From the desk of Pierre Beaudry

PART I

THE EGYPTIAN PYRAMID
AND
THE ARCHYTAS DOUBLING OF THE CUBE

by Pierre Beaudry 9/15/2009

PART I

[**HOW THE ARCHYTAS DOUBLING OF THE CUBE WAS DERIVED FROM THE SHADOW OF THE EGYPTIAN PYRAMID.** Class to the LYM Bogotá, Wednesday, Sept. 13, 2006.]

INTRODUCTION

There are three important things that I want to stress with respect to the function of shadows as footprints of universal physical principles. No doubt this will perplex some of you, but bear with me during these few hours. You won't have any difficulty in understanding Lyn's metaphors. They are always elevating. First, think back about {***Sphaerics***} as the dramatic crucial setting for axiom busting experiments of discovering universal physical principles in Plato's Cave. Second, think of {***Sphaerics***} as the complex domain of stereographic projections of the LaRouche-Riemann continuous manifold onto the discrete manifold. And thirdly, think of {*Sphaerics*} as the domain of LaRouche physical economy where universal physical principles interact between astrophysics, classical musical composition, artistic composition, and universal history.

Then, once you have established that concept in your mind, climb on LaRouche's shoulders and look forward to the next 50 years of your lives as the opportunity to integrate {***Sphaerics***} into the LaRouche Eurasian Landbridge program based on the Principle of the Peace of Westphalia, the principle of the {***Advantage of the other***.} It is only from that vantage point that you will be able to understand why Lyn has been saying that we are living in the shadow of the Great Pyramid. Indeed, Lyn has been saying that for years, but have you ever wondered what was the significance of that metaphor?

To me this meant that, on the one hand, universal history of ideas had been set within the boundary conditions of Egyptian {***Sphaerics***}, as they were initially developed by ancient astronavigators over 5,000 years ago. This also meant that the shadow cast by the Great Pyramid, as Thales found its height by relating it to his own shadow, was designed as an instrument for measuring the power of the human mind of all future generations; that is to say, for measuring the power of creativity, as opposed to the power of manipulation, like British Freemasonic mathemagicians do by mystifying the Great Pyramid as an artifact for the cult of the dead. That, I think, is the point to bring home with respect to the sophistry of oligarchical manipulation in ancient history. We must declare war on this sort oligarchical dominated sophistry and rid our schools of these oligarchical high priests! I hope that, with the help of the LYM, Lopez Obrador will be successful in accomplishing that mission in Mexico, today, by reviving the principle of the {***Pursuit of Happiness***} for all of Ibero-America.

So, to me, the ancient Egyptians were not mathemagicians like British historians would have you believe, but rather, astronavigators who had mastered a very ancient science of shadow reckoning, which they passed down to generations after them, and whose knowledge was stored in the science of {***Sphaerics***} that Thales, Pythagoras, Archytas, Plato, but also Aristarchus, Hipparchus, Apollonius, Archimedes, and Eratosthenes used, and was passed onto us as the new receptacles for the benefit of future generations.

However, for centuries, Aristotelian and Euclidean sophists have buried {***Sphaerics***}, and have replaced them with flatland geometrical traps, which have dominated our education system. So, the tradition embodied in the shadow of the Pyramid reflects the body of a science that must be revived and be reconstructed, in order to demonstrate how the {***necessary Egyptian predecessors***) of Archytas, for example, had begun to discover the elementary steps that led this Pythagorean genius, Archytas, to establish the universal physical principle of cubic power in the manner that Gauss polemicized in his dissertation of 1779. This is the reason why Lyn insisted that the LYM should study Archytas and Gauss together as opposed to the sophistry that we get in our universities, today. So, we shall follow the old LaRouche principle, which says:

"{***Believe nothing that for which you cannot give yourself a constructive proof***.}"

My purpose here, today, is to help you do that by building a practical physical model for the Archytas construction that you can use for organizing on the campuses of Universities. It is only after you have gone through such a step-by-step geometric construction that you will be able to demonstrate that the conical function of the Archytas model, as Lyn identified it in the Briefing of June 6, 2006, actually embodies an extraordinary congruence between the arithmetic-geometric mean of Gauss and the Lydian interval modality of the well-tempered musical system of Bach. In other words, the Archytas model for doubling the cube is a {***Sphaerics***} instrument for measuring the creative powers of the complex domain. So, let's follow what Lyn said on June 6.

In an answer to a question on the Archytas doubling of the cube and the arithmetic-geometric mean, by our French LYM member, Sebastien, Lyn gave the following lead to investigate the conical function with respect to Archytas:

"{***Question: Yeah, Lyn, this is Sebastien from France. I had a question about the doubling of the cube. Because we worked on that a little bit, and concerning one thing, which is, when you want to go through that, you have to develop this idea of geometric mean, and arithmetic mean. And the problem is that often, people don't really understand this principle of geometric mean and arithmetic mean. Because it's like a definition coming on the table; and then you have to go through that, to understand the doubling of the cube - and people stop to these concepts. And I was wondering, because the fact is that looks like definition, actually.***

And I was wondering also, we were discussing about music, also, and the fact that in Greek culture, science and music were very related. And the fact that all these ideas of geometric mean and arithmetic mean came from this idea of music. It was just something I was wondering. So, I don't know if you can elaborate on that , and my question is, because this reflection bring to me the idea that maybe geometry comes from music. I don't know if it's actually, but it's something very real for me.
So, if you can develop--.

"{***LAROUCHE: All right, go back and – don't try to interpret it from the standpoint of the subject of geometric and arithmetic mean, as ordinarily discussed. That's something in there, but don't focus on that. Like the first question that came in today, on this question of the emotion in music. It's the same problem.***

Now, the best way to do this, is again, like the Rabelais problem: you must situate your mind and your emotions in the right place, and you must put your ideas and your emotions together in the right place of reference.

Now, let's take this case of this arithmetic-geometric mean, not as such, but lets take the case of the doubling of the cube, as such. Now, you have a case, in which the last great praise of this work of Archytas in doubling the cube, in ancient times, was by Eratosthenes of Egypt. Now, Eratosthenes came from a culture which is a maritime culture, on the coast of what was then Egypt, and he studied and was a product of the Platonic Academy. He went back to Egypt as a tutor to the candidate for the pharoahship, and became the leading scientist in the world at that time.

Now, he was the one who praised the significance of the Archytas doubling of the cube, of the Delian problem. Now, how did he do that? How did he see this? Well, he had a predecessor who delt with conical functions. Ahhhh!!! Of course, the conical function is the key to the doubling of the cube. And always focus on that. It's the conical function.

***Because what's the difference between a spherical or a {Sphaerics} function, a four-square function, such as a Euclidean function, and the ability to solve that problem? You cannot solve this problem with Euclidean geometry. You can only solve it within the domain of {Sphaerics}. Now, what's the key? The function of the conical function. That's the pivot of this*. […]**}" (Lyndon LaRouche, Morning Briefing, June 6, 2006)

