ELEMENTARY CONSTRUCTIVE GEOMETRY

Circular Action Basis of Geometry

by Pierre Beaudry

Acti_n in the universe is never arbitrary, it is always determined. That is, it has a lawful cause. And that is why I shall begin this study of geometry by repudiating the current fallacies about teaching geometry in our schools, where teachers impose on children a system of axioms, or rules, which are given as self-evident and not to be questioned.

For instance, children are told that the foundation of geometry is based on the Euclidean plane, which is defined as a collection of lines; the line is a collection of points, and finally the point is the smallest irreducible entity in itself. This is all nonsense. In order for anything to make sense in the universe, you must account for its existence, and these axioms cannot account for their own existence. Where do points and lines come from? How were they created? These questions I shall now begin to answer.

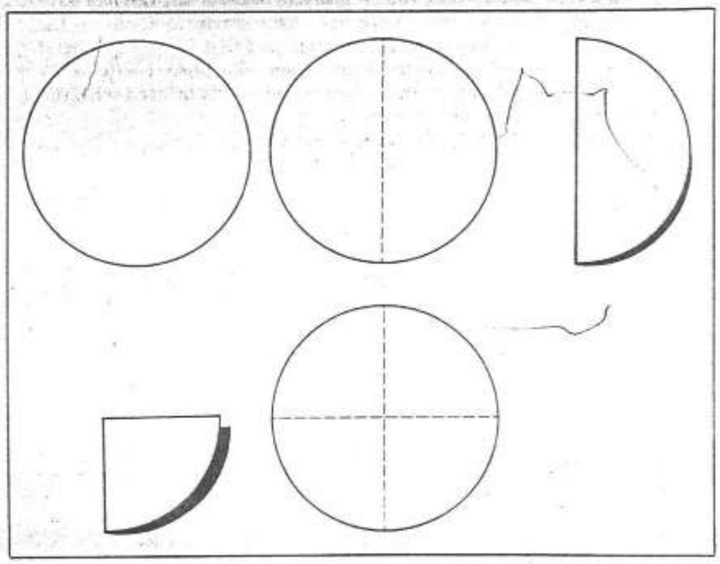
Circular Action

I intend to demonstrate here, and subsequently in other geometric problems, the hypothesis that Lyndon LaRouche has posed: Everything in God's universe is the result of circular—more precisely—spiral action. And only by beginning with circular action can we find causality in the universe. That is creativity. It follows from this that since man is created in the image of God, everything He creates in that image is also the result of circular action.

Third, it also follows from this, that things have no existence by themselves. Planes, lines, points, etc. are not self-evident things. They are singularities, moments of discontinuity of rotational action. They are the mere traces of the limiting and bounding process of rotation. They are what Plato called the shadows on the wall of the cave, the images of a higher projection.

Creating the Plane

What happens when you make circular action? That movement circumscribes a certain amount of space, a certain area, and bounds it, giving it closure. In fact, you have just created a circular plane! Here, something very unique has happened, that is not immediately visible. Consider the relationship between the circumscribing action and the area being bounded by it. Upon reflecting on itself, the motion



The foundation of all geometry is circular action, that is self-reflexive, and generates the plane, the line, and the point.

of rotation has accomplished the most fundamental creative action. It has created the largest possible area by the shortest possible means. If you measure the contour of the circle with a string, and form any other closed shape, say a square or a triangle, the enclosed area will always be less than that of the circle. This is the minimum/maximum principle discovered by the great Cardinal Nicholas of Cusa during the Italian Renaissance. The principle by which you accomplish the maximum amount of work by using the minimum amount of action is called the least action principle, and was named by Wilhelm Gottfried Leibniz.

Creating the Line

Fold the circle on itself by repeating the same circular action a second time, and you will have created the straight line. The line is not an infinity of points, it is the intersection of two planes formed by a double self-reflexive action of the circle upon itself.

Note that the bounding condition here is strict. I cannot make an arbitrary fold, which would give just any line. I have covered the maximum area by the minimum action, and thus produced the maximum line (the diameter).

By perfectly fitting the circle on itself, I also discover measure—the power of two. This is interesting, since in visible space, there is no greater division than dividing by half. Did you ever notice that any three-dimensional thing you look at, the maximum you can see is half of it?

Creating the Point

Make a third circular action at a right angle to the other two, by again folding the circle on itself. You have just created the point at the center of the circle! The point is therefore not defined as the 'smallest entity in itself' and is by no means self-evident. The point is the result of a triply self-reflexive action of the circle on itself, the intersection of three planes forming a sphere.

Note that in the first circular action, you could have chosen any size among an infinity of rotations. Similarly, in the second, any fold among an infinity of folds would have given you a diameter. However, in the third case, you can accomplish only one action. The bounding process of these three actions is now at its minimum, or if you wish, at its maximum, which is the same. Thus, the point is the singularity of the minimum/maximum activity of a triply self-reflexive rotation of the circle on itself. This is the foundation of geometry.

Knowing the Universe

The Arithmetic and the Geometric Means

by Pierre Beaudry

In my last column, I began to discuss how geometry by circular action is the mode by which the universe is continuously created. This means that the universe is not discrete in itself, but is continuously developing, growing and creating the means by which it furthers its own development. This world view spans over 2,000 years of fundamental research and discovery initiated by Plato and the Egyptian priesthood of the Temple of Ammon before him, and carried forward through history by such great scientists as Nicholas of Cusa and Leonardo da Vinci during the Italian Renaissance, Monge and Carnot of the 19th century Ecole Polytechnique, and the German school of geometry from Leibniz to Gauss, Steiner, Riemann, and others.

During the course of this series, I shall touch upon key geometrical constructions as developed by these authors and show how their approach has been one of constructing the process of discovery itself. As LaRouche has written on geometry, "You don't know anything until you have constructed it." So if you want to know anything about the universe, get yourself a circle and start folding.

Generating the Circle

In last week's article, I constructed Figure I by a triply self-reflexive action creating four singularities, or discontinuities, by the least action principle of rotation. By the first circular action, I generated the circle; the second rotation created the straight line, and the third produced another line intersecting the first, thus creating the center point.

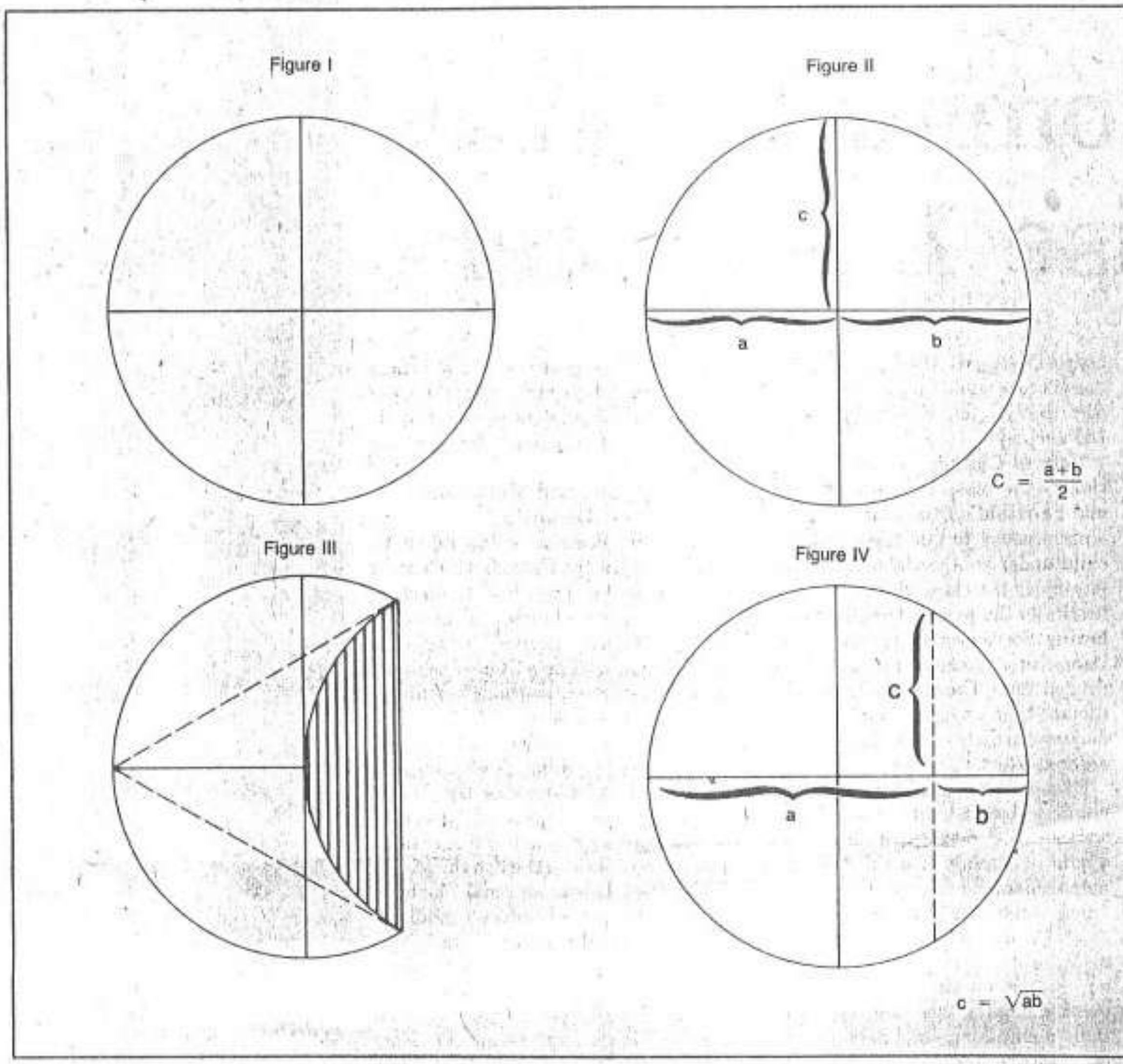


Figure I shows triply self-reflexive circular action. Figure II, and IV demonstrate the arithmetic and geometric mean, while Figure III demonstrates how to derive the equilateral triangle.

The questions the reader should now try to answer are: What is the next (fourth) rigorous self-reflexive action of the circle? And what does it create that didn't exist before? Bear in mind that the last singularity created (the point), which subsumes all previously nested singularities, is the determining factor in achieving the next step.

The Arithmetic Mean

In the Timaeus, Plato describes how God, the Composer of the universe, created all things beautiful by using harmonious proportions. "Two things can only fit together beautifully," he says, " when a bond is created between them by a third. . . the best bond is that which fuses into one both itself and the things which it binds together. . . . Thus, wherever there are any three numbers or surfaces, or volumes that have such a mean that the first is to the mean what the mean is to the last, . . . then they all come to be the same." Let us demonstrate this.

By folding the circle on itself three times, as in Figure II, you can observe that the right angle circular action expresses the power of two (or division by half). This is the arithmetic mean. This mean exceeds the first by the same quantity as that by which it is itself exceeded by the last. Such as in the case of an arithmetic progression like 2,3,4, . . . The mean $3 = 2 + 4/2$. Thus the formulation for the arithmetic mean: $a + b/2$.

The Geometric Mean

Now, I shall answer the question I left unanswered above. Since the last singularity discovered by a triple rotational action of the circle is the central point, such a point will determine the fourth circular action as follows.

Rotate the perimeter of the circle on its center, i.e. by folding half of a diameter on itself. By folding the remaining portions of the perimeter also on the center, and in line with the said diameter, you will have created an equilateral triangle! (See Figure III.) This is the first polygon composed of seven new singularities—three edges, three vertices, and one triangular face. Later, I will show that the equilateral triangle is the "building block" of the entire Euclidian universe, and serves as the basis for constructing the five Platonic solids.

It should be noted here that the formation of the triangle is the only rigorous next step in a continuum of circular action. I stress this because many are tempted to join the four points on the circumference and construct the square. This is a cute trick but it's wrong. It does not comply with the necessity of the least action principle, that is, of a complete action of the entire circle.

By folding the circle on its center, I

have also generated the geometric mean. This has the very unique characteristic of expressing the proportionality of growth in the universe. But let me first consider the formulation for the geometric mean: the square root of a,b (Figure IV). The geometric mean is proportional to the first and the last by the root of the product of their multiplication. Thus, the geometric progression 2, 4, 8, . . . where the mean 4 = the square root of 2 × 8.

In the case of the equilateral triangle, the geometric mean divides the diameter in the proportion of 1/4 where a = 3 and b = 1. By multiplying a by b, the length of the geometric mean c will be the square root of three. This beautiful proportion is such that a/c = c/b. Note that the arithmetic mean in this case is 3 + 1/2 = 2.

