLING OF THE CUBE WITH A CYLINDER, A TORUS, AND A CONE.**

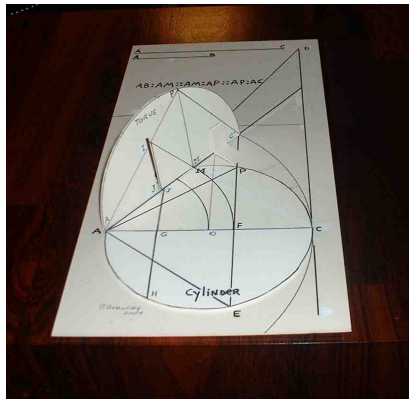
1. Draw the base of a Cylinder circle ABMPCEH (18 cm diameter) with the profile of the Great Pyramid APE inscribed into it and glue it on a rectangular white cardboard (45 cm X 23 cm) to form the cylindrical base for the Archytas Model.



**Figure 8. [Cylindrical Base for the Archytas Model.]**

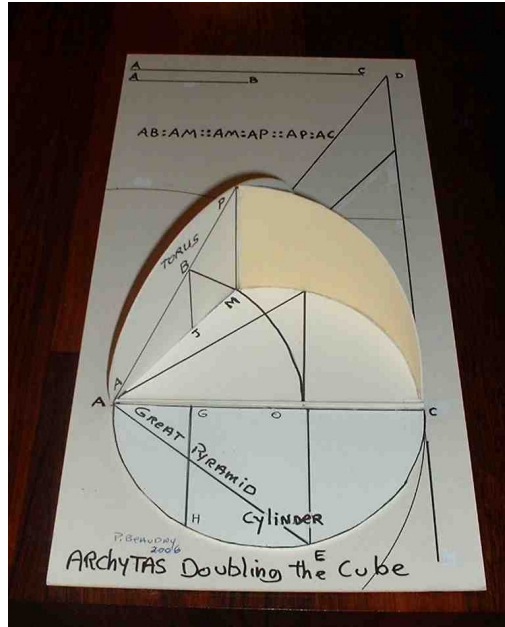


2. Draw onto the cardboard base, from points A and C a large scalene triangle ACD (18 cm x 36 cm x 31.176). Build an ACD cardboard scalene triangle with these precise measurements. This triangle represents the vertical cut of a right scalene Cone, which you can rotate around the fixed axis AC and which will intersect the Torus and the Cylinder at a unique unknown point.
  
3. Cut a half-circle APC' (18 cm diameter), which represents the rotation of a Torus around the fixed point A, and moving around and across the base of the Cylinder from point C to point M. Mark on the half-circle the angular profile (38-degrees) of the Great Pyramid, APF, as already identified on the base circle. The vertical line MP locates at P the singular point at which all three volumes, the Cylinder, the Torus, and the Cone must intersect and lock themselves together. Note in **Figure. 10** the multiply connected circular action converging onto point P. This reflects the axiomatic change between the two manifolds of doubling the square and doubling the cube.

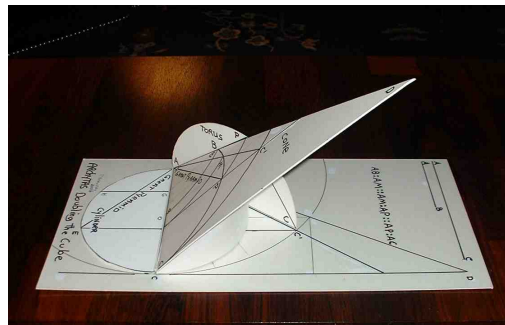


**Figure 9. [Torus half-circle.]**





**Figure 10. [Torus and Cylinder in locked position.]**



**Figure 11. [Intersection Between the Cylinder, the Torus, and the Cone.]**

This is the simplest three-step construction for the Archytas Model, which is also coherent with the Thales Theorem and the Mesolabe of Eratosthenes that I would like to introduce at this point in our discussion. But before going ahead, are there any questions?