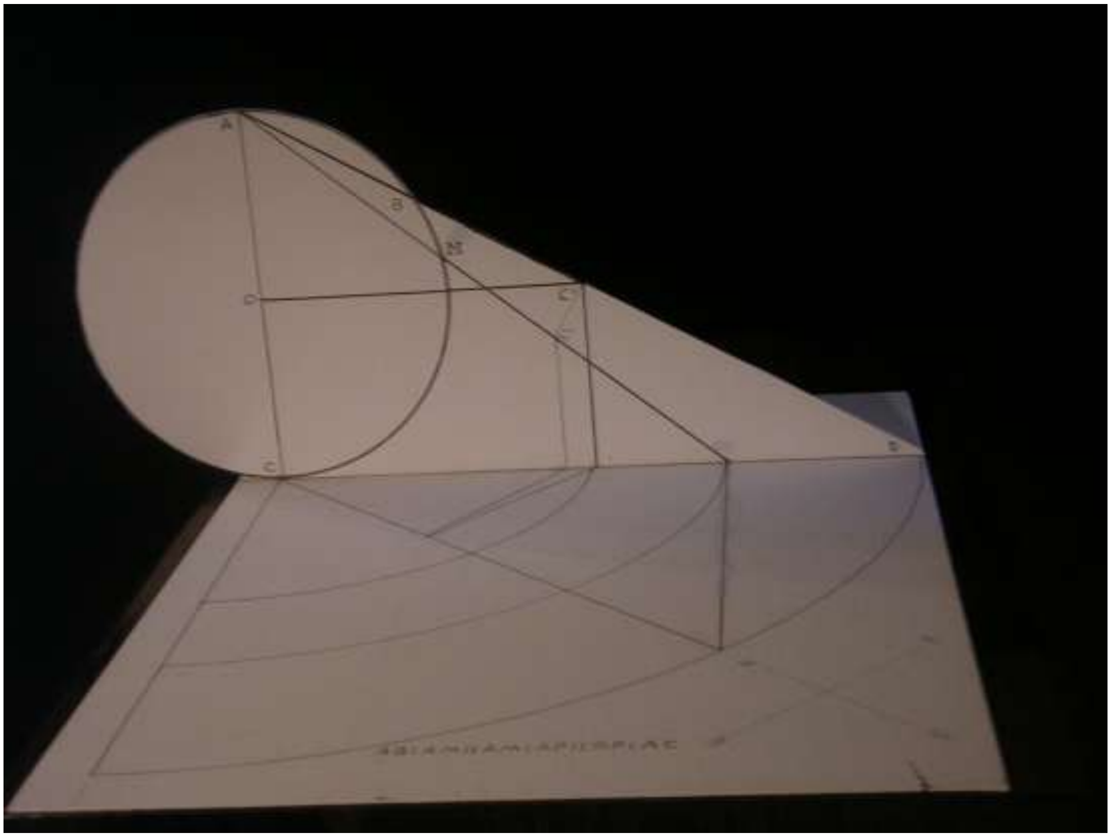
1. HOW THE EGYPTIANS FIRST ESTABLISHED THE DOUBLING OF THE CUBE.

So, let's look first into how the Egyptians established the doubling of the cube by a double conical function and see why Lyn is saying that this conical function is the key to the doubling of the cube by Archytas? And, second lets investigate why Lyn says that this conical function of the domain of {***Sphaerics***} is the pivot of the difference between the flatland of Euclid and the discovery of Archytas? Let's start with the Egyptian principle of proportionality. I have developed some aspects of that in my article on {Pythagoras Sphaerics}, in the 21st Century, Summer, 2004. But I had not discovered then that the Great Pyramid included in its very construction frame the idea of doubling the cube. So, if you refer to this article on page 53, you can verify the following proportionality:

{***The height of the great Pyramid of Egypt PO is to its apothem PE as two mean proportionals are to the doubling of the cube***.}

Now, as you all know, the geometric problem that Archytas had to resolve was formulated as follows: {***find two mean proportionals between two extremes in the ratio of two to one***}. In order to find those two mean proportionals, the Archytas construction for the doubling of the cube required a cone, a torus, and a cylinder. But no mention of the Great Pyramid was ever made. However, this Greek discovery was based on the more ancient Egyptian discovery as its {***necessary predecessor***}. So, let's look at how the Egyptian discovery must have been made.

Take **[Figure 1]** and study it for a moment without the knowledge that you already have of Archytas and Gauss, but keep those two friends sitting in the back of your mind for a while. Now, let's do a purely Egyptian experiment at the astrophysical location of the Great Pyramid itself, as if you were to be preparing to establish the plan of that great monument, yourself. This experiment is very simple and it merely requires that you establish your latitude is accordance with the altitude of the North Star. This is a very elementary but very crucial observation, which established the foundation of astrophysics in Egypt at about 3000 BC. This Egyptian experiment could also be executed from Houston, Texas, which is at about the same latitude of 29.97 degrees. Now, what is **[Figure 1]** telling us with respect to the latitude of the Great Pyramid?

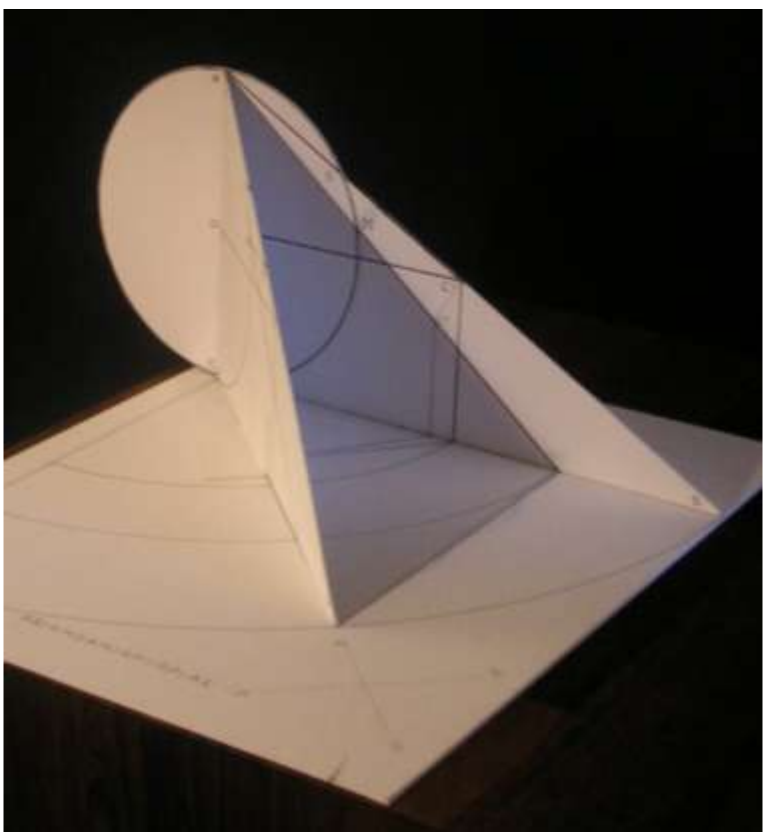


[Figure1. Projection of the scalene triangle of 60, 30, 90 degrees.]

1. First of all, situate yourself on the plateau of Giza, in Egypt, at the time of the construction of the Great Pyramid, and imagine yourself projecting through the celestial North Pole A of a transparent sphere ABC, an imaginary light ray AB, whose length is ½ of the diameter AC of that sphere, and where angle BAC must form an angle of 60° degrees, which is half of the apex angle of a cone of 120 degrees rotating around the sphere. Thus, the projection is a scalene triangle of 30, 60, and 90 degrees, the basic triangle used by Plato for the construction of three regular solids.

   [Construct a baseboard 33 cm by 43 cm, and a scalene triangle of 18 cm x 31 cm x 36 cm. The triangle must be the perfect half of an equilateral triangle. Glue triangle ACD behind a circle of diameter AOC of 18 cm. Mark on the circle line AB, which is half the length of AC, and where the angle OAB is 60 degrees.]