In only one case will the geometric mean express what is called a golden proportion. That is the case where the mean will exceed the first, by the same growing proportion as that by which it is exceeded by the last, in such a way that the first plus the mean will be equal to the last. Such is the case of the Fibonacci series 1, 2, 3, 5, 8, 13, 21, . . . etc.

As discovered by Kepler, the universe as a whole is dominated by the golden proportion. In addition, every living organism, as illustrated throughout the works of Leonardo da Vinci is defined by a growth process that converges upon the golden section.

In a future column, I will demonstrate how the arithmetic mean serves as the function of addition in generating arithmetic series, the projection of which comes from the helix on a cylinder, while the geometric mean defines the function of multiplication as it will generate an exponential series through the projection of the spiral on a cone.

Problem of the week: any triangle has one and only one circle circumbscribing it—i.e., touching all three vertices. Given any triangle, how can you construct its circle? Send in your constructions and answers to me in care of the Tribunal.

Knowing the Universe

Cusa and the Infinite

by Pierre Beaudry

This column is for anyone excited by the power of "imparting and receiving profound and impassioned conceptions respecting man and nature." These lessons on constructive geometry do not presume any expertise in mathematics or physics. All questions, comments, and contributions are welcome.

Cardinal Nicolaus of Cusa was one of the greatest thinkers of the 15th century Golden Renaissance. He was an inspiration to many, including Leonardo da Vinci. In his book "On Learned Ignorance," Cusa says: "God has implanted in all things a natural desire to exist with the fullest measure of existence that is compatible with their particular nature." Since our nature is Reason, man's fullest measure of existence is his desire for absolute truth. This noble desire—never fully realized, since only God is absolute—becomes substantial as man becomes more God-like through mastery of the laws of the universe. As an aid, Cusa used geometrical analogies that approximate the infinite—what he termed the absolute maximum.

The human mind tries to reach the infinite by finite means. But there is no gradation from the finite to the infinite. Cusa says, "The relationship of our intellect to the truth is like that of a polygon to a circle; the resemblance to the circle grows with the multiplication of the angles of the polygon . . . but no multiplication, even if it were infinite, of its angles will make the polygon equal to the circle."

The Actual Infinite

How then does Cusa approach the infinite? He begins by relating the straight line to the curved line. He writes: "Now, if the curve of the circumference becomes less curved as the circle expands, the circumference of the absolutely greatest possible circle will be the smallest possible curve; it will be, therefore, absolutely straight. The maximum and the minimum are, therefore, so identified that we most clearly perceive that in the infinite there is the absolute maximum of straightness with the absolute minimum of curve. A study of the accompanying figure will dispel all possible doubt on this point" (see **Figure I**).

"We see that the arc *C-D* of the larger circle is less curved than the arc of *E-F* of the smaller circle, and that *E-F* is itself less curved than the arc *G-H* of a still smaller circle; the straight line *A-B* will therefore be the arc of the greatest possible circle."

Note also that if you proceed inversely toward the absolute maximum curve, it will become the point of tangency of all the circles. Thus the point and the sphere coincide!

Cusa shows that by circular action, an infinite line can also be transformed into a triangle, a circle, and a sphere (see **Figure II**). He writes: "If then we have a line *A-B*, and if while the point A remains fixed, the line is moved till it reaches *C*, a triangle is described; if the movement of that line is continued till it returns to *B*, a circle is described. With *A* still fixed, let us suppose *B* again moves till it reaches *D*, which is directly opposite the initial point *B*; *A-B* and *A-D* form one continuous line and a semicircle is described. Let us suppose next that the diameter *B-D* is fixed, and that the semicircle is turned completely round; we have then a sphere, and the sphere is the ultimate and total actualization of a line, for no more perfect figure is able to be produced from the sphere.

"If therefore, the finite line is potentially all the above figures and if in the infinite line all the potentialities of the finite line are actualized, then it follows that the infinite line is a triangle, a circle and a sphere. . . ." It is also a beautiful proof of the existence of God!

The Point of Infinite Similitude

Let us now reflect on these fundamentals. In our first two lessons, we created the plane, the line, the point, and the equilateral triangle, by circular action alone. Now, these four circular actions do not simply accumulate onto one another indefinitely. At the moment of forming the point as the center of a triply self-reflexive action of the circle on itself (**Figure III**), something happens which deserves further consideration.

First, when talking of infinity, we must avoid a bad infinite, which occurs when we attempt to find a gradation between the finite and the infinite. For example, what is the greatest possible number of the series of regular integers, 1,2,3,4,5 . . .? I know that no matter how large a number, it is only one finite number added to the previous series, and I can add another finite unit to it.

And there is the question of the infinite potential of circular action. What do I do with what is left out as I choose one circular action, and not another? For instance, in the first action that produces the circle, I have left out an infinity of larger or smaller circles. Similarly, in the action of folding the circle on itself to create the line, I could not retain the potential of an infinity of diameters.

However, a third circular action of folding half the circle onto itself, creating a central point, is the only possible action, and no potentiality is left out, either. It is seemingly a dead end—but:

Since an infinity of diameters touching the surface of the sphere (even pos-

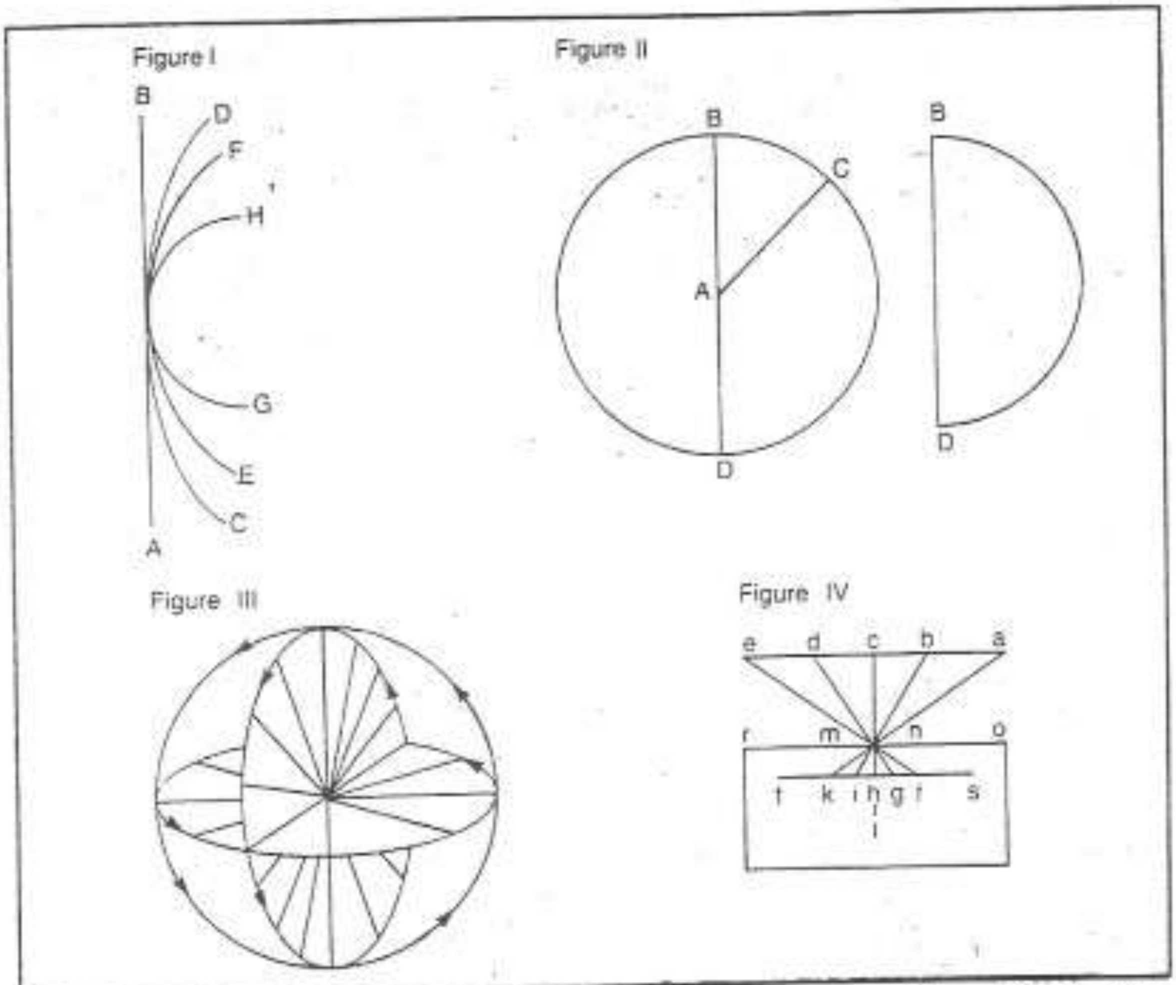


Fig. I shows the absolute maximum of straightness—approaching infinity—has the absolute minimum of curve. Fig. II shows how an infinite line can be transformed into a triangle, circle, and sphere. Figure III is the triply self-reflexive action of the circle on itself, forming a sphere. Fig. IV shows how infinity is concentrated in a single point, in this schematic of a camera, 500 years before it was invented!

sibly an infinite sphere) all go through that one point, there must be something very special about it. That single point contains, in itself, the infinity of space encompassed by the sphere. Amazingly, I have condensed all of infinity in one point! One moment I was confined to one and only one action, and now, by that very action, I have captured an actual infinite. The sphere and the point are identical, since any other point in the universe projected, even from the bad infinity of the plane, could be mapped through one and the other.

This may seem outrageous, but it's true, because the central point in the sphere acts as a point of infinite accumulation, or better, of a point of infinite similitude. Thus, as a singularity of infinite radial projection, this center point is an **actual infinite**—all potentiality is realized within it. This is the concept of transfinite that Georg Cantor, in the 19th century, developed from this idea of Cusa.

Da Vinci and Infinite Perspective

Leonardo da Vinci applied this conception to the study of light propagation, human vision, and perspective (see **Figure IV**). Leonardo wrote in his notebooks, "An experiment, showing how objects transmit their images or pictures, intersecting within the eye in the crystalline humor, is seen when some small round hole penetrates the images of illuminated objects into a very dark chamber. Then, receive these images on a white paper placed within this dark room, and rather near to the whole, and you will see all the objects on the paper in their proper forms and colors, but much smaller; and they will be upside down by reason of that very intersection. . . . Let *a,b,c,d,e,* be the object illuminated by the sun and *o,r,* the front of the dark chamber in which is the said hole at *n,m*. Let *s,t,* be the sheet of paper intercepting the rays of the images of these objects upside down. Because of the rays being straight, *a* on the right hand becomes *k* on the left, and *e* on the left becomes *f* on the right; and the same takes place inside the pupil."

You have just read a precise description of how the color camera works—500 years before it was invented!

Knowing the Universe

The Trinity and The Triangle–Part 1

by Pierre Beaudry

Last week, we raised the question of the infinite as it was discussed by the 15th-century scientist, Cardinal Nicolaus of Cusa. He showed in his book "On Learned Ignorance" that the absolute maximum, or God, can be conceived through geometrical analogies, such as the absolute maximum line—one which becomes straighter as the circle it is part of becomes infinitely larger—becoming the triangle, the circle and the sphere. Now, I will show how Cusa conceived the maximum line as the absolute maximum triangle, to exemplify the Holy Trinity.

The contradictory aspects of a straight line which is a circle, or a line which is a triangle, show the limitations of an imagination used to grasping finite things—shapes, sizes, lengths, in themselves. But our intellect can grasp an infinite maximum line, that is be simultaneously an infinite triangle.

Infinite Triangle

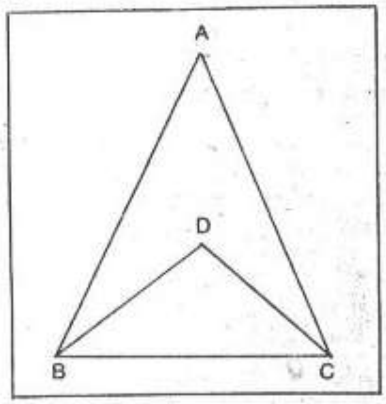
Cusa shows this in the following way (**See figure**): "Any two lines of a triangle in the sensible order, when joined together, are greater than the third line, the more the angle they form becomes acute.... [less than 90 degrees]. The lines B-A and A-C when joined together are much longer than B-C because the angle B-A-C, is more acute. Inversely, the greater the angle, B-D-C, for example, the less B-D and D-C when joined, exceed B-C in length, and the smaller the surface is. If then, we were to suppose that the angle were one of 180 degrees, the entire triangle would be reduced to a simple line."