[In response to a question on the “mean proportional”.] The best way to understand what mean proportionality signifies is to first avoid looking at them as mere straight lines. The idea of mean relates to the idea of growth. The straight lines of **Figure 4**, for example, are mean proportionals representing the sides of cubes, whose volumes double the areas of each other, and as such, reflect how cubes grow to be double of one another by some conical angular function. That is to say, when you wish to make something grow to become the double of its original size, the idea of mean is introduced to express the half way *{measure of change}* in that process. In that sense, a mean is always a *{measure of change}* by half. So think of the mean as the midway or halfway *{measure of change}* in some form of growing action in the physical universe. Take a physical spiral action,

for example. If one full unit of spiral action is measured as the motion of an octave around the surface of the cone, in a ratio of 2/1, the geometric mean is going to be the half way interval of action around the surface of that cone. On the other hand, the arithmetic mean should be considered as the midway section taken from the height of the cone between the two same octaves. So, from that standpoint, arithmetic and geometric means are not flatland expressions of the mechanical Euclidian plane, but reflect a dynamic *{measure of change}* within the stereographic domain of Sphaerics. From that standpoint, a mean is also the non-entropic measure of a process, which has reached a well-ordered growth point from which the action to be completed can be pre-determined.

Now, from the standpoint of principles, it is the discrepancy between those two means that is interesting, because it identifies a geometric discontinuity, a sort of margin of error that exists between the two and which is never zero. It is that margin of error that can be brought to the smallest possible infinitesimal, which indicates the shortcoming and the failure of a geometric determination of a dynamic process that Leibniz had expressed as the delta of his catenary function, and that Gauss had later called the arithmetic-geometric mean. From the standpoint of Archytas, the arithmetic-geometric mean idea is implicitly built-into his construction and should be looked at, strictly speaking, as Lyn had already proposed earlier, that is, from the standpoint of how Eratosthenes rediscovered Archytas from the vantage point of a conic function that he also expressed in his Mesolabe. This also points to a higher concept of proportionality, which cannot be measured mathematically, but can be understood from the standpoint of a Riemannian tensor.

For example, we know that there is an incommensurable proportionality between the different geometrical domains of the line, the surface and the solid. Similarly, there is a higher form of proportionality, which acts as a bridge between incommensurable magnitudes such as the Abiotic, the Biotic, the Cognitive, and the Divine. For instance, we can relate to the divine proportion of the universe as a whole, as the fourth domain that Lyn referenced, by considering that the Abiotic is to the Biotic as the Biotic is to the Cognitive in the same proportion that the Cognitive is to the Divine. We know that the universe as a whole is well ordered in such a non-entropic manner and with some special cases of interpenetration between the different domains. So, it is this higher domain that determines the lower ones, and it is therefore from that level that the notion of mean proportional must be ultimately understood.

### **3. THE CONSTRUCTION OF AN ANCIENT SOLAR CALENDAR.**

The triple intersection between a Cylinder, a Torus, and a Cone does not represent these perceived objects as such, but the idea of a self-developing triply connected motion which is related to the Egyptian Sphaerics idea of establishing a first science of astrophysics. From that vantage point, my hypothesis is the following: The construction of the Great Pyramid, itself, as derived from the Twelve Starred Dodecahedral Sphere, had not only been built with the intention of creating an astronomical observatory, but also with the intention of establishing a *{dodecahedral solar calendar}* that was to

reflect the normalization of the twelve months of the year, the twelve intervals of the solar system as related to the twelve intervals of the musical system, and the twelve divisions of the Equinoxes between night and day. These are the main features that are represented by the common angular measurements of the Egyptian Starred Dodecahedron and the Archytas Model for doubling the cube as in **Figure 7**.

Construct the following measuring device, an Egyptian Angular Ruler, just to relive how much fun the Egyptians must have had in constructing their Great Pyramid from Sphaerics. Glue on top of each other two self-similar Equilateral triangles and a Great Pyramid triangle whose apothem is the length of the equilateral side of the two other triangles. Lock them together in such a way that the Pyramid triangle lays between the other two and such that the whole angle measures  $60 + 15 + 22.5 = 97.5$  degrees. If you put them together in accordance with this Egyptian Angular Ruler, all of the required angular measures will be found in the Archytas Model for doubling the cube, that is: 75, 60, 22.5, 15, and 10 degrees. These are also the elementary angular measurements for the Egyptian Dodecahedral Sphere! Thus, the Egyptian Angular Ruler may be used for both constructions. This brings out an important point with respect to British empiricism.