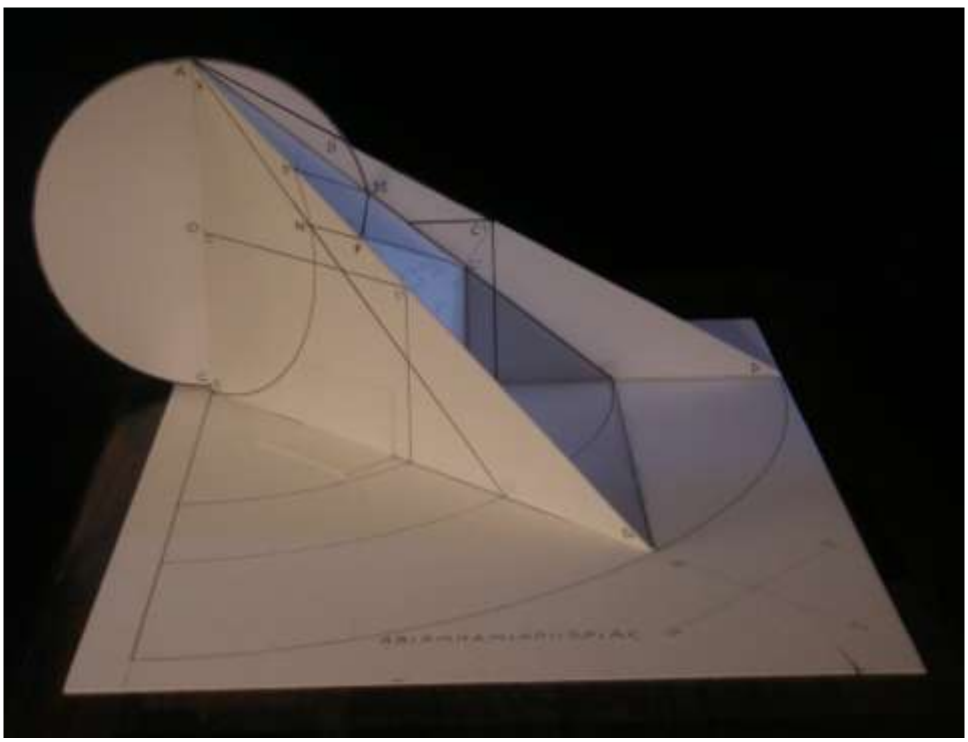
The simple reconstruction of this important astrophysical event is crucial for two reasons. One, the projection of a ray from the celestial North Pole A to point B represents the angular elevation of the North Pole, which determines the latitude of the Great Pyramid on the plane of Giza. By doing this, the Egyptians had established their precise astrophysical location with respect to the moving Axis of the Universe as a whole. This is an extremely important conceptual and emotional moment for any early astronavigator. Two, this event also represents an elementary partitioning of the sphere into six equal parts, forming a cuboctasphere from which may be generated three of the five Platonic Solids: the cube, the octahedron, and the tetrahedron. By continuing the projection of that ray, AB, through the sphere and extending it to the level of the plane at D, the basis for the greatest astrophysical observatory in history is locked into position from that latitude of 30° degrees. In a moment, I will show you why this projection is essentially Egyptian in character because it establishes the original design for the Great Pyramid itself. Any questions?



[Figure 2. Rotate the first projection by 45 degrees around the sphere.]

2. Secondly, rotate the scalene triangle, ACD, by an angle corresponding to 1/8th of the sphere, that is, by 45° degrees in the plane, making sure

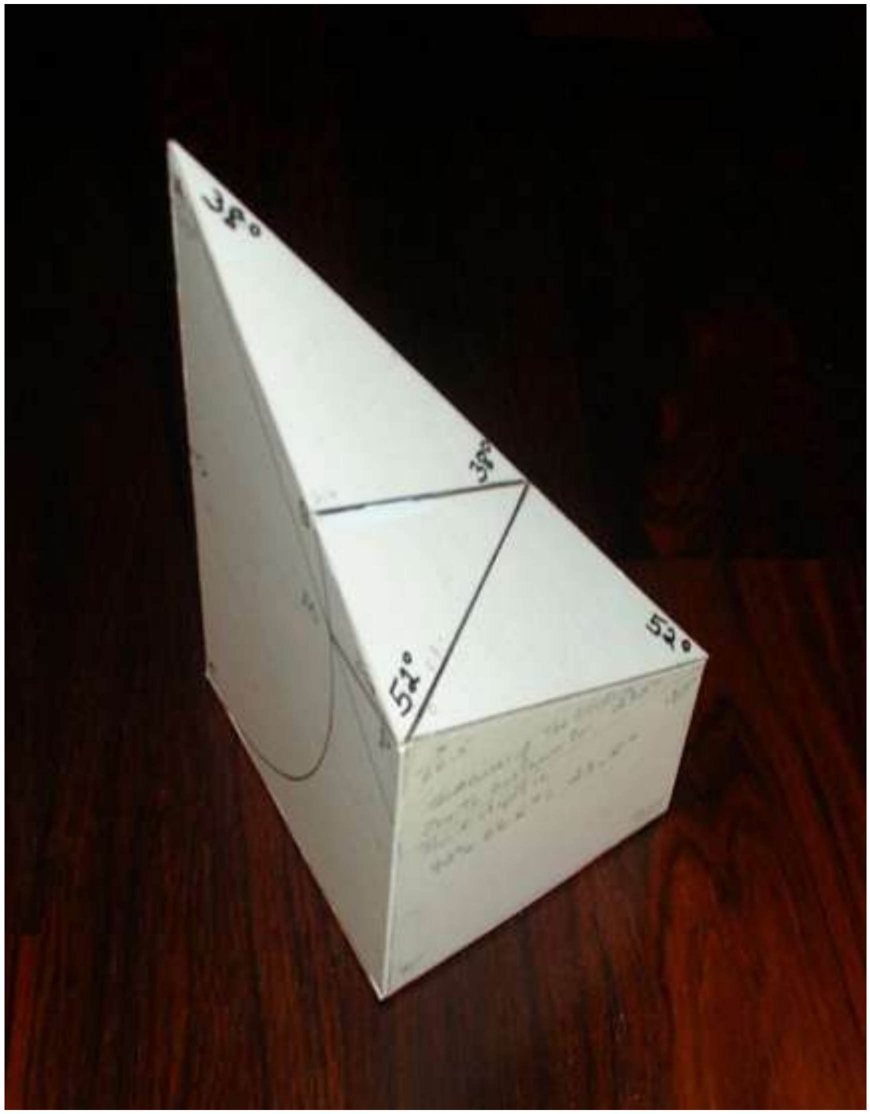
that the rotated triangle ACD’ has pivoted on the hinge, AC. Think of this rotation as generating a cone whose apex is at point A. Next, project a second source of light at right angle to the initial triangle ACD. You don’t see that source of light but only its shadow, as if on the dimly lit wall of Plato’s cave. Here, an apparent insignificant non-visible event occurs, a minuscule anomaly, which causes a perplexing paradox. This second light projection creates three intersecting things: 1) the precise mean proportional AM for the doubling of the cube whose side is AB, 2) the precise angle MAP (38 degrees), which is half the apex angle of the Great Pyramid, and 3) the precise angular projection of the Ecliptic (23.5 degrees). That is an extraordinary anomaly. As a result of this double projection, from the Celestial North Pole and from the Ecliptic, the angles formed between the two triangles, at the level of the plane and at the level of the hypotenuse (apothem), are axiomatically different. This axiomatic difference represents two different levels of power of the human mind. This is the conical function anomaly that locates, within a single shadow, the non-visible axiomatic difference of passing from the doubling of the square to the doubling of the cube. Any questions?



[Figure 3. Projection of harmonic proportions of the shadow.]

3. I have inserted within the shadow area a solid shadow, a {stereo-shadow} as the Greeks would say. When D’CD forms an angle of 45° degrees in the plane, then PAM forms an angle of 38° degrees at the

level of the hypotenuse (apothem for the solid). This ambiguous transformation of a 45° degree angle into a 38° degree angle results in the creation of a right triangle, which corresponds to half of the meridian triangle of the Great Pyramid of Egypt. This confirms that the meridian design of the great pyramid itself embodies the solution for doubling the cube. Thus, {***the height PO of the Great Pyramid is to its apothem PE as two mean proportionals are to the doubling of the cube***}.



[Figure 4. Solid shadow.]

[Construct the five-sided solid shadow of the Great Pyramid. The angular base of the shadow must be 45 degrees and the angular summit of the shadow must be 38 degrees. The numbers that I have used for this model are respectively AB = 9 cm, AM = 11.34 cm, AP = 14.29 cm, and AC = 18 cm.]

Thus, the Egyptian doubling of the cube is simply a derivative of two astrophysical observations that had to be made at the site of the Great Pyramid in order to establish its architectural design. Those two conic projections, from the North Pole and from the Ecliptic, generate the frame-shadow of the Great Pyramid whose triangular meridian angle, PAM, shows that the two proportional segments, AM and AP, respectively represent the sides of two cubes whose volumes are in the ratio of 2/1. So, it becomes clear that this where the Archytas construction took its origins. Observe the double proportionality of Figure 3:

AB : AM :: AM : AP :: AP : AC.