"In consequence," concludes Cusa, "this supposition, though unrealizable in the material order, can assist you in your mounting to the intelligible order, where what is impossible in the material order is seen not only as a possibility but as an absolute necessity." In reality, or in the actual infinite, which is the same, the line and the triangle could never be contradictory or mutually exclusive.

In his *Timaeus* dialogue, Plato addresses this very directly. He says that whenever you are looking at an ephemeral object, you should never refer to it as "this" or "that" but as "thus" (such-like). The reason for this is simple: Things in themselves have no permanence, they are always subject to the process of becoming. The question therefore should not be "What is it?," but "How has this become what it is, and how can it become something else?"

So there is only an apparent contradiction in saying that there are three Persons in One God. Cusa makes this point very clear. "From our previous consideration," he writes, "we know, insofar as it is humanly possible, the true triangle and the infinite line; from this knowledge then, we shall, in learned ignorance, acquire a knowledge of the Trinity. For we see how the infinite triangle differs from the finite triangles; in finite triangles we find one angle, then another and finally a third; and because these angles are really distinct from one another, they can only form in the unity of the triangle a unity of composition; whereas in the infinite triangle we find one angle which is three without being numerically multiplied. The most learned Augustine, on that account, justly remarks: 'From the moment you begin to count the Trinity, you depart from the truth.'"

This also encompasses the idea expressed by St. Augustine and Cusa of the consubstantiality of the Trinity. The Spirit (universal law, the Logos) proceeds from both the Father and from the Son (Filioque), making the Son as well as the Father recipients and instruments of infinite love, the Spirit. Every human being, made in God's image, is the recipient of this infinite love as well; and each individual human must foster further progress of humanity, as he himself moves toward this infinite.



This triangle shows that the more acute the angle, the greater the length of any two sides of a triangle. Thus, the length of B-A-C, whose angle BAC is acute, is longer than B-D-C, whose angle BDC is less acute.

Problem of the week: There are five and only five Platonic solids: The Tetrahedron (4 triangles); the Octahedron (8 triangles); the Cube (6 squares); the Icosahedron (20 triangles); the Dodecahedron (12 pentagons). How can you construct all five solids by circular action of the equilateral triangle? Send your solutions, and any other comments to me in care of the Tribunal.

Knowing the Universe

How to Circumscribe a Triangle

by Pierre Beaudry

This column is for anyone excited by the power of "imparting and receiving profound and impassioned conceptions respecting man and nature." These lessons in constructive geometry do not presume expertise in mathematics or physics. All questions, comments, and contributions are welcome. Please send them to this author, in care of the Tribunal.

This week, I would like to bring forth the geometric problem we posed in the Tribunal (Vol. 1, No. 3), and solve it. The problem was: Any triangle has one and only one circle circumscribing it—i.e., touching all three vertices. Given any triangle, how can you construct its circle?

You can easily discover how to do this if you think through the implications of creating a straight line by circular action in the manner of Cusa. (The straight line is created simply by folding the circle perfectly on itself.)

In order to find the truth about any phenomenon, you must begin by hypothesizing. Our general hypothesis is that everything created has been created by circular action. From this standpoint, we consider the creation of the triangle as the result of circular action and also as an instrument for further circular action. If this be true, then the question must be: How many forms of circular action can the triangle generate? And how can this be decided?

Finding Singularities

Well, circular action always leaves traces of itself behind, so to speak, after it has accomplished work. We call these traces singularities. They are the clues that guide us toward discovery of something new. In the case of the triangle, the traces or singularities, of previous circular actions are already there. There are three vertices, three sides and one plane triangular surface. This is all we know about the triangle; but this is enough to find what we are looking for.

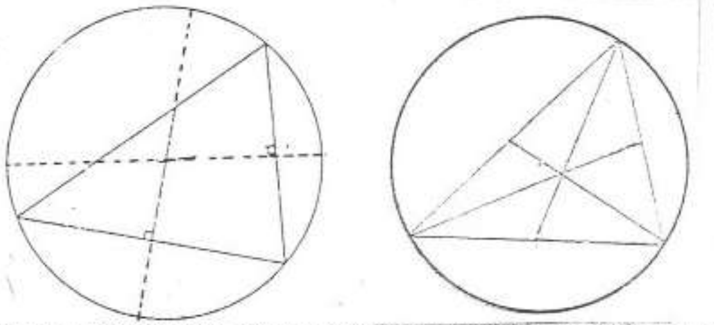


Figure 1: Fold the edges of any triangle one onto another, to bisect all three angles. The intersection is the geometric center of the triangle. The center of the circumscribing circle is obtained in Figure 2: By folding the vertices of the triangle one onto the other, two by two, the intersection is the center of the circle.

The next step is to ask how many types of circular actions can I produce with a triangle? A glance at the number of singularities will tell me there are three sets of three circular actions each. These are: 1) folding triply all vertices onto one another, two by two; 2) folding triply all edges (sides) onto one another, also two by two; and 3) folding triply the face of triangle on itself or on its center.

Proving Our Hypothesis

Let's start here and see where we end up. First draw a triangle on a piece of paper and cut it out. Now, let's apply these three circular actions. If we begin with the last case and work our way back, we see that we cannot fold the triangle on itself just yet because we have no center of the triangle, or any other singularity to guide ourselves. So we must investigate the edges and the vertices.

Take then the second case, and fold the edges onto one another. What do you find? You will discover that the folds bisect all three ~~angles~~ bases and that their intersection will be the geometric center of the triangle. (**Figure 1**) Is this the center of the circle I am looking for? No, because the bisectors (folds) dropped from the vertices to the center of the triangle are of different lengths. Well I only have one hypothesis left, and that is to fold the vertices on themselves.

Take all three vertices of the triangle (**Figure 2**), and fold them one onto the others, two by two. Each perpendicular fold will be the mid-point or the arithmetic mean of each side. The intersection of the three folds will be the center of the circle you are looking for. Following the same procedure, you can find the center of any given circle by rotating any two chords, each one on itself. The perpendicular bisector of any chord of a circle passes through its center.

This little problem involves much more than we can cover in one weekly column, so we shall pursue it next week. Meanwhile, the problem for next week is: given any triangle, find one circle where all perpendiculars and bisectors coincide. By the way, Mark Fairchild, the Democratic nominee for Illinois Lieutenant Governor, is the only person who sent in a correct solution to today's problem.

correction: Fold the edges so that you connect the vertices to the mid-points of the bases.

Knowing the Universe

Triply Folding the Triangle

by Pierre Beaudry

In our last column, we showed how folding the vertices of the triangle one upon another gives you the center of a circle that circumscribes the triangle. This shows that all triangles are related to the circle, as folding is circular action. However, this construction reveals another very interesting characteristic. If you join the center of the circumscribed circle to each of the three vertices, as in **Figure 1**, you will discover that any triangle will be composed of three isosceles triangles (a triangle with 2 equal sides). The reason is that the distance from the vertices to the center corresponds to the radius of the circle.

Note that an angle is not simply the space between two lines coming together. An angle is the result of the power of two, rotating one line onto another. Now look at **Figure 2.** If you join together the mid-points A,B,C, you will form an inscribed triangle which is a reversed mirror image of the larger. The circle circumscribing the inner triangle will be half the size of the larger one. Again, this demonstrates the constancy of the power of two in circular action.

The Equilateral Triangle

A third kind of circular action of the triangle is formed by folding an equilateral triangle on itself. By folding triply the entire face of the triangle on itself, you have created four self-similar equilateral triangles. Note that the perpendiculars and the bisectors are identical, as opposed to the previous cases, and that the center of the circle is also the geometric center of the triangle. By folding each side onto another, the bisectors/perpendiculars create three isosceles triangles which in turn create six smaller triangles, each of which has a hypotenuse twice the size of the smaller side. You can readily see here how all sorts of triangles can be created from one another.

The equilateral triangle is unique, in that it is the universal model. In fact, if an equilateral triangle is inscribed onto a sphere, touching its three vertices, you can turn that sphere in any direction and you will get an infinity of all possible triangles. Thus, all triangles are equilateral! Another way to do this is to conceive that an infinity of equilateral triangles can be generated by circular action, each of which can be viewed as merely one of an infinity of vertical cuts through the apex of an "equilateral cone" inscribed in a sphere.

We have seen that in the equilateral triangle, the bisectors of the angles are identical to the perpendiculars, and the center of the circumscribed and inscribed circles are the same as the geometric center of the triangle itself. These three characteristics create unity, equality, and connection within the equilateral triangle.

Why can't this be the case for any other triangle? How, in any triangle, can the discrepancies of the bisectors and perpendiculars be unified? In other words, how can you make the perpendiculars and the bisectors coincide in any triangle?

Poncelet and Brianchon Theorem

There is a beautiful theorem by Poncelet and Brianchon (**Figure 3**) that shows how you can synthesize these differences. If you find a circle of an inscribed triangle, touching its three vertices, which are also the intersection points of the perpendiculars dropped from the vertices of the circumscribed triangle to its sides, then you have found the circle through which all bisectors and perpendiculars coincide. To construct this you fold each side of the triangle on itself such that the perpendicular folds are in line with each of their opposite vertices (which are not bisectors). This will give you an inscribed triangle P,Q,R. You can easily see that points P,Q,R, lie on the same circle as points J,K,L, the mid-points (bisector points) of each side! The circle also goes through points A',B',C' which are also the mid-points of the segments AD,BD,CD. The two circles are also in the proportion of two to one.

Again, this shows the infinite synthetic power of circular action using the power of two. Thus, for any triangle, all perpendiculars and bisectors coincide in one and the same circle, which is the inscribed circle of the equilateral triangle.

Figure 2

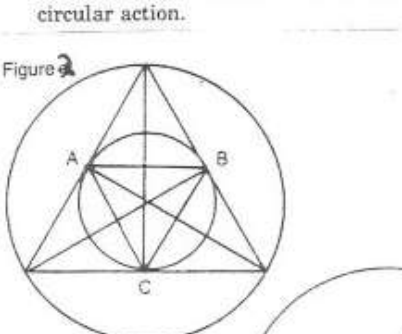
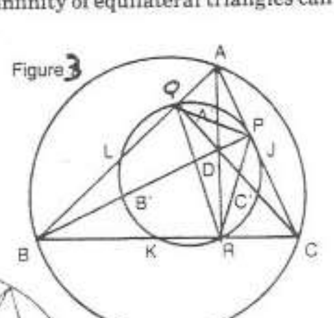


Figure 3



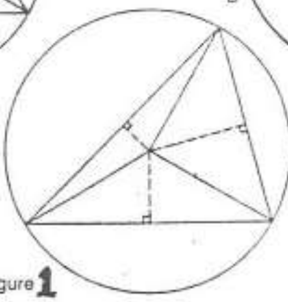
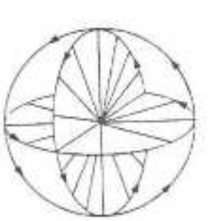


Figure 1

Figure 1 shows any circumscribed triangle. If the center of the circle is joined to each of the three vertices of the triangle, the triangle will be composed of three isosceles triangles. ***Figure 2*** *shows four self-similar triangles formed by triply folding the face of the equilateral triangle on itself. The perpendiculars and bisectors are identical. In* ***Figure 3,*** *we see a circle of an inscribed triangle, touching all vertices, whose intersection points are perpendiculars from the vertices.*



Knowing the Universe

Isoperimetric Theorem In The Triangle

by Pierre Beaudry

This column is for anyone who is excited by the power of "imparting and receiving profound and impassioned conceptions respecting man and nature." These lessons in constructive geometry do not presume expertise in mathematics or physics. All questions, comments, and contributions are welcome. Please send them to the author, in care of the Tribunal.

We have discovered previously that all the singularities of a triangle (lines, points, surface) are constructed by the perimetric actions of two circles—one circumscribed and the other inscribed.

I shall now begin to demonstrate that, first, in relation to one another, these two circles are the absolute maximum and absolute minimum circles of all polygons; and second, that the circumscribed and inscribed circles of any polygon of the same perimeter, converge upon the golden section or the isoperimetric circle of the equilateral triangle.

The maximum-minimum theorem, better known today as the isoperimetric theorem, was discovered by Cardinal Nicolaus of Cusa in the 15th century. This revolutionary idea helped to launch the Golden Renaissance, which saw that science is inseparable from art; and this theorem served as the foundation for Gottfried Leibniz's Principle of Least Action.

In the first article in this series, I showed that the maximum-minimum principle defines the circle as the maximum area enclosed by the minimum perimeter. (In economic terms, this means the minimum amount of activity required for the maximum amount of productive work demonstrated by advances in technology.)