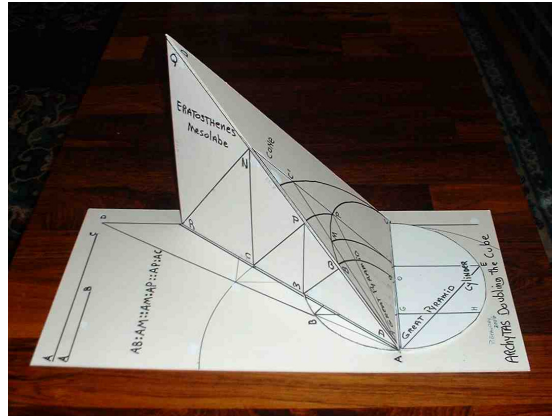
It is notorious that British Pyramidiots are fanatics about the measurements of the Great Pyramid made originally by Flinders Petrie and Piazzi Smyth at the end of the 19<sup>th</sup> century, and consequently, they have been perpetuating the fallacy that the only way to come up with the right measurements of the Pyramid is to be physically on location, and measure the nearly 5,000 year old remains of those stones. This might appear to be perfectly sound British education, and I am sure it is completely convincing to a pragmatist of any nation. However, it is wrong, because the only way to know the true angular measurements of the Great Pyramid is through Sphaerics. And if you don't know Sphaerics, you don't know what the pyramid builders were all about and what they were really doing.

It is only through the power of cognitive closure of constructing the 10-circle Egyptian Sphere that the precise plan of the Great Pyramid can be made to reflect the true angular measurement of the monument itself, after 5,000 years of deterioration, and not in the dusty plane of Giza. Otherwise, you end up believing the recent fallacies of the current Cambridge luminary, John Romer, who made the claim that all the pyramid builders were interested in doing was to fit stone on top of stone perfectly, like dead bodies on top of dead bodies, in accordance with the perfection of British mathemagics. The Egyptian Starred Dodecahedron proves the point, and so does the Archytas doubling of the cube. The point being that the British claim for the angle of the base stone of the Pyramid to be 51.50' degrees is false by the standard measurements of Sphaerics. It does not compute because in order to fit the sphere, the apex of the Great pyramid must be 75 degrees and not 76 degrees as the British claim. So much for the disease of British empiricism.

Finally, the triply connected circular motion that locks the Cylinder, the Torus, and the Cone together at point P in the Archytas Model, shows a number of crucial discoveries whose traces have been covered up by the sands of time. I will recall two of

them at this point. First the Connection between the Archytas Model and the Eratosthenes Mesolabe whose purpose was to discover a series of multiple mean proportionals. Secondly, I will show you how the Egyptians may have derived the non-visible ecliptic pathway of the sun during a single year from a projection of the Archytas Model.

#### 4. THE ERATHOSTHENES MESOLABE



**Figure 12. The Eratosthenes Mesolabe.]**

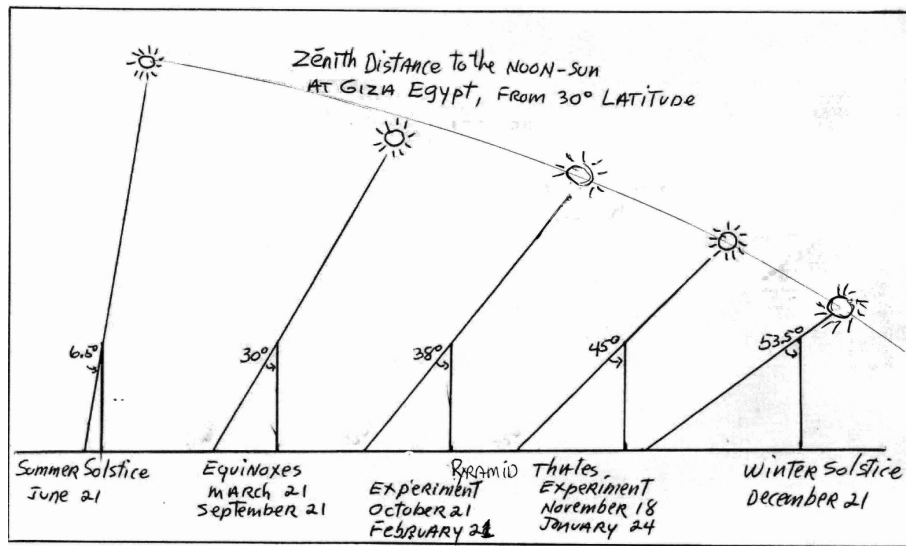
The Mesolabe of Eratosthenes had the purpose of illustrating how to construct an instrument, which established any number of mean proportionals such that they reflected the Great Pyramid Triangle, the Thales Theorem, and the Archytas doubling of the cube. We have already shown how the Great Pyramid and the Archytas construction are related to each other. I will now show you, briefly, how the same construction also relates to the Thales Theorem.