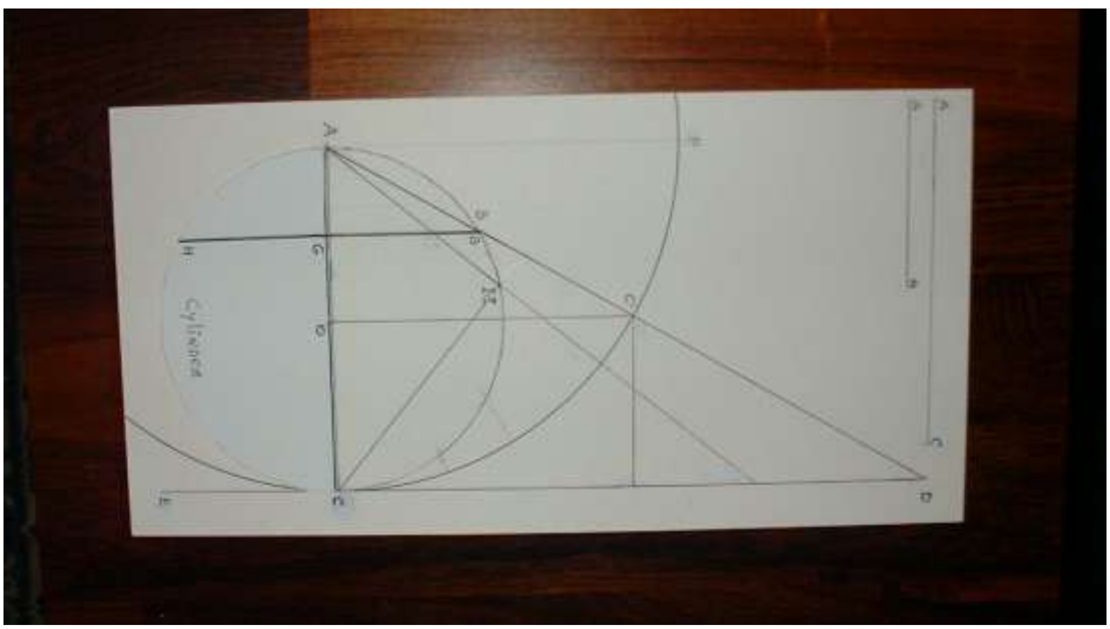
Ironically, this Great Pyramid triangular frame-shadow of 90°, 52°, and 38° degrees, with its harmonically conjugated segments, AB, AM, and AP, not only reflects the power of successively doubling the cube, but also reflects the golden section, the Great Pyramid paradox of squaring the circle, and the 256 series behind the well-tempered musical system. If you refer to the illustration of my 21st Century article, page 53, you will recognize that we have just constructed the Queen's Observation Shaft. Any questions?

In retrospect, it is important to reflect on the fact that a true axiomatic perplexity took over the class last night, at the very point when it was discovered that the Equinox projection of the Sun was casting a shadow at 23 ½ degrees. This angular projection, which falls at right angle to the initial projection from point A establishes the proportion whereby, {***the height PO of the Great Pyramid is to its apothem PE as two mean proportionals are to the doubling of the cube***} Thus, this perplexity coincided with the axiomatic change of doubling the cube. The point to stress is that such a state of perplexity is an essential component of this discovery of principle. Such a state is by no means to be avoided. When this happens, the will of the individual should immediately avoid the tendency to fall back onto his previous axioms, postulates, and definitions, which no longer work, and inverse the process into investigating what might be behind the anomaly.

For example, instead of looking for the angle of 23 ½ degrees, as a thing in itself, look for the astrophysical principle that generated such an angle. What is the concept that generates such an astrophysical singularity during the year? The answer is to be found in the principle that fixes that angle of intersection of the Sun with the Celestial Equator at the Equinoxes. During the entire cycle of the Ecliptic, there are only two days when the year is divided into two equal parts at the same time that the day is divided into two equal parts. Those are the Equinoxes. The interesting thing is that these are also the only two days of the year when the Sun casts a noon-shadow of 30 degrees at the site of the Great Pyramid! Thus, it should be an awesome surprise to discover that this intersection of two projections of 30 degrees and 23 ½ degrees both relate to the doubling of the square and the doubling of the cube!

2. CONSTRUCTING THE ARCHYTAS DOUBLING OF THE CUBE.

The first thing you must avoid when you are first introduced to the Archytas doubling of the cube is the trap of fumbling all over the cone, the torus, and the cylinder, as if they were things in and of themselves. They are not. They are visual traps. What you must do, immediately, is to look behind the visible domain and reach out for the principle of what Lyn has always identified as multiply connected circular action. The reason you want to concentrate on intervals of circular action is because they always express the principle of least action in some form of proportionate way. This is the polemical point of Gauss against the false "shadows of shadows" of imaginary numbers concocted by D'Alembert, LaGrange, and Euler.



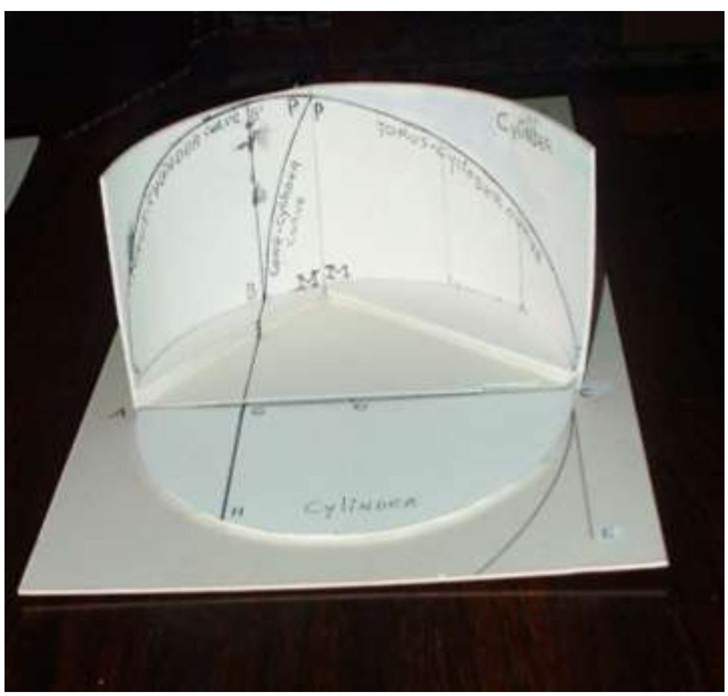
[Figure 5. Baseboard of the Archytas construction.]

[Start with white cardboard material, and establish a baseboard 23 cm by 45 cm. Draw the appropriate circle and conic triangle lines exactly as in the previous Egyptian model. The crucial difference, here, between the two models is that Archytas replaced the double projection of the Egyptians the interaction of two bold curves which require the intersection of a Cone, a Cylinder, and a Torus. So, instead of drawing the great circle of a sphere, start by drawing the base of a cylinder with an 18 cm diameter.