The Absolute Maximum-Minimum

Nicolaus of Cusa polemicizes that the foundation of geometry is the Augustinian conception of the Holy Trinity! Everything in the universe is a trace or a reflection of a Trinitarian God. "On the subject of the ever-blessed Trinity," writes Cusa in his book On Learned Ignorance, "there is a point that is well worth noting, which is that the maximum is a trinity and that anything beyond a trinity in the maximum, e.g. 4, 5, or more, would be in contradiction with its simplicity and perfection. The simplest figure to which any polygon can be reduced is the triangle, which is, in fact, the smallest polygon that can exist. . . .

"Therefore, what unity is in numbers, the triangle is in polygons, and just as every number is reduced to unity, so polygons are reduced to the triangle. The maximum triangle, therefore, with which the minimum coincides, compresses all polygons, for the infinite triangle is to all polygons as infinite unity is to all numbers."

What Cusa identifies here is truly fundamental. He is not considering any finite triangle as such, but rather circular creative action as trinitarian action in the universe as a whole. In other words, a triangle is nothing in itself but traces of minimally and maximally triple self-reflexive circular action.

If you examine **Figure 1,** you will discover that the equilateral triangle inscribed in a circle represents the smallest possible area of any inscribed polygon of the same perimeter (isoperimetry). Its circumscribed and inscribed circles determine the maximum and the minimum in the proportion of 2/1.

The Isoperimetric Triangle and Square

To prove that no other polygon can express the unity of the maximum and the minimum as the triangle does, develop the following experiment. Take a string, connect the two ends and form an equilateral triangle that you will have traced on a flat piece of paper. Draw the circumscribed and inscribed circles. Then take the same string and make a square out of it which is to overlap the triangle, but without touching its circumscribed circle. You will discover that the square will have a smaller circumscribed circle, and a larger inscribed circle than that of the triangle! (**Figure 2**) The same holds true for any other polygon!

The Isoperimetric Circle

Note that if you compose, with the same string perimeter, a series of polygons of five sides, six sides, and so forth, their respective circumscribed circles will become smaller and smaller, while their inscribed circles will become larger and larger. These two series of circles will tend toward one unique circle, which is called the isoperimetric circle of the equilateral triangle.

Cusa wrote a wonderful treatise called "On Geometric Transformations" where he develops this theorem. (**Figure 3**) "Polygonal figures . . . when they have the same perimeter, given equal sides, are called isoperimetric. As is well known, of all the isoperimetric figures, the triangle has the smallest area. And because the more angles the isoperimetric figure has, the more area it encloses, the circle has the greatest area of all isoperimetric figures. One cannot get it by multiplying the angles, just as one can also not get a maximum with a number. No polygon can have a rational ratio to the isoperimetric circle.

"But because the difference in surface areas of isoperimetric figures corresponds to the difference of the radii of their inscribed circles (which was already previously known), then neither the inscribed circle, which is smaller, nor the circumscribed circle, which is larger, will have a rational ratio to the isoperimetric circle. The difference in the radii of the aforesaid circles is greatest with a triangle, and it becomes smaller, step by step, with other polygons. The radii coincide, given an isoperimetric circle, because the inscribed circle, the circumscribed circle, and the circle itself coincide here. So what is to be investigated is the art through which we can succeed in establishing coincidence and in reaching our goal."

Part of the goal is to construct a proof of this. But a problem arises here which shows again the limitations of our finite imagination. If we were to attempt the foolish task of constructing the largest possible polygon, it would never be any closer to the circle. In fact, the more you go in that direction, the farther away you are from the circle. There can be no rational ratio between a straight line and a curved line.

Cusa's construction for the proof of the isoperimetric circle avoids this trap of attempting to "square the circle," and establishes a proportional relationship between the radius of the circle and the side of the triangle. He writes, "The radius of the isoperimetric circle to this triangle has a ratio to the line, that connects the midpoint of the circumscribed circle to the point that divides the triangle's side in quarters, as five does to four." This proof also establishes that the isoperimetric circle is a "golden circle"; its radius is in a golden-mean proportion (5/3) with the radius of the inscribed circle of the triangle.

Correction: Due to a pasteup error, last week's figures were in the wrong order. Respectively, Figures 1, 2, 3 should have been 2, 3, 1. In Figure 3, singularity Q should appear on fold C,D,Q, and point I should have been J. We apologize for any confusion we may have caused our readers.

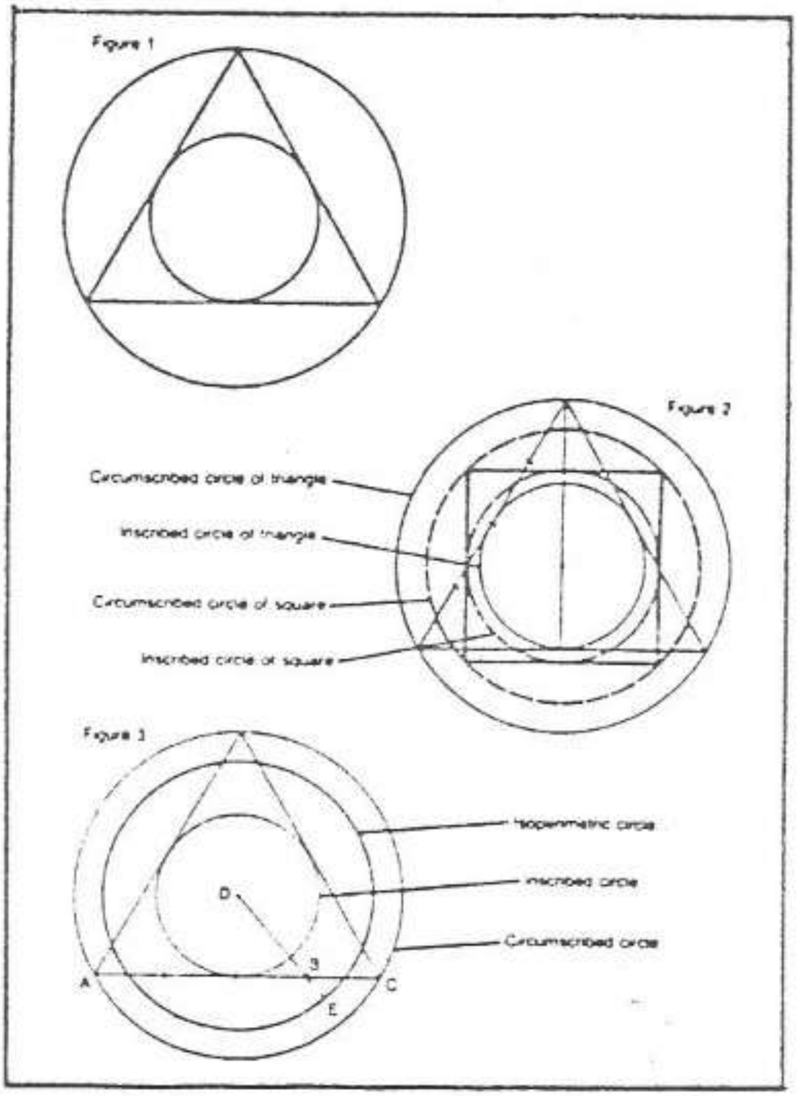


***Figure 1** shows the equilateral triangle inscribed in a circle, the smallest possible area of any inscribed polygon of the same perimeter (isoperimetry). Its maximum and minimum are in the proportion of 2/1. **Figure 2** shows the circumscribed and inscribed circles of the equilateral triangle are a maximum and minimum as compared to the corresponding circles for a square (dotted lines). **Figure 3** shows Cusa's construction of the isoperimetric circle for an equilateral triangle: $BC/AC = 1/4$. $BE/BD = 1/5$.*

Knowing the Universe

Construct the Isoperimetric Theorem

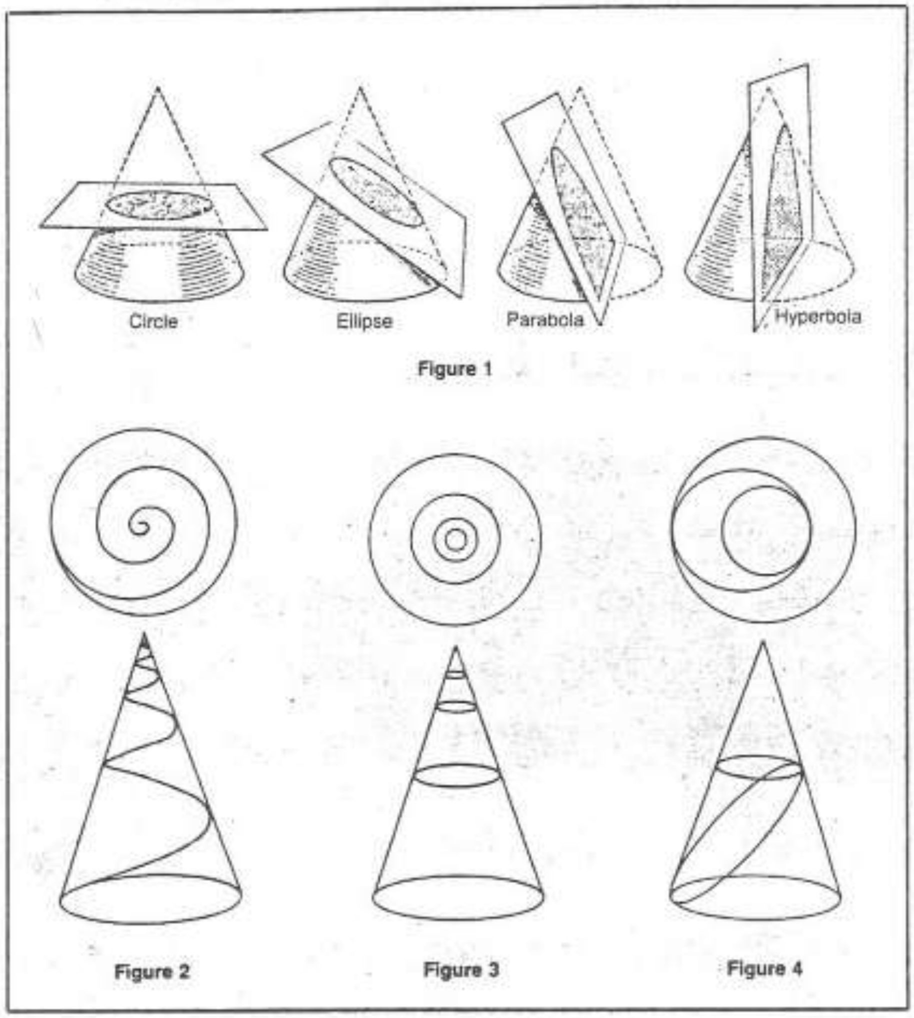
by Pierre Beaudry

This is one of a series that first appeared in the Illinois Tribunal. For the interest of Michigan Tribunal readers, the previous parts of this series will be reprinted in this paper, in coming weeks.

1. Conical Self-Similar Spiral Action. In this series, I have developed a constructive, synthetic geometry, based on simple triple circular action associated with the hereditary principle of least action, as discovered by Cardinal Nicolaus of Cusa during the Golden Renaissance. Such a principle, called by Cusa, the **Maximum/Minimum Principle,** is the very foundation of constructive geometry. And for reasons which will become more and more evident as we go along, this principle is incompatible and irreconcilable with Euclidean deductive geometry, which is based on unjustifiable assumptions of axiomatic arithmetic and algebra.

More specifically, and without the burden of measurement, we have discovered in past weeks that, by circular action alone (folding): 1) the absolute minimum polygon is the triangle; 2) the circumscribed and inscribed circles of the triangle are the absolute maximum and minimum circles; 3) the maximum and minimum circles encompass all polygons whose circumscribed and inscribed circles converge upon a "golden circle" called the isoperimetric circle.

In our last column, I have shown how Cusa establishes this as the Isoperimetric Theorem, but I have not given a construction to prove it. I will now give the reader all that is needed to conceive and elaborate a synthetic model for an actual construction of Cusa's isoperimetric circle. Such a model, which I will construct in coming weeks, shall demonstrate that polygonal singularities generated so far by simple triple self-reflexive action of the circle find their "raison d'etre" ultimately in the higher geometry of the complex plane generated by triple conical self-similar spiral action.