In accordance with the Thales Theorem, triangle **NCA** of **Figure 12** gives **NC : PM :: PA : NP**. Following the same principle of simple proportionality, **AB : AP :: AM : AC**. The same also applies for triangle **QRA** where **QR : NC :: QP : PA**.

With these Thales proportionalities, you can then derive the following series of multiple mean proportionals: That is, the quadruple mean proportionals **BM : PM :: PM : PC :: PC : NC :: NC : NR :: NR : QR**. These multiple mean proportionals represent the same Archytas double mean proportionals whereby the series of cubic doubles are: **BM=1, PM=2, PC=4, NC=8, NR=16, QR=32**. This **256** series also doubles the cube because it reflects the case where the height of the Great Pyramid is to its apothem as two mean proportionals are between two extremes in a ratio of 2/1. Now, let's investigate the solar calendar. These different approaches from Thales to Archytas to Eratosthenes represent the same dynamic required to make axiomatic changes from the line to the surface to the solid. This was the universal expression of Sphaerics that the Greeks inherited from the Egyptians.

## 5. THE LOCK PRINCIPLE OF THE ANCIENT ECLIPTIC SOLAR CALENDAR.

So, from the standpoint of Sphaerics, you can see that the intention of the ancient builders of the Great Pyramid was not simply to pile stones upon stones with extraordinary precision, but also to establish a science of astrophysics, and in their query, they were trying to discover universal physical principles. In other words, the doubly connected circular motion of the Archytas Model represented for them an attempt at locking into place an ancient solar calendar. How did that work? How can you discover such a *{lock principle}* just by studying the shadows of the sun? Well, let's start with the solar shadows at the location of Giza.



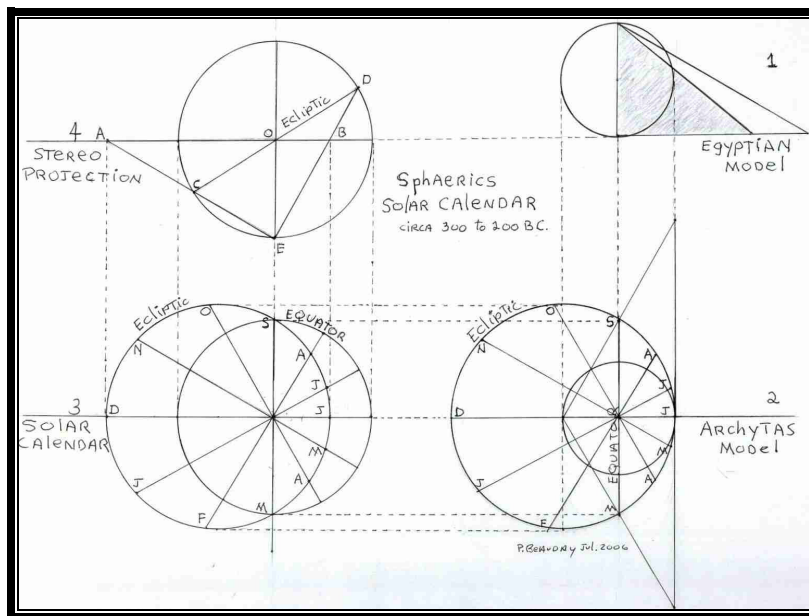
**Figure 13. [Solar Shadows in Egypt at Latitude of 30 degrees.]**

The crucial discovery about a solar calendar is the relationship between one's Zenith location and the North Star. So, with the knowledge of this doubly fixed relationship, you cannot get lost, because these two fixed points gives you a bearing on a number of stars in the heavens which return back in relatively the same position during a period of about a lifetime, that is about 72 years, when the Precession of the Equinoxes moves about one degree in its great cycle of 25,920 years. It is not known when exactly this relationship was discovered and who discovered it, but it is estimated that the early astronavigators knew about it, and established their traveling patterns from it. Whoever discovered it, however, made a beautiful Promethean discovery, because it gave man, for the first time in history, the mental power of knowing where he was located, exactly, anywhere on the planet, and was able to obtain angular measurements from any other place, with respect to his location, and enabled him to travel to and from any location he wished.