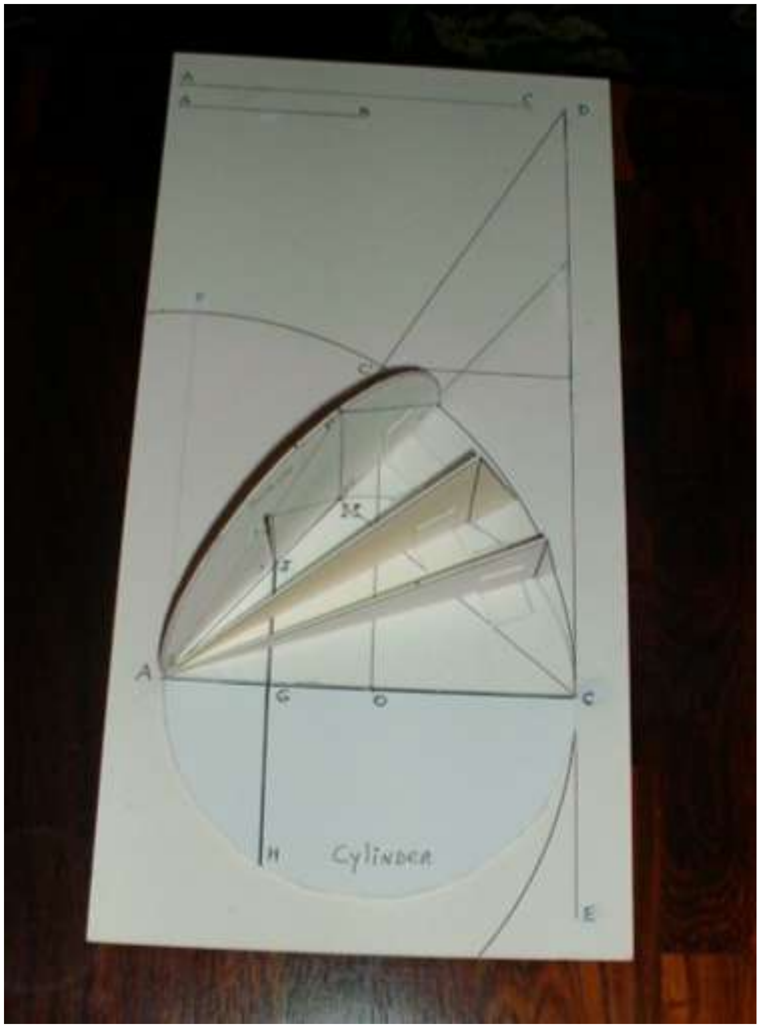
Next, construct two parts of a half cylinder whose height is 9 cm. Draw on each part the letters identifying the different points of the base board, that is, A, B, M, C, then line BB' and line BJGH. After that, construct the two bold curves of the Torus-Cylinder and the Cone-Cylinder and establish point P as the intersection singularity of four degrees between the two double curves.]

Think of the intersection of those two bold curves as the traces of two circular actions generating a single unity of effect that is stereographic in character, that is to say solid in character. Then, think of the Torus-Cylinder Action separately from the Cone-Cylinder Action. They are both constructible only by circular angular rotation. The reason you want to start with the construction of the Cylinder is that this is the only one of the three solids that is not moving, and it is the only solid on the surface of which the two bold curves can be traced.

So you must build in your mind a thought object, a {***Geistesmassen***}, or a {***stereo-idea***}, as Plato called it. Remember you are not dealing with a visual object, but with a conceptual object, which was also an attempt at establishing a solar calendar by means of which one could map the different positions of the Sun during the year with respect to the Celestial Equator. Therefore, it is not difficult to imagine that the circular tracing of the Torus outside the base of the Cylinder could represent the Ecliptic pathway of the Sun during the year, and that the tracing of a circular cut of the Cone, at the intersection of point P, could represent one of the two points of the Equinoxes. Here, I am just throwing a seed to the wind to see if it will grow. But that would require another class. For the time being, concentrate on the Torus-Cylinder action.



[Figure 6. Torus-Cylinder curve on a half-Cylinder.]



[Figure 7. Torus half circle and two animation tracers.]

THE TORUS-CYLINDER ACTION

[Construct the half circle of the Torus as you did the Cylinder base, with a diameter of 18 cm. The two circles have the same diameter. Observe first that the tracing of this curve is occurring in your mind only. This is not a visible sense-perception curve. It is very important to make that difference because when you investigate the question of the Ecliptic, that is, the pathway of the Sun, that pathway is not a visible curve either, but a conceptual curve.]

Starting from an initial position at AC, rotate the Torus half-circle one full circumference around the fixed pivot point at A. Trace a quarter circumference on your baseboard and note that the Torus half-circle motion intersects the Cylinder at every point of its wall. Construct the stereographic image of that Torus-Cylinder curve. The motion of the Torus is leading toward a point of singularity, point P, which is the only point that intersects simultaneously the three surfaces of the Cone, the Torus, and the Cylinder! There is no other point, anywhere in that whole complex construction which intersects the three surfaces all at once. Everywhere else, the surfaces of the three solids only meet two by two. That is why it is important to trace the two bold curves separately. So, the question is, can you find in the Torus-Cylinder Action any reason to stop at point P? The answer is no.

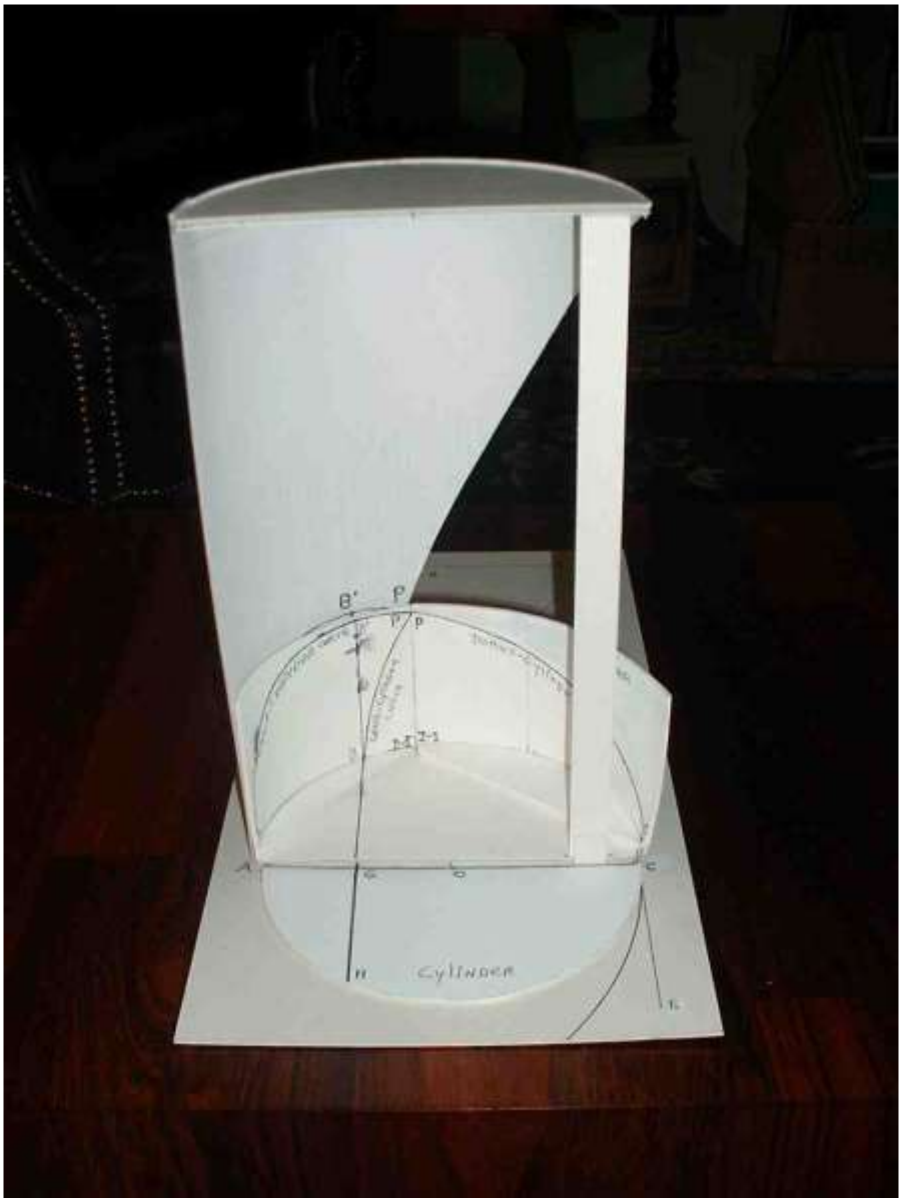


Figure 8. [The two bold curves]