2. Plato's Cave. In Book Seven of The Republic, Plato shows how the human mind must free itself from the illusions of the shadows of reality and turn toward the reality which is casting them. He asks us to "picture men dwelling in a cave with a long entrance open to the light on its entire width. Conceive them as having their their legs and necks fettered from childhood, so they remain in the same spot, able to look forward only.... [P]icture then a light burning higher up at the entrance behind them ... projecting on the wall of the cave the shadows from the reality outside.... Then in every way such prisoners would deem reality to be nothing else than the shadows of the artificial objects." Plato's myth of the cave is to-

Figure 1. The Conic Sections. Parallel projections of a right cone onto a movable plane, and rotating the plane, gives shadows of a circle, ellipse, parabola, and hyperbola.

Figure 2. The cone is generated by a continuously expanding rotational action around a center moving in a straight line, creating a logarithmic spiral.

Figure 3. Perpendicular cuts through the cone, where the spiral has accomplished a full rotation, generates a series of circles whose projections on the plane forms shadows of a series of concentric circles.

Figure 4. At the intersection of a tilted plane cutting the cone from one level circle to a smaller level circle, the ellipse serves as a sort of mean between these two circles. In the plane, the shadows of the two circles appear concentric, while the ellipse overlaps them at the two extreme points of the minimum and maximum circles.

tally appropriate for the subject at hand. Lines, points, polygons, circles, etc. are mere shadows of a higher reality which must be sought.

However, when the prisoner is compelled to turn around toward the source of the light, no matter how painful this may be, he will be able to discern that the shadows he formerly saw were, in fact, mere illusions; but now that he has turned to a reality which is closer to the truth, he is able to return to the cave and free others from their illusions.

I have repeated many times now, that in geometry the mind must conceive reality from the standpoint of triple least circular action. Such least action is never expressed by a straight line action—as a pathway between two points—but by a curved action acting on the straight line at a right angle. Only in such a case will the curved action be minimal while the straight line produced by it will be maximal, the two converging toward the unity of the maximum with the minimum.

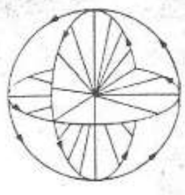
3. The Conic Sections. Imagine, first, the parallel projection of a right cone onto a movable plane. By rotating this plane, as in **Figure 1**, the projection will form, successively, the shadows of a circle, an ellipse, a parabola, and of a hyperbola. All of these cuts belong to different families of curve, but all are generated by a continuous circular action.

First, the circular action that creates the cone itself is called the spiral. As you can see in **Figure 2**, the cone is generated by a continuously expanding rotational action around a center which is moving in a straight line. For the purpose of our eventual construction of the isoperimetric theorem, the reader should consider here only the relationship of the spiral to the circles and the ellipse. The specific spiral I am using is called the logarithmic spiral, which makes a full revolution every time it reaches the halfway mark on the axis of the cone. The reason for this is, that the ratio of the maximum and minimum circles I am looking for has to be in the proportion of two to one.

Second, observe that when you follow the path of the spiral, going up the cone, you can make perpendicular cuts through the cone where the spiral has accomplished a full rotation. This will generate a series of circles whose projections on the plane will form shadows of a series of concentric circles (**Figure 3**). The shadow of the spiral in the plane also shows that it is the self-similar spiral action that connects all of these circles.

Third, the ellipse has a very special relationship to the circle in the sense that it connects two circles at different levels on the cone. In **Figure 4**, note that in the intersection of a tilted plane cutting the cone from one level circle to a smaller level circle, the ellipse serves as a sort of mean between the two circles. In the plane, the shadows of the two circles appear concentric while the ellipse overlaps them at the two extreme points of the minimum and maximum circles.

The reader now has all that is necessary to conceive and construct the Isoperimetric Theorem of Cusa. The problem of the week is thus the following. All polygonal figures of the same perimeter, and given equal sides, are called isoperimetric, and will all converge toward a circle where the minimum and the maximum (inscribed and circumscribed) circles of the equilateral triangle coincide. Demonstrate by constructing a conic projection how you can find an isoperimetric circle in the plane.



Knowing the Universe

Model for the Isoperimetric Theorem

Pierre Beaudry

Modern science began with the work of Nicolaus of Cusa. In 1440, Cusa published "On Learned Ignorance," where he presented a discovery which modern science calls the Principle of Least Action, and which mathematicians call the "isoperimetric theorem." Lyndon LaRouche has noted, "Cusa presented a way of thinking about physics, which set the stage for the later work of such leading figures as Leonardo da Vinci, Kepler, and Leibniz. Every step of fundamental progress in experimental science since, has centered around discovering mistakes, called 'anomalies,' in generally accepted scientific doctrine." This week, we shall show how to construct a geometric model to resolve and supersede a grave anomaly discussed by Cusa.

The Fundamental Anomaly

First, let me caution the reader about three essential aspects of anomalies. An anomaly should first, embarrass the human mind's pretension of ever achieving absolute knowledge about the truê universe. Second, anomalies should force the human mind to reconsider entirely the underlying assumptions that led to the crisis-point and change the parameters that led it to that point. Third, anomalies occasion optimism, because they provide opportunities for new breakthroughs in science to improve the human condition. That is why the reason behind such a compelling exercise must be nothing else but the love for the Good.

In our last column, I stated the problem this way: all polygonal figures of the same perimeter (isoperimetric), and with equal sides, will converge toward an isoperimetric circle where the maximum and the minimum will coincide. However, as Cusa states, identifying the anomaly, "The relationship of our intellect to the truth is like that of a polygon to a circle: the resemblance to the circle seems to grow with the multiplication of the angles of the polygon. . . . But no multiplication, even if it were infinite, of its angles will make the polygon equal to the circle."

Our mind here seems caught in contradictions. How is it possible that a straight line coincides with a curved line? How could a polygon ever become a circle? Can these very embarrassing questions be solved? Cusa directs us where to seek the answers. He says that "because no rational ratio can exist between these magnitudes [straight and curved lines], the secret here must hide itself in a coincidence of the extremes. And because this coincidence occurs in the maximum . . . it must be sought for in the minimum—that is in the triangle."

Equilateral-Isoperimetric Action

The **minimum** is always expressed by the closure of perimetric action (creating a boundary or perimeter) while the **maximum** is the enclosed area of confinement. The perimetric action of the circle which produces the equilateral triangle (see **Figure 1**), by folding the circumference of the circle onto its center, divides the whole area of the circle into two enclosed areas: One is the minimum area inscribed by the triangle, as defined by the inscribed circle; the other is the maximum area circumscribed by the circle. The ratio of the maximum and minimum circles is 2/1.

Figure 2 shows that when you inscribe a square whose perimeter is the same as the triangle, the inscribed circle of the square is larger than the triangle's inscribed circle, and the square's circumscribed circle is smaller than the triangle's circumscribed circle! By multiplying the sides of our initial triangle, using the same perimeter, the reader can imagine that an iterative (repeated) series of inscribed circles of the polygons would grow larger toward the maximum, while an iterative process of circumscribed circles would grow smaller toward the minimum.

Somewhere, between the minimum and the maximum, we know by reason that these two interdependent, iterative progressions must coincide in a limit circle. But we also know that it is impossible for a polygon to ever become a circle. So what is the nature of the problem here?

First, we have postulated the actual existence of the absolute maximum polygon. This absurdity lies in two fundamentally wrong underlying assumptions. One is that straight-line action is primary in the universe. Second, straight lines are presumed to be composed of an infinity of points. Hence, the sides of an infinitely large polygon must be so small that they become points forming the circle itself. This is the insanity of considering things in themselves, such as lines and points, and considering them to be self-evident. We are ready to root out such assumptions!

Let us then change this false premise of "straight line/point measure" and resolve the anomaly by a new measure which is more primary: conical self-similar spiral action.

Isoperimetric Spiral Action

Consider that concentric circles in the plane are shadows resulting from triple self-similar spiral-action on the cone. The power of two expressed by the proportion of 2/1 in the ratio of the inscribed and circumscribed circles of the equilateral triangle in the plane is the result of the action of the spiral accomplishing one full revolution by going halfway up the cone, as we showed in our last column. The beginning and the end levels of the spiral action locate two circular cuts, A and B, on the cone (see **Figure 4**). If we now replace the "measure" of equilateral isoperimetric action by isoperimetric spiral action, we shall begin to solve our problem, and be able to construct a model for the isoperimetric theorem.

You can observe that the difference in surface area of isoperimetric polygons depends on the difference in area of the radii of inscribed circles.

In turn, the difference in area defined by inscribed and circumscribed circles, which decreases with each step of a new polygon, is caused by a corresponding decrease in the amount of spiral action going up the cone. This can be measured by the range of an ellipse acting as a mean between two circles. Thus, the construction of such a theorem is possible by generating the iterative process of a family of ellipses in the cone where each ellipse becomes the mean between the inscribed and circumscribed circles of each isoperimetric polygon. We shall therefore replace the polygons by ellipses! The entire family of "isoperimetric ellipses" will converge in a continuous fashion towards a limit circle in the cone, which is the isoperimetric circle!

You can do this by constructing any cone (here an equilateral cone), and projecting the diameters of the circles upward until the parallel projections meet the surface of the cone. In order to show this (Figure 3) it is sufficient to simply draw a frontal elevation of the cone and locate the key intersection points and lines that are generated from the circular plane.

From these points of connection with the cone, draw cross sections perpendicular to the axis of the cone, as in A and B. These respective cuts represent the inscribed (A) and circumscribed (B) circles of the equilateral triangle. If you join the extreme points of each set of projections from the inscribed and circumscribed circles, by oblique lines (such as line A,B), this will generate as many ellipses across the cone. All of these elliptical cuts will cross one another at different points in the cone and will end their rotational action in the circle C,D. The level circle (C,D) at that point of connection with the ellipse of the equilateral triangle at (F) locates the isoperimetric circle!

If you trace that circular cut through the cone and then project it back down, you can draw the isoperimetric circle of the equilateral triangle onto the plane. Figure 4 shows you can also do this by projecting directly the perimeter of the inscribed circle from the side of the equilateral triangle up into the cone and locate immediately point (F) as the second focus of ellipse A,B. Point (O) is the first focus.

The following set of figures have been computer generated by Mark Fairchild, Democratic Lieutenant-Governor nominee of Illinois.

FIGURE 1-- Shows the circumscribed and inscribed circles of the equilateral triangle. Their ratio, two over one, represents the absolute maxiumum and absolute minimum of all isoperimetric polygons.

FIGURE 2-- Shows that, given the same perimeter, any polygon such as the square, will have a larger inscribed circle and a smalled circumscribed circle than those of the equilateral triangle. The inscribed circles of any series of polygons will tend toward the minimum.

FIGURE 3-- The concentric circles in the plane are the inscribed and circumscribed circles of the equilateral triangle, the hexagon, and the dodecahedron. Their respective diameters are projected up onto an equilateral cone. The diagonal cuts such as A, B, are elliptical cuts connecting respective sets of inscribed and circumscribed circles. All the ellipses rotate at different rates until they reach their limit in circle C, D, the isoperimetric circle where the maximum and the minimum coincide.

FIGURE 4-- The isoperimetric circle of the equilateral triangle can be otherwise obtained by a very close approximation (the difference between .6045 and .6000 in the conic elevations of the isoperimetric circle and that of the circular cut passing through the second focus (F) of the ellipse, respectively.) To obtain this, just project the radius of the inscribed circle up into the cone to focus F of ellipse A, B. At that level, the circular cut C, D, can be projected back down onto the plane as a good approximation of the isoperimetric circle.