The location of Giza in Egypt is very special from that vantage point, because it is at latitude of 30 degrees, which means that the Zenith distance of the Great Pyramid from the Pole Star is the angle of an equilateral triangle, that is, 60 degrees. This astrophysical

location was very conducive to discovering the Platonic Solids and developing the kind of Sphaerics that we are presently investigating. For example, it was more natural for the Egyptians than the Greeks to discover the Starred Dodecahedral Sphere and its derivative Five Regular Solids.

But, lets look at the angular positions of the noonday sun during the year at the location of Giza and see if we can derive something from it that will lead us to discover an idea for a **{locking principle}** for the solar calendar. See **Figure 13**. Bear in mind that the complete cycle of the ecliptic is not visible, and that the pathway that the sun travels through during an entire year is but a visual illusion. Even the ancients knew that when they looked at the motion of the sun, they had an understanding that this was a reflection of the Earth's motion. So, they could imagine themselves located on the surface of the sun and look at the motion of the Earth against the sphere of the universe. I have shown how that would work in my first Pythagoras pedagogical. [ ] They were quite aware that their sense perception was lying to them. So, as Carnot put it later, **{their purpose was to generate ideas by means of the senses, of acting on the soul by the organ of vision.}** What does that **Figure 13** tell you?



**Figure 14. [The Archytas Model projected as an Ecliptic Calendar.]**

Next, take **Figure 14** and relate all of the shadow projections from one to the next, in the ordered sequence of 1,2,3,4. This is an elementary form of projection from a continuous manifold to a discrete manifold. You have to think of these projections as stereographic and orthographic; that is to say, as a solid proportional projection and parallel right angle projection at the same time. These types of projections can be traced back to between 200 and 300 BC to the works of Hipparchus and Apollonius.

**STEP ONE:**

Study closely the Archytas Model of **Figure 14** and you will find that it can easily be transformed into a Solar Calendar. The Egyptian Model for doubling the cube, which we have constructed together, a few months ago, can be rotated to project directly onto the Archytas Model for doubling the cube as if it were viewed from above. Then, change that projection of the Archytas Model into the projection of a Solar Calendar. This involves two important transformations. One is the transformation of the Torus Circle into an Ecliptic Circle divided into twelve months. Note how the summer months are shorter than the winter months in order to give an approximation of the changes in the length of the days during the relevant period. The second is the transformation of the Cone Circle into a Celestial Equatorial Circle divided into two pairs of six months from the Equinox Point of September (S) to the Equinox point of March (M). The most important feature of this double transformation is the locking in of the four points of the Solstices and the Equinoxes into a solar calendar. Think of the two intersections of the circles as corresponding to the precise moments when the noonday sun casts a shadow of 30 degrees on the plane of Giza.

Next, project the Archytas Model 2, into the Solar Calendar 3. The coupling of the Ecliptic Circle with that of a Celestial Equatorial Circle, as shown in the Solar Calendar 3, provides the key elements for the construction of an Astrolabe. The next step requires normalization with an imaginary celestial sphere.

## **STEP TWO:**

Project the Solar Calendar 3 from a Stereographic Celestial Sphere. This is the most exciting part of the construction of a Celestial Calendar. From the angular determination of 30 degrees at Giza, all it took was a correction to 23.5 degrees for the ecliptic and establish an accurate base for an astrolabe. This involved the ability to solve several paradoxes about the pathway of the sun, which are both exciting and mind-boggling. The first is, how can I trace the Ecliptic pathway of the sun when, in reality, it is a reflection of the pathway of the earth? Another paradox is the one where the sun travels outside of the Celestial Equatorial Circle, that is to say, outside of the Universe.

So, you can only solve these paradoxes by extending the proportionality of the process of stereographic projection outside of the sphere. This means that we are not dealing with a real paradox but with a geometrical device that will permit us to map, in accordance with the angular positions of the stars on the surface of the imaginary celestial sphere, the two different hemispheres. Note how every point on the surface of the northern hemisphere gets mapped on the internal surface of the equatorial circle, while every point on the surface of the southern hemisphere gets to be mapped on the external surface of the same equatorial circle. I am sending you separately the last series of Figures to illustrate the Astrolabe of Hipparchus. The stars projected on such a complex equatorial circle are not located in the night sky as in accordance with sense perception, but in proportion with the angular stereographic projection of the spherical almucantar and azimuth circles.

[INCOMPLETE]