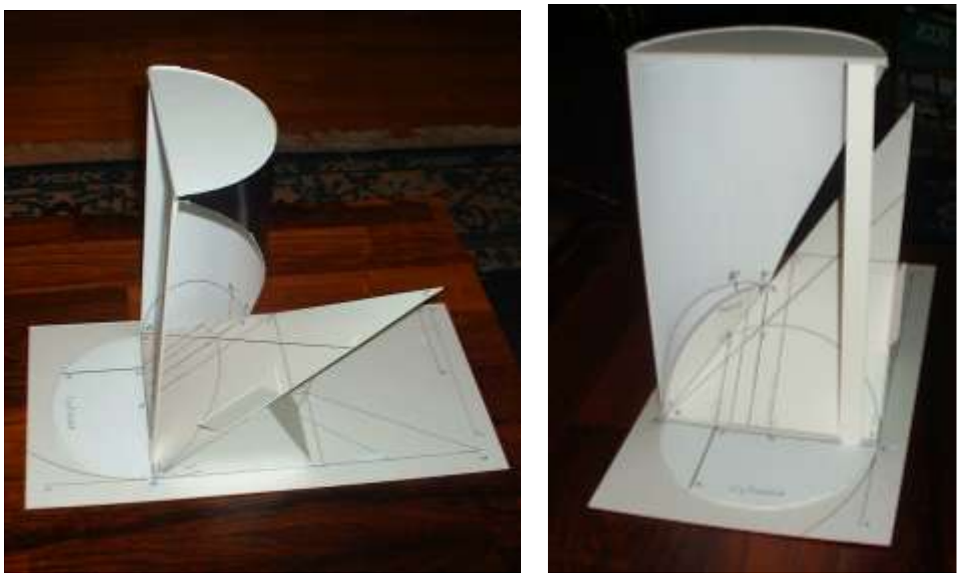
When you rotate the Torus half-circle, perpendicular to the base circle of the Cylinder, you see in your mind the trace of a bold curve, as it has been called, on the surface of the Cylinder. The curve has been called "bold" because it is daring, because it is the coastline that rises "bold" between the two domains of Euclidean Flatland and the domain of Pythagorean Sphaerics. This bold curve is a double Torus-Cylinder curve; meaning that it traces the shadow-contact of the two surfaces as the motion of the Torus half-circle constantly intersects the fixed surface of the Cylinder in its angular rotation. As you follow that trace on the Cylinder, imagine that the same trace is moving slowly on the circumference of the Torus half-circle, from C toward P. Now, consider this curving action as an {***axiomatic change indicator***}, for it has no other meaning than to trace the shadow leaving point C and moving toward the singularity of point P because it is tracing the pathway of the axiomatic change between the doubling of the square and the doubling of the cube.

You can easily establish this curve by having the Torus half-circle stop anywhere you choose along the base of the Cylinder and trace two other different positions on two animation tracers that will reflect the positions of two different points on the Torus-Cylinder curve. [Explain this with Figure 7.] Bear in mind that what is special about that point P is that it is the point where the Torus and the Cylinder meet the {***conical function***}. Ahhh!!! This is what Lyn said was the key. So, there you have it. There was a reason for the Torus-Cylinder Action to stop at P, and that reason is to be found in the cone, that is in the second action. So, there must be a second {***bold curve***}, generated by the Cone-Cylinder Action of the {***conical function***}, and which will contain the reason why the Torus-Cylinder curve must stop at P. This is where the Egyptian missing link comes in. How can we construct that?

It is essential to discover this point by a construction process and not simply assert its existence by intuition. The reason why it is necessary to go through the constructive proof of a discovery is because a so-called intuitive proof is a cover for a false underlying assumption. This is the reason why Bernoulli, for example, wrote a letter to Newton, urging him to send him his method of construction for the catenary curve, because Newton was flaunting his solution without showing what was behind it.

Of course, Newton never sent him his method of construction for the catenary because he had copied the answer from the back of Leibniz's book. So, you see, Newton was hiding behind his "intuitive proof" because he had not found it by construction and he wanted to use that cover as a means of exercising authority and power over people. I am sure you have never encountered any such situation, yourselves. At any rate, that is the underlying assumption which is hiding behind a so-called "intuitive proof," and that is why you must always provide a constructive proof in everything that you do. Now for the Conic-Cylinder action.

THE CONIC-CYLINDER ACTION



[Figure 9. Scalene conic section and the Cone-Cylinder curve on a half-Cylinder.]

Lastly, let us construct the bold curve of the {***conical function***}. Rotate the scalene conic section through the entire Cylinder. That 180-degree rotation generates a half cone whose axis is hinged at AC in the plane. Now, as you elevate the tip of the scalene-conic section around the axis AC, the quarter conic rotation traces a curve along the entire surface of the half-Cylinder up to a maximum point at D’ at 90 degrees. Cut the Cylinder along that Cone-Cylinder curve. This last step shows why the whole process of the double circular action stops at point P on the Cylinder. It is because this angular elevation is a mixture of 45 and 38 degrees, that is, {***the angular difference between doubling the square and doubling the cube!***} That was the crucial singularity to be discovered that connects the Archytas construction with the Egyptian construction. That is the singularity where the crucial discontinuity of a change of power between the plane and the solid becomes intelligible to your mind’s eye.

As a result, point P becomes a {***quadratic singularity point***} intersecting four surface contacts: two between the Torus and the Cylinder surfaces and two between the Cone and the Cylinder surfaces. In fact, this is the only {***quadratic singularity point***} in the entire Archytas model. This point of discontinuity could also be likened to a thermodynamic phase-space transformation point between solid, liquid, and gas. As Lyn often demonstrated, and as the current crisis-point in history also shows, a point of high density of singularities represents the turning point of a physical axiomatic change. So, the significance of point P is that it acts as a catastrophic shock-effect point, or as a turning point of opportunity, at any rate, as a change of power, a Riemannian change of geometric manifold. That is what point P is all about. It is an axiom busting point of four degrees, something like a four-degree osculation that Leibniz talked about in his {***Acta Eruditorum***} papers.

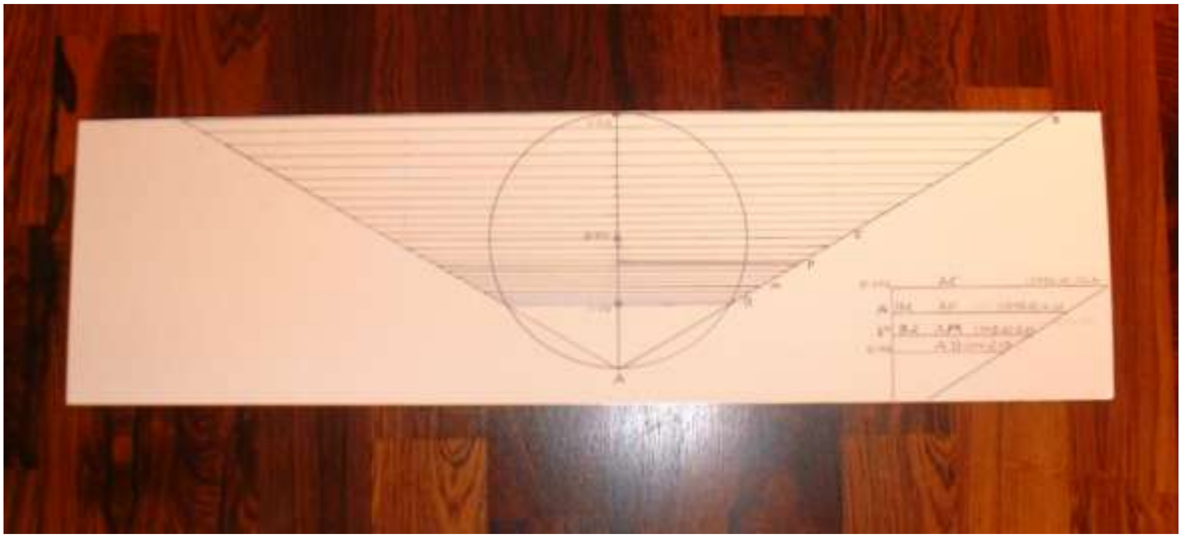
Now, take an example from universal history. Take a look at the ecumenical alliance between Charlemagne, Harun al-Rashid, and the Jewish Khazar Kingdom and consider that all three came to such a singularity point of transformation at about 800 AD, and the Eurasian Landbridge they organized against the Venetian Bankers and the evil backers coming out of the Byzantine Empire. This is the same thing that Lyn is calling for in his call for ecumenical combined efforts between Eurasia and the Americas for the transformation of Africa today; that is to say, how to devise a grand design of political and diplomatic effort which would get South Africa to become the motor of development of all of Africa, in collaboration with the sovereign nation-states of the Americas. That new historical curvature has to be built from the outside and from a mission oriented purpose of developing the principle of the {***Advantage of the other***} among all of the sovereign nation-states concerned.