Figure 1

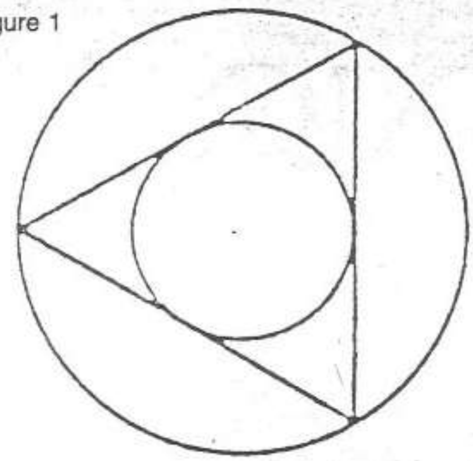


Figure 2

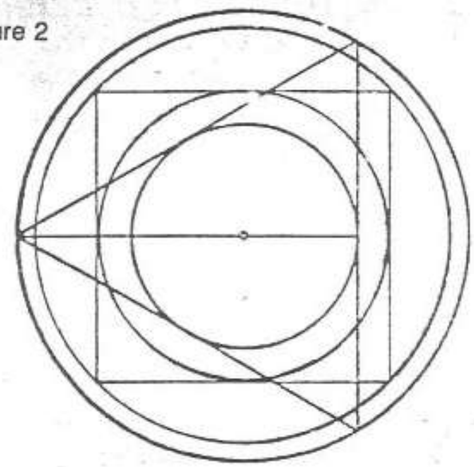


Figure 3

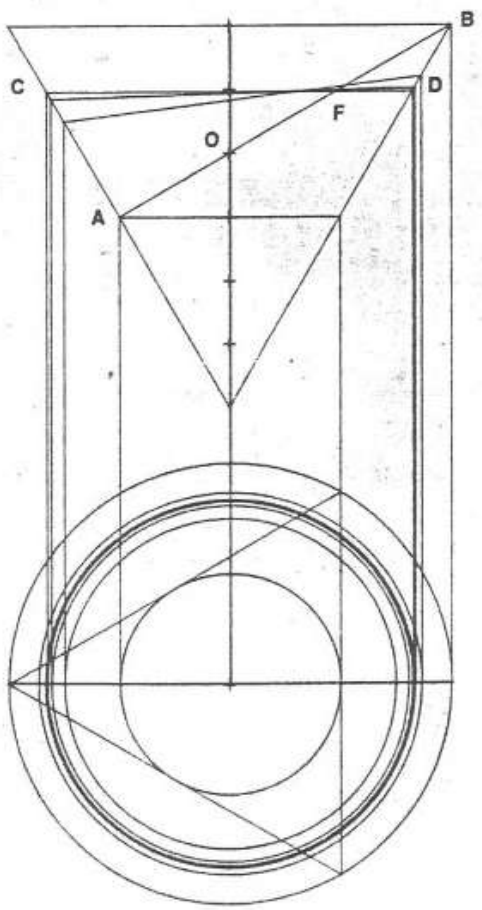
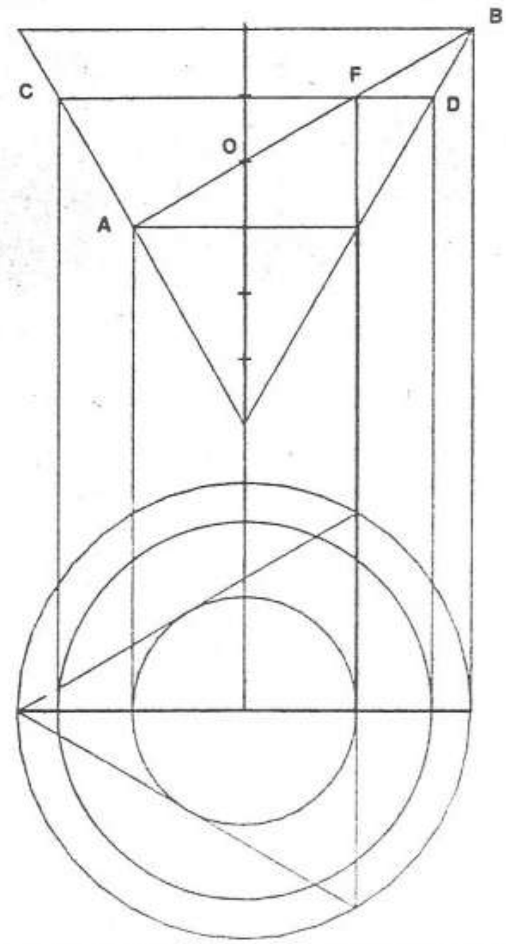


Figure 4



Perspective and the Filioque

by Pierre Beaudry

The Triumph of Poetry

O death where is thy sting, thy victory?
By man cam'st thou, by man art thou destroy'd,
Nor doth the grave retain thy memory,
For in Thy death are all who live o'erjoy'd.
How are the dead rais'd up? the fool will say.
Yet is what's sown made quick, except it die?
What's in corruption sown, on judgment day,
In incorruption shall be rais'd on high.
Nor shall we sleep, but we shall all be chang'd,
At once, as in the twinkling of an eye,
When 'neath His feet all foes have been arrang'd,
And death, the last, beneath His feet shall lie.
Thus death is swallow'd up in victory,
As I this banner raise to poetry.
—Will Wertz, July 21, 1986

During the period of the Council of Florence (1438), the painter Piero della Francesca, who attended the Council under the guiding spirit of Cardinal Nicolaus of Cusa, developed the geometry of perspective as an explicit construction of physical space-time, to express the universality of the Divine Trinity, and in so doing, to elevate man to the dignity of Christ, thus creating a Renaissance.

In this column, I shall initiate a new series of geometry problems relating to the furtherance of the discovery of perspective by Piero della Francesca. In following weeks, I shall demonstrate how the invention of the harmony of proportions defined by perspective in painting also applies to music, poetry, and architecture, and cannot be fully grasped without a fundamental understanding of the Filioque, as developed by St. Augustine and Cardinal Nicolaus of Cusa.

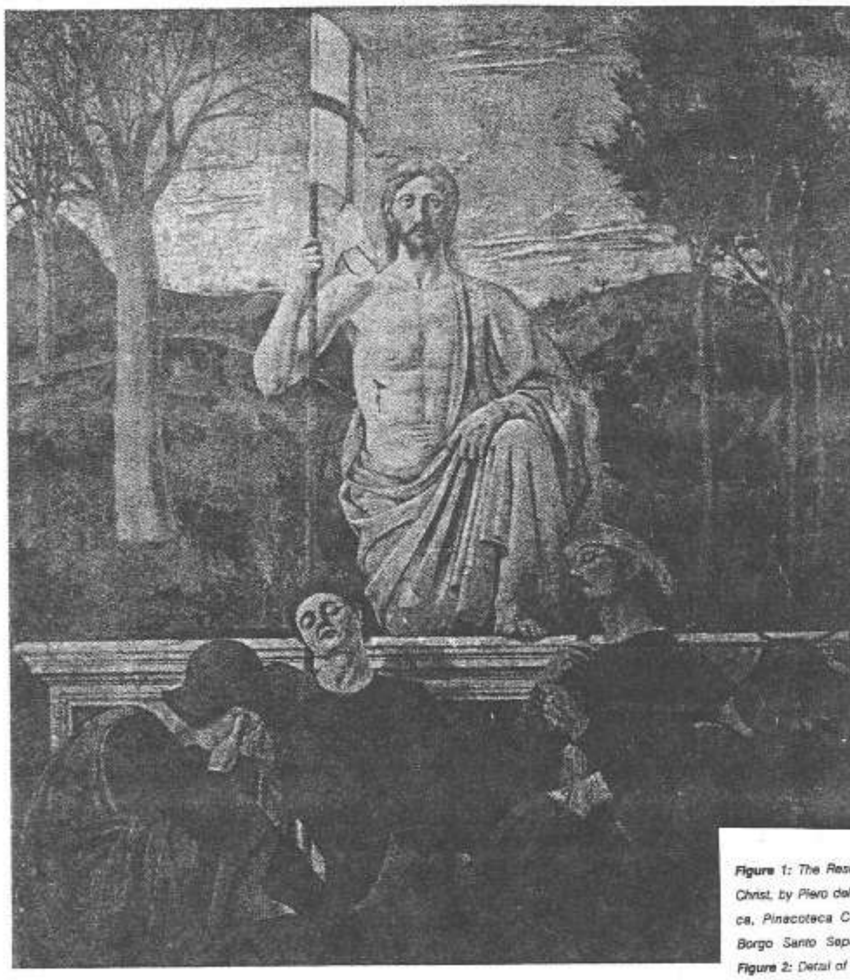


Figure 1: The Resurrection of Christ, by Piero della Francesca, Pinacoteca Communale, Borgo Santo Sepolcro, Italy. Figure 2: Detail of the head of Christ.

Piero's 'Resurrection'

Piero's "Resurrection" (**Figure 1**) was painted around 1460, and was later brought into the Palazzo Communale, the town hall of the Conservatives in the small town of Borgo Santo Sepolcro, where Piero shared political responsibility with them to create the Italian Renaissance.

The subject of the "Resurrection" is not chosen simply because the town is called Santo Sepolcro (Holy Sepulchre), but most emphatically because it embodies the very idea of founding a Renaissance. The resurrection is the triumph over death, over the Black Plague of the Dark Ages, the victory over the bestiality of men toward men. It is the triumph of Christ giving man access to immortality, giving birth to a new man, who, in the imitation of Christ, becomes the Renaissance man, a true universal.

Thus, man is elevated to the dignity of Christ. His character now is to be measured by his ability to be God-like, through his divine spark of reason. **That is how perspective is invented.** Central perspective reflects the Renaissance man's ability to master physical space-time, and thus show, through universal harmonic proportions, that he is created in the image of God the Creator. Let us now turn toward Piero's "Resurrection" and see how this conception is applied.

If you begin scrutinizing the painting, and looking into the history of Piero's work, two very interesting features will strike you as odd in his treatment of the painting. One is that the sleeping soldier facing you (second from the left) is a self-portrait of Piero! This must have amused and intrigued people who immediately recognized him. In fact, this brings a question to the observer's mind, since the painting is dominated by the powerful central figure of Christ. Why would Piero attract the attention of the spectator away from the subject of the resurrection? Or does he?

Second, a keen observer will notice that there are two perspective centers in the painting, one at the level of the tomb where Piero's head is reclining (which corresponds to the level of the observer's point of view), and the other is located at the level of Christ's eyes. In fact, Christ's head could not be seen in such a frontal position from the point of view of the observer. From that lower point of view, Christ's head should be seen slightly from beneath. How can such a master of perspective as Piero make such a mistake? Or is it a mistake?

Only One Set of Laws

Quite a number of art historians, such as Nico Fasola, have totally missed the point, by falling into a gnostic interpretation of the two centers of perspective. They contend that these two centers, correspond to two different laws in the universe, the law of God, and the law of man, and between God and man, there is no common ground. For such gnostics, God is the absolute unknowable, unattainable except through the mysteries of the occult Gnosis. As for man's law, man is a creature of pleasure and pain; the law is the law of survival of the fittest.

This is exactly what Piero polemicizes against in the "Resurrection." **There is a common ground** between God and man, which is Christ. How, then, can one, and only one lawful order in the universe be established as one unified law of both God and man? How can the two different centers of perspective become one, and express this fundamental unity of Christ, who is both God and man? The answer can only be found when man is elevated to the dignity of Christ, that is when he has, in the imitation of Christ, defeated death. Piero develops an incredible polemic to get this point across.

Observe the painting again, this time more closely. Look at the four soldiers. Two of them are struck by the vision of Christ resurrecting: One cannot stand the sight, and covers his eyes, while the other is so stunned that he is falling on his back. Of the other two soldiers, one is asleep leaning against his lance, and the last (Piero) seems to be sleeping also. But is he really sleeping? Look closer still.

What is Piero leaning against? The staff held by Christ after he has risen from the tomb. How can this be? How can Christ put his staff against the back of Piero without waking him up? The perspective projection of the shadow from Piero's head onto the tomb also indicates clearly that he cannot be reclining against the tomb. How then can Piero be asleep? He is not. He is pretending to be asleep and that is why, by painting his own recognizable traits in the painting, Piero forces the observer to focus for a moment on the tremendously imposing figure of Christ, but only to have him shift his concentration in the next instant, and force him to think of Piero's "imaginary sleep," and then return his attention to Christ, and again to Piero, and so forth. Why would Piero do that?

Unity, Equality, and Connection

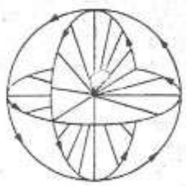
The spirit that unites God and man is expressed in Piero's "Resurrection" by central perspective coming out of the painting itself from the eyes of Christ as the measure of all things. You can observe that no matter where you stand before the portrait of Christ (**Figure 2**), He is always looking at you. If you move to the right or to the left, His compassionate and determined look follows you. Everybody, everywhere, is a captive of His perspective, as if all things dwell in Him.

However, the humanity of Christ resurrected is not simple mortal humanity. It is humanity glorified, which Piero lawfully contrasts with the mortal nature of the humanity of the stunned and sleeping soldiers at the foot of the tomb. Such a glorification of human nature cannot be achieved by man without being elevated to the dignity of Christ himself. It is for this reason that Piero introduces another center of perspective, whose horizon is located at the level of the tomb, the level of mortality and death which is being crushed by Christ's powerful foot.

Moreover, since man can be elevated through his divine spark of reason, he can also rise to immortality. Thus, through reason, man must be uplifted and united to the divinity of Christ. This is why these two perspective centers must become one. The level of mortality must be raised to the level of Christ's glorious perspective.

Piero accomplishes this by a very subtle subterfuge. If you were to imagine transposing the head of Piero onto the position of Christ's head, and conversely transposing the head of Christ in the place of Piero's head, you would have **reestablished one unified central perspective!** In fact, if you examine closely the perspective of shadows on both Piero and Christ's necks, as well as the reclining position of Piero's head and the frontal view of Christ's face, and you will see this even further. The painting becomes one of the most exciting examples of the Filioque.