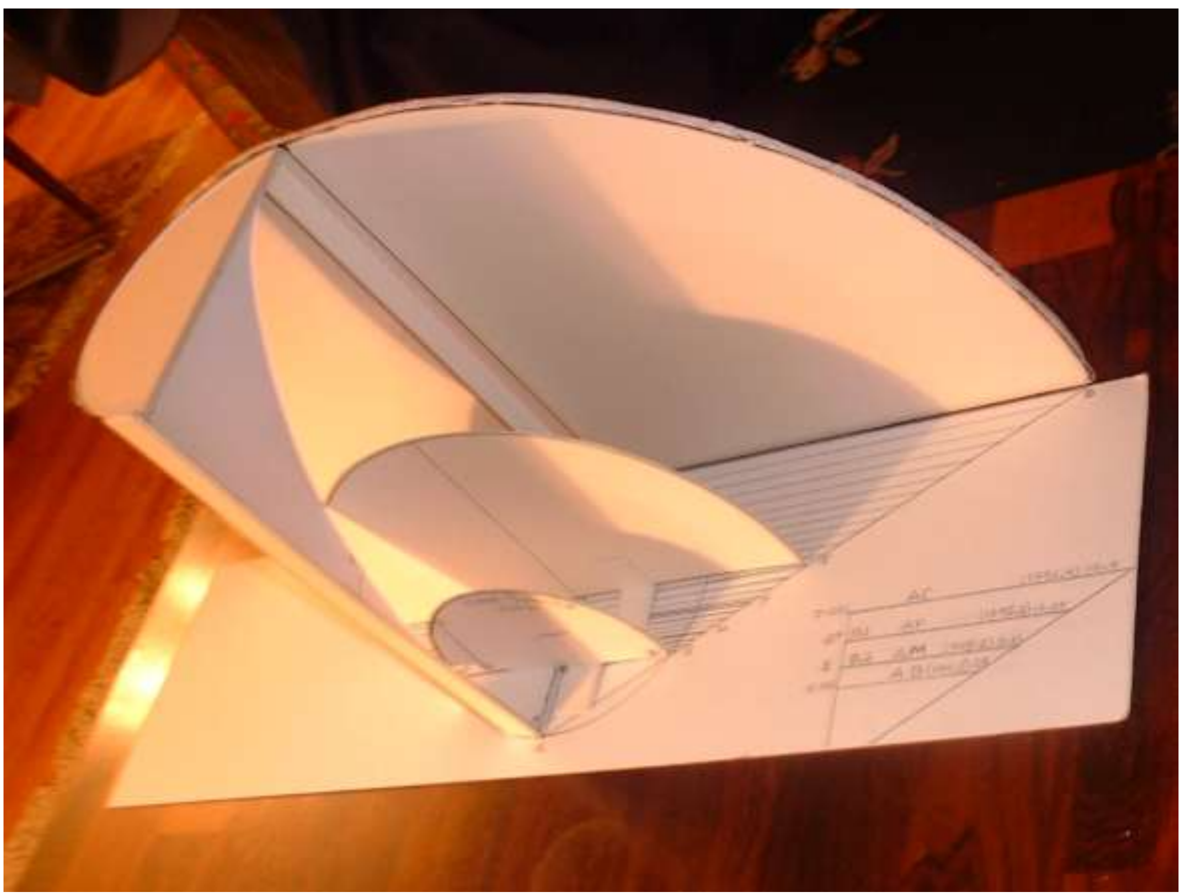
This {***agapic***} policy was expressed by Charlemagne, Harun al-Rashid, and their mutual Jewish ambassador-merchants in a form of interest-free gift economics of exchange of ideas and goods for the development of mankind. The key was that the leaders of the three great religions, Christianity, Islam, and Judaism, had all agreed to expand their trade and development between Africa, Europe, and Eurasia on the basis of excluding usury! That small little intersecting point between the three was the function of point P in the Archytas construction, that is, the point that brought unity to the whole process. If I had more time, I would show you how, by translating the Archytas construction into an astrophysical calendar, point P which can be located in two different places on the cylinder, was probably also used to express the point of the Equinoxes where the Ecliptic pathway crosses the Equator. (See my class with the Philippines LYM. 06, Part III.) Thus, pedagogically, the function of point P comes to represents a significantly high density of singularities in the history of mankind, which is not to be underestimated at this strategic historical juncture.

The creation of this higher manifold of the Charlemagne Eurasian Landbridge is the equivalent of introducing a new dimensionality in the political affairs of the world. This is the political equivalent of reaching point P to determine the higher geometry of cubic roots. Most emphatically, this is the same axiomatic change that Gauss introduced in passing from the domain of the square roots to the domain of the cubic roots in his 1799 polemic against d'Alembert, Euler, and Lagrange.

Thus, Archytas established the two mean proportionals that were required to be found between two extremes in a ratio of 2/1 in order to double the cube. Such is the {***quadratic proportionality***} of the conic section where AB : AM :: AM : AP :: AP : AC. That is the pivot of the {***quadratic conical function***} that Lyn identified as the key that unlocks the Archytas theorem, and which brings it in congruence with the Pyramid of Egypt, where {***the height of the Great Pyramid is to its apothem as two mean proportionals are to the doubling of the cube***.}

THE ARITHMETIC-GEOMETRIC MEAN AND THE LYDIAN INTERVAL.





[Figures 10 and 11 The two Archytas mean proportionals as Lydian Intervals.]

The figures show the general conical function determining the Cone-Cylinder curve by means of Logarithms. Although Archytas did not know the science of logarithms, as Napier discovered them, and published them in 1616, it is clear that the principle of proportionality established by Thales, Pythagoras, and Archytas for the study of {***Sphaerics***} was the very same that was later perfected by Kepler, Napier, and Leibniz.

Think of the arithmetic-geometric mean function as the shadow projection in the continuous manifold of a process of changing the rate of change in the physical economy, that is, a process of determining a higher rate of changing the change calculated by both the arithmetic half and the geometric half of a logarithmic spiral action. In other words the arithmetic mean is $(A + B)/ 2$, and the geometric mean is $(\sqrt{A \cdot B})$. In {***So, You Wish to Learn All About Economics***}, Chapter III, Lyn illustrated this idea by the iteration of a series of elliptical cuts defining smaller and smaller conical volumes of the logarithmic spiral action. In 1984, when his book was published, Lyn identified the foci of the ellipses, which called for the calculation of the iteration by way of the harmonic means that is, by $2(A \cdot B)/ (A + B)$. Now, Lyn is using a different iteration, which he identified with the Gaussian arithmetic-geometric mean. So I will proceed with his latest proposal, unless he indicates otherwise.

I know of two ways of illustrating this Gaussian process. One is by constructing two different spirals: an arithmetic spiral (Archimedean spiral), which reaches the half way-mark up the axis of the cone at the arithmetic mean, and a logarithmic spiral (Equiangular spiral) whose halfway mark is the half-cycle of the spiral action around the cone, which marks the geometric mean.

The other method is to use simply a logarithmic spiral and identify the halfway vertical interval of spiral action representing the arithmetic mean, while the halfway spiral action around that volume of the cone represents the geometric mean. This difference corresponding to half a tone of the well-tempered system is fairly small and requires a four-step-iteration of the arithmetic-geometric process of change. In this process, the irregular elliptical change in curvature is modified in an accelerated way. I encourage you to build your own models to understand how this works.

The point is that the accelerated {***changing of the change***} defines a quantum of action, which is expressed by the smallest value of “delta” in the Leibniz Calculus. This infinitesimal crack in the universe is a similar expression of the infinite as the one reflected by the isoperimetric elliptic-circle of Cusa’s {***Isoperimetric Principle***}. It is through such small intervals of action that you can recognize the presence, in the simultaneity of eternity, of a universal physical principle acting on the universe as a whole.