If man is to be truly human, he must, therefore, in the imitation of Christ, rise to the likeness of God Himself! Such a connection, as in the case of a discovery, has the emotional effect of creating a sort of light being switched on in the mind. That is the expression of the divine spark, of the Spirit making the connection in the creative mind. As Piero's great teacher Cusa writes in "On Learned Ignorance": "It was a comparison drawn from things finite that led our saintly doctors to call the Father Unity, the Son Equality, and the Holy Ghost Connection." As for anything being created, its beginning is caused by the Father, its completion is caused by the Son, and the fitting within the lawful ordering of the universe is caused by the Holy Spirit. "These traces of the Trinity are to be found in all things," concludes Cusa.



Knowing the Universe

Steiner and the Universe's Harmony

by Pierre Beaudry

Primary action in the universe is demonstrably circular. Furthermore, as Nicolaus of Cusa elaborates in his book "On Learned Ignorance," circular action is "trinitarian": Everything that is created in the universe is traceable to minimally and maximally triply self-reflexive action.

This does not mean that the number 3 is some kind of magic number, or that the Holy Trinity can be reduced to a triangle, or any other nonsense of that sort. What Cusa means is that the Trinity takes effect in every created being, in every creative action which rigorously applies the principle of least action (maximum-minimum principle). For instance, in creating the straight line and the point by circular action, the effect of the Trinity is located in the response of a child when, after working through the geometric construction himself, he exclaims out of joy, "Wow! It works!" At that moment, a light turns on in the head, where Unity, Equality, and Connection come together. Such an emotional gestalt is the most precious expression of the "spark of reason" which proceeds directly from the Father (Unity), the Son (Equality), and the Holy Spirit (Connection). Now you are beginning to understand why the Russians have to steal our blueprints!

Synthetic geometry, otherwise known as constructive geometry, is impossible without the Trinity, without the Filioque! Nothing can ever be created in the universe without the procession of the Holy Spirit who causes, out of love, the harmonic fitting within the lawful ordering of the universe, whose beginning is caused by the Father, and whose completion is caused by the Son.

I shall now set up an experiment for the reader to grasp this point in a most sensuous and overwhelming way.

Perspective: Infinite EquilateralPerimetric Action

During the Italian Renaissance, Filippo Brunelleschi, Piero della Francesca, and Leon Battista Alberti discovered perspective and used it as a rigorous scientific construct for proving the lawfulness of Cusa's maximum-minimum principle of trinitarian circular action. A powerful example of this is Piero's painting "The Resurrection," which I discussed in my last column. Let us now investigate how the process of such a discovery could have occurred in their minds.

First, imagine yourself to be standing in the middle of an infinite plane, where you see nothing around you but an infinite horizon. Conceive of this plane as an infinite triangle, so large that its three sides and angles become one and the same infinite straight line (the horizon line). This, in turn, coincides with the perimeter of an infinite circle which, by rotation on itself, forms an infinite sphere. Perspective approximates this process in what Cusa calls a contracted maximum, as opposed to the absolute maximum, which could not be visualized.

Second, imagine that before you, everything is in the process of becoming, of being transformed, that nothing is fixed, and that the infinite plane is acting on itself globally as well as locally so that you are witnessing an infinite process of integrated triply self-reflexive, equilateral-perimetric action in the infinitely small, as well as in the infinitely large.

Third, observe that such infinite self-similar circular actions in the plane produced an infinity of interconnected singularities such as lines and intersection points, forming an infinity of integrated self-similar equilateral triangles which are everywhere close-packed—that is, so compressed that no empty space can be found anywhere between them (see **Figure 1**).

You can see that this generates the entire geometry of perspective where the whole of space is harmonically divided into infinitely receding intervals. Note that if you move any point on that plane you will also transform the whole.

For example, by moving point A of the triangle A,B,C, (an equilateral triangle in perspective) toward point P, the vanishing point, A will tend toward the infinite minimum by smaller and smaller increments, while points B and C will simultaneously tend, on both sides, toward the infinite maximum by larger and larger increments. When point A reaches point P, both points B and C will reach infinity, and all three points of the triangle A,B,C, will coincide with the infinite straight line of the horizon.

Finally, extract any equilateral triangle from that infinite plane (**Figure 2**) and note that it is composed of fundamental harmonic proportions which have been formed by the least action principle (minimum-maximum principle) of equilateral-perimetric action. Such harmonic ratios not only define order in a lawful universe, they also prove that the universe grows **harmonically** by such a principle.

Now, you are ready to enter the world of Jacob Steiner (1796-1863), the most remarkable geometer of the 19th century.

If you examine Figure 2, you will note that all the singularities—circle, triangle, lines, and points A,B,C,D,E—have all been produced by perimetric action (folding). Points A and E result from the folding of the circle on itself; point C comes from folding A and E onto one another; point D comes by folding C over E, and B is generated by folding A on D.

Figure 1

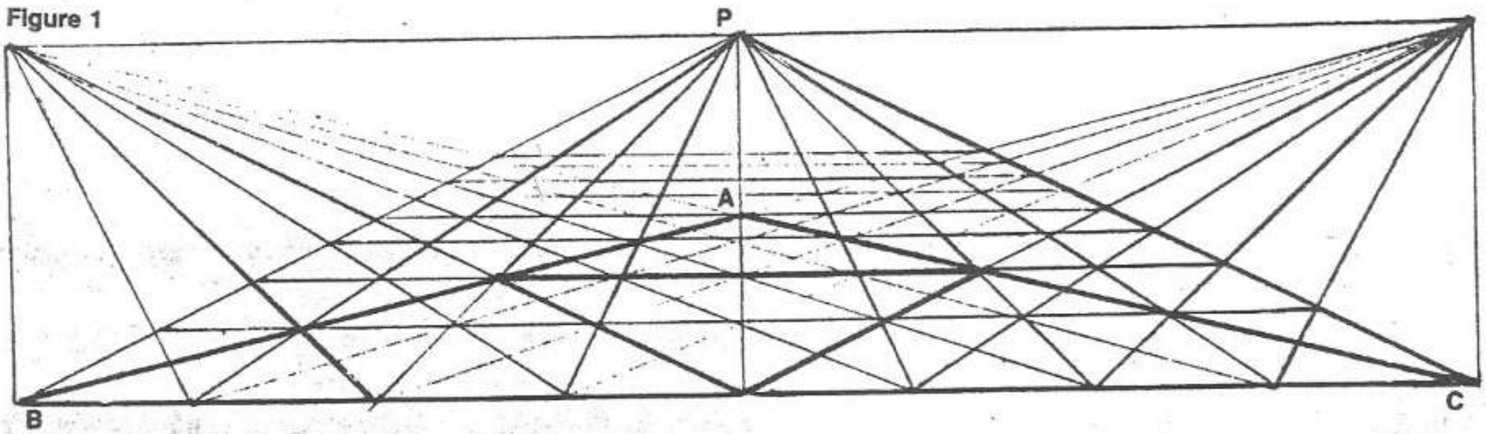
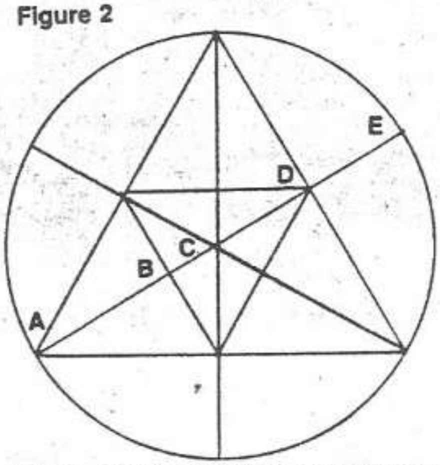
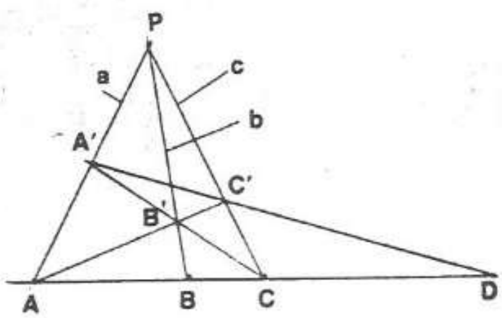


Figure 2



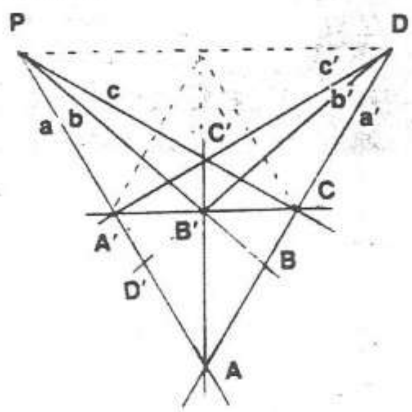
AE/CE=CE/DE=AD/BD= 2/1 (The Octave)
AE/AD=AC/AB= 4/3 (The Fourth)
BD/CD= 3/2 (The Fifth)
BE/BD= 5/3
AD/BE= 8/5

Figure 3



AB/BC=AD/CD

Figure 4



AB/BC=AD/CD
AD′/D′A′=AP/A′P

Each one of these points created by perimetric folding is the product of the power of least action, that is the power of self-reflexivity/ self-similarity, the power of two. Since these points of intersection have been generated at different moments in a sequence of developing a physical space-time continuum, they will divide the diameter A,E in several segments (not fractions) whose ratios will be interwoven within a range expressing an order of becoming of such a physical-space-time.

Growth by the power of two will therefore show the following internal differentiation of harmonic ratios. AE/CE = CE/DE = AD/BD = 2/1. In musical terms, this is the octave; then, AE/AD = AC/AB = 4/3, which is the musical fourth; and finally, BD/CD = 3/2; BE/BD = 5/3; AE/BE = 8/5, generating the musical fifth (3/2), and the beginning of the Fibonacci series. By taking all of the ratios of this triple equilateral-perimetric action (that is those formed by the intersection points A,B,C, and D) and combining them into one single cross-ratio invariant, you will obtain the universal harmonic range of Steiner, where AB/BC = AD/CD.

Steiner's Complete Quadrilateral

Steiner's harmonic range can be easily constructed in the following manner with the use of a straight edge alone. Take a pencil and a straight edge, and draw on a sheet of paper a line with three points on it. For any three arbitrary points A,B,C on a straight line, there will always be a fourth point D which is harmonically conjugated with them (**Figure 3**). Project onto A,B, and C, three rays a,b,c, from any arbitrary point P in space. Have ray a at A bounce back anywhere through rays b and c, generating points B′ and C′. Then have ray c at C bounce back similarly through rays a and b through B′, generating point A′. By drawing a line from A′ through C′, you will intersect the baseline of A,B,C, to find point D. The cross-ratio of these four points will be such that AB/BC = AD/CD. Such a geometric construction is called a **complete quadrilateral,** formed by the four points A,A′,C,C′ joined by diagonals. If you want to prove to yourself that this is truly the case for any projection of point P, repeat the same construction over your exercise, but choose a point P anywhere under your initial line formed by A,B,C.

What Steiner constructed without a compass, and without a circle, is the equivalent of what we have done before by folding (rotation) alone! How so? Well, imagine again that you are standing on that infinite plane formed by an infinity of close-packed equilateral triangles. Such triangles, viewed from where you are, will appear to you in different shapes and sizes because of the receding of the plane in perspective. You have therefore all around you an infinity of triangles of all possible shapes. From this vantage-point, all triangles are equilateral or part of an equilateral triangle! What Steiner has constructed as a complete quadrilateral is nothing but a part of an equilateral triangle, and this can be demonstrated in the following way:

Construct a complete quadrilateral by projecting from any arbitrary two points P and D any four rays a,c,a′,c′, which cut one another by pairs in four points A,A′,C,C′ (see **Figure 4**). Join the four points A,A′,C,C′ by two diagonals intersecting at point B′. If you drop ray b from P through B′ to line a′ you will have formed the harmonic range AB/BC = AD/CD by the complete quadrilateral. Similarly, by dropping ray b′ from the point of projection D through B′, to line a you will have the harmonic range AD′/D′A′ = AP/A′P. Although I have chosen here an equilateral triangle for pedagogical purpose, any projection of points P and D will yield the same result. The dotted lines show the missing portion of the equilateral triangle.

Every line of Steiner's constructions should be viewed as the equivalent of a fold obtained by circular action in the plane, as well as rays of light cones projected in perspective. These two simultaneous views are not opposed in any way if you integrate conic projections within a sphere and consider the plane as a vertical cut through the apex of one of those cones.