If one were to establish a similar conic projection of the well-tempered logarithmic spiral to the Archytas cone, as illustrated in [Figure 9], then one would be able to establish a very similar relationship between the arithmetic-geometric mean of Gauss and the Lydian intervals of the well-tempered musical system of Bach. This is an amazing case where you can see the interaction of the Gaussian arithmetic-geometric function and the Lydian function of Classical musical composition.

If you use the two Archytas extremes in proportion of 2/1, that is, A = 18cm and B = 9cm as the two values for the arithmetic and geometric means, and proceed to establish the arithmetic-geometric mean function between them, you shall arrived at the elliptical-circular value of 13.110837…cm. It is very interesting to note that this value, established on the apothem of the cone, corresponds to the singularity of the Lydian interval of a minor third in the well-tempered system, when the logarithmic proportionality is developed within the elliptic function of the internal conic sections of that same cone. Conversely, if you use the musical octaves in proportion of 2/1, that is to say, A = 15.6cm., and B = 7.8cm., and proceed to establish the arithmetic-geometric mean between those two values, you shall arrive at the elliptic-circular value of 11.3629…cm which is approximately the value for the side of the cube whose volume is double the initial cube whose side is 9cm.

This is a very special {***cross-proportional-singularity***} of the arithmetic-geometric mean which intersects the Classical artistic and the scientific domains, such that finally, this Archytas construction, originating from ancient Egyptian {***Sphaerics***}, takes us full circle into the domain of elliptic functions of Gauss. It demonstrates how, through however unevenly and dimly perceived the shadows of its projection may be on the wall of Plato's cave, the external beacon of light casting the shadow of the Great Pyramid of Egypt down to us, today, is a reflection of the most powerful historical singularity of creative knowledge that the human mind was capable of producing more than 5,000 years ago. In so doing, such an ancient Egyptian thinker as Imhotep had set the stage of history and had defined the measuring instrument of change by which the battle for liberating the human mind would be fought for all centuries to come. Among the rubbles of civilizations past and future, this original Archytas construction, born out of the shadow of the Pyramid of Egypt, shall stand as a testimony to the endurance of the human mind's quest for truth and for its unceasing commitment to recognize its own optimistic spirit in reaching out for the development of future generations. Thank you for your attention and your generous perplexity. Any questions?

THE ARCHYTAS MUSICAL FUNCTION FOR DOUBLING THE CUBE.

8/22/2009

How do you determine the doubling of the cube by using only the Archytas conical function? The short answer to this question is by determining the logarithmic range of two mean proportionals of the equal-tempered system between the octave of C-256 and C-512.

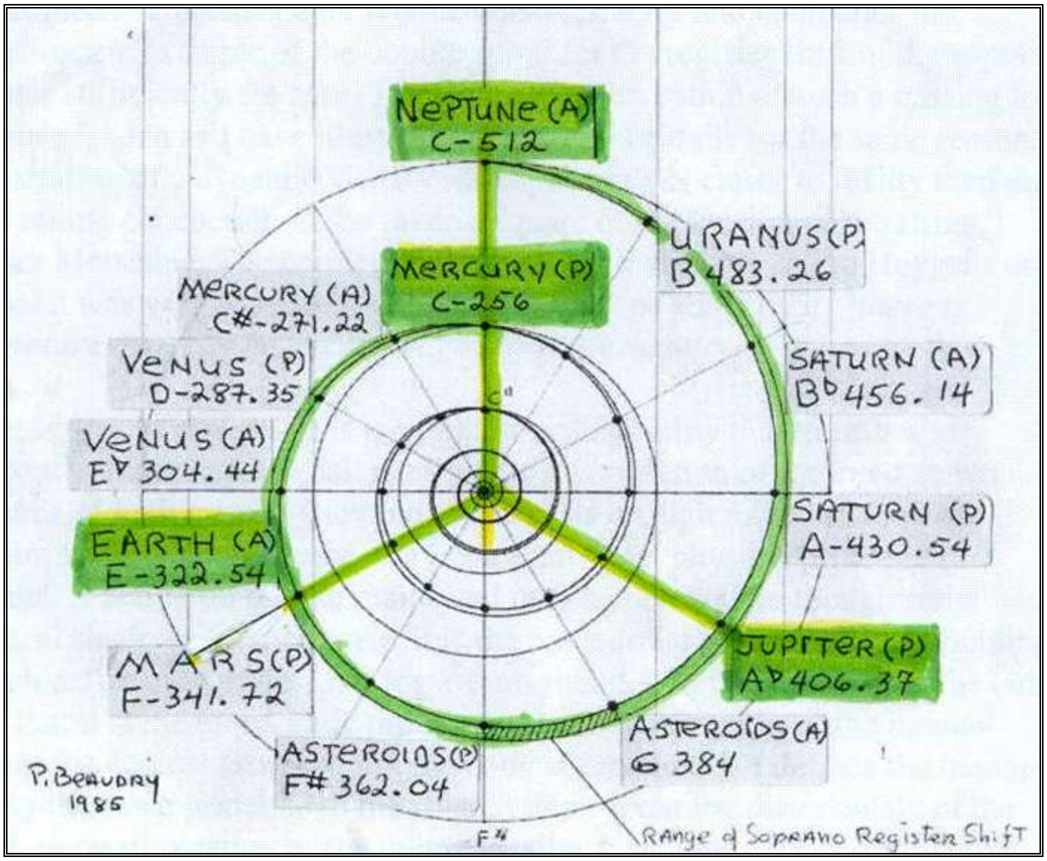


Figure 1. Correspondence between the conical projection of the equal-tempered musical system and the planets aphelion and perihelion.

As curious as the case may appear to be, the simplest way of solving the problem of doubling the cube is a musical idea that goes back to Pythagoras, Plato, Archytas, and Hippocrates. It is the equivalent of seeking the two mean proportionals within a musical octave of the equal-tempered system. Take, for instance, the octave between C-256 and C-512. The two mean proportionals will be E-322.54 and Ab-406.37.

Those two mean proportionals, E and Ab, also correspond to the aphelion of Earth and the Perihelion of Jupiter when you establish that the perihelion of Mercury and the aphelion of Neptune mark the octave.

Planets	Astronomical Units	Log 10^x	Added Constant	Multiple Constant	Cycle Equivalent	Musical Cycles	Planets
Mercury	(P) 0.310	−0.5086	+2.496	×128.8	255.97	C = 256	Mercury
	(A) 0.470	−0.3279	" "	" "	279.25	C# = 271.22	Mercury
Venus	(P) 0.715	−0.1457	" "	" "	302.72	D = 287.35	Venus
	(A) 0.725	−0.1397	" "	" "	303.49	E♭ = 304.44	Venus
Earth	(P) 0.983	0.0074	" "	" "	320.52		
	(A) 1.017	0.0073	" "	" "	322.42	E = 322.54	Earth
Mars	(P) 1.379	0.1396	" "	" "	339.46	F = 341.72	Mars
	(A) 1.661	0.2204	" "	" "	349.86		
Asteroids	(P) 2.2	0.3424	" "	" "	363.32	F# = 362.04	Asteroids
	(A) 3.6	0.5563	" "	" "	393.13	G = 383.57	Asteroids
Jupiter	(P) 4.95	0.6946	" "	" "	410.95	A♭ = 406.37	Jupiter
	(A) 5.45	0.7364	" "	" "	416.33		
Saturn	(P) 9.006	0.9545	" "	" "	444.43	A = 430.54	Saturn
	(A) 10.074	1.0032	" "	" "	450.69	B♭ = 456.14	Saturn
Uranus	(P) 18.288	1.2622	" "	" "	484.05	B = 483.26	Uranus
	(A) 20.092	1.3030	" "	" "	489.31		
Neptune	(P) 29.799	1.4742	" "	" "	511.36		
	(A) 30.341	1.4820	" "	" "	512.37	C = 512	Neptune

Figure 2. Planetary orbits and the equal-tempered system established by Bill Bohdan.

The longer answer is by constructing the conical function of the logarithmic system of musical equal tempering. This is constructible with a straight edge alone. I will show this construction to anyone who wishes to travel to Jupiter and back under G-1 Acceleration/Deceleration?

FIN