From such rigorous application of Cusa's ideas, the reader can discover that there is no room here for arbitrariness. This kind of space which the French geometer Carnot called "geometry of position" is entirely harmonic, as opposed to the Cartesian, Jesuitical-gnostic space where objects exist by themselves, independent of one another and without relations to the whole, or to the Trinity. In constructive geometry, not one point can be moved from its position without transforming the whole, because no point is a thing it in itself; each is caused by the triply self-reflexive connection of the Holy Spirit!

Problem of the week: 1) Given three arbitrary points A,B,D, find the fourth harmonic point C. 2) Given three arbitrary points A,C,D, find the fourth harmonic point B. 3) Given three points A,B,C, where segment A,B = B,C find point D.

Knowing the Universe

Steiner's Synthetic Geometry

by Pierre Beaudry

Last week I introduced in this column the construction of Jacob Steiner's complete quadrilateral, to illustrate how the entirety of physical space-time, in the large as well as in the small, is not merely harmonic, but also expresses harmonic growth. This conception is entirely coherent with Cardinal Nicolaus of Cusa's minimum-maximum principle (least action) derived from Saint Augustine's Trinitarian conception of God acting upon the universe. Such a complete quadrilateral construction is the basis for the perspective elaborated by Filippo Brunelleschi, Piero della Francesca, and Leone Battista Alberti, during the Golden Renaissance.

Problem Solving.

Last week's problem consisted of finding a construction for a complete quadrilateral where, for any three arbitrary points A,B,C, projected from any point P onto a straight line, there is always a fourth point D which is harmonically conjugated with them such that AB/BC = AD/CD (see **Figure 1**). In order to fully grasp the ontological implications of such a least action construction, you must focus your attention on two fundamental considerations.

First, you must look at this complete quadrilateral and its harmonic range as a universal expression of physical space-time derived from the triply self-reflexive process of equilateral perimetric action, generating the harmonic singularities of the equilateral triangle.

Second, the universal application of complete quadrilateral harmonics creates a very special kind of space where everything is interlocked, such that not one interconnected point or line can be moved without transforming the entirety of space. Thus, any harmonic range of that space acts as a command function for positioning the whole, which generates the entire geometry of perspective in the plane.

From this standpoint then, let us solve the problems we posed in last week's column. They were as follows. 1) Given three arbitrary points A,C,D, find the fourth harmonic point B. 2) Given three arbitrary points A,B,D, find the fourth harmonic point C. 3) Given three arbitrary points A,B,C, where segment AB = BC, find the harmonic point D. The reader should take some paper, pencil, and a straightedge and work through these constructions.

Problem 1). This construction (Figure 1) simply consists in projecting from any two points, P and D, four rays intersecting each other by pairs in four points A,A′,C,C′, which, when joined by two diagonals intersecting at point B′, will form a complete quadrilateral. If you drop a fifth ray from point P through B′ to point B, you will form a harmonic range with the four points A,B,C, and D, such that AB/BC = AD/CD. You can prove the correctness of this construction by generating another quadrilateral under the same line A,C,D, using any other point of projection P′. This is called the proof by complete quadrilateral.

Problem 2). (See **Figure 2**.) Project from any point P three rays A,B, and D, on a straight line. Then draw a diagonal from A anywhere through B′ to point D′. Project from point D a ray through B′ to A′. Join points B and D′ by a line crossing D,B′ at intersection point C′. If you then drop a ray from P through C′, you will find the fourth harmonic point C on line A,B,D. These four points will form the harmonic range AB/BC = AD/CD.

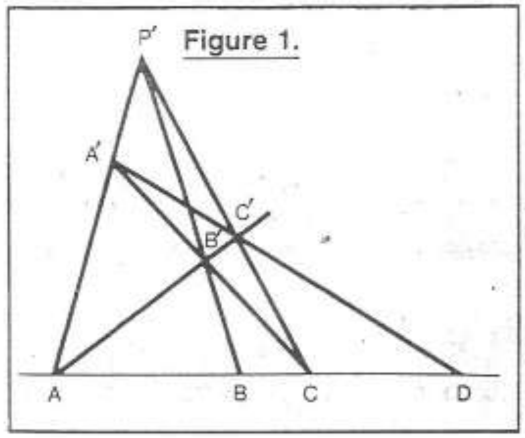


Figure 1.

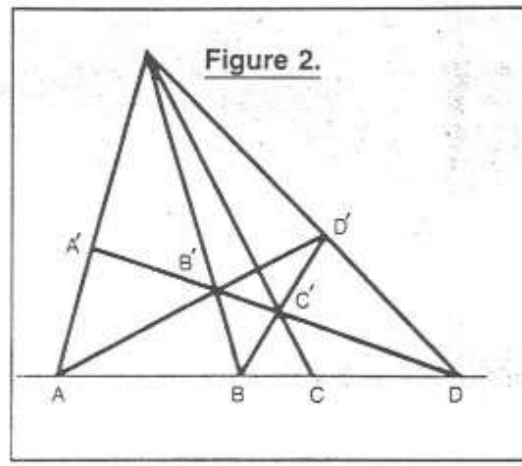


Figure 2.

The Infinite Complete Quadrilateral

Problem 3). Of these three problems, this is the most fascinating and most fundamental (see **Figure 3**). Draw from any point P three rays A,B, and C, in such a way that B is the midpoint of A,C. Then generate two diagonals from A and C intersecting anywhere through line P,B, to get points A′,C′. If you join A′,C′ by a line, you have constructed a parallel line! Where then is point D?

This may quite destabilizing, since the fourth harmonic point D is no where to be found intersecting line A,B,C. Where is it? If point D is nowhere to be found, does this mean that Steiner's harmonics do not apply universally? This is indeed quite disturbing. Is this an exception to the rule?

Well, the situation is quite different from what we perceive to be the case. But this is not an anomaly, or an exception. On the contrary, this is the rule—the actual cause of all finite complete quadrilaterals! This is the [illegible]nite complete quadrilateral, where [illegible]ne A′,C′ intersects line A,B,C at infinity where point D is to be found. In point of fact, all parallel lines intersect in that point at infinity, just like lines PA,PB,PC are all parallel lines meeting at infinity in P!

Thus, the infinite complete quadrilateral generates an infinite harmonic range where AB/BC = 1/1 = AD/CD = infinity/infinity! This is in complete concordance with Plato and Cusa: The one (Unity-Equality-Connection) is the infinite (the absolute maximum).

Infinite Division and Multiplication

The infinite complete quadrilateral we have just discussed can generate within itself infinite series of harmonic division and multiplication.

1) Harmonic division: (Figure 4) Given that you can generate parallel lines intersecting in D at infinity, the absolute maximum, consider that you can now divide any portion of the plane into an infinite series of harmonic ratios tending toward the absolute minimum. By means of the previous construction (Figure 3), divide segment A,B, into an infinity of harmonic ranges.

First draw line A′B and project from P a ray through E′ to E. You then have a harmonic range AE/EB = AC/BC. Subsequently, draw line A′E through F′. By again dropping a ray from P through F′ to F, you obtain a new harmonic range where AF/FE = AB/EB. Such an iterative process harmonically divides line AC, in such a way that AB/AC = 1/2, AE/AC = 1/3, AF/AC = 1/4. . ., and can fill the entirety of space with an infinite series of rational ratios.

Note that this iterative process is dominated by the power of two (least action) generating the midpoint B, and subsequently a parallel line. Such power of two in generating parallel lines will become even more evident in the case of multiplication.

2) Power of multiplication. Given three parallel lines (see **Figure 5**), find any multiple, say **n** multiple of segment AB. Draw through any point P two rays cutting the two lower parallel lines at A′,B′ and A,B. By drawing a line from B through A′ to P′, and then another line from P′ through C′ to D, you have a second multiple of A.B where A,B = B,C. If you continue this process by extending a line from C to P, and then another from P′ through C′ to D, you have a second multiple of A,B where AB = BC = CD. . . By continuing this iterative process, you can see how you can find any multiple of A,B.

By constructing these few elementary problems himself, the student can begin to master the rigor of synthetic geometry. The mastery of such a method gives the student a pleasant way to understand mathematics by happily avoiding the arbitrary road of axiomatic arithmetic and algebra.

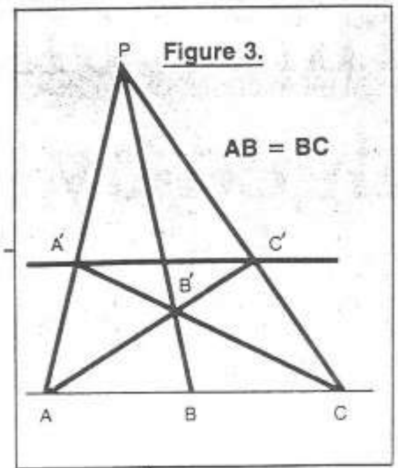


Figure 3.

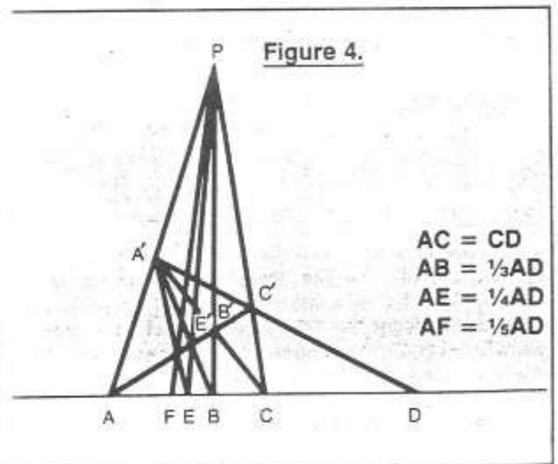


Figure 4.

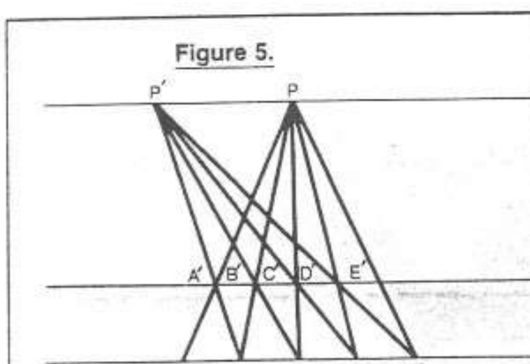
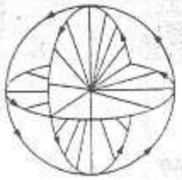


Figure 5.

Knowing the Universe



The Five Platonic Solids

by Pierre Beaudry

Generally, there are two methods by which you can construct the five Platonic solids. (See **Figure 1**.) The first method consists in fitting together the required number of polygons such as triangles, squares, and pentagons, and gluing them together to construct 1) the tetrahedron, 2) the octahedron, 3) the hexahedron (cube), 4) the icosahedron, and 5) the dodecahedron. The problem with this approach is that the polygons are considered to be the "basic building blocks" for the five Platonic solids. That is wrong. Not only are polygons not basic, they are nothing in themselves. They are mere singularities, just like lines and points. Therefore, since the justification for their existence cannot be found in them, they cannot be considered "basic," and the reason must be found somewhere else.

The second method which we shall use here is the method Leonardo da Vinci used for his illustrations for Fra Luca Pacioli's book "The Divine Proportions." This method is simple, clear and has the advantage of constructing the five Platonic solids by circular action alone.

The point of method is crucial here, because you can spend years (and some people do) scrutinizing the simple plane figures (polygons), and do a lot of fancy compass work constructing them, without ever understanding the underlying principle of least action (minimum-maximum) that justifies their existence. Therefore, polyhedrons (solids), polygons (faces), edges (sides), and vertices (points), are mere traces—discontinuities—of the least action work of circular action.

Leonardo's Method

Let us proceed first by establishing the following minimal conditions. With only a piece of the plane, say a sheet of white paper, construct the five Platonic solids by circular action alone. You are not allowed a compass, or a straight edge, not even a circle!

Begin by thinking through how, with a mere sheet of paper, you can create an equilateral triangle (see **Figure 2**). You must define, first of all, the boundary conditions for the creation of seven singularities forming the triangle (three vertices, three edges, and one triangular face).

You must first start with triple self-reflexive circular action. The piece of paper from the plane is the result of the first circular action. The second circular action (folding) defines one edge and two vertices of the equilateral triangle. A third circular action, by folding this first edge on itself (or folding the two vertices B,C onto one another) generates a fold which is a perpendicular elevated from the midpoint of that edge, B,C. You now have all you need to construct the equilateral triangle (Figure 2).

If you fold one of the two vertices, B or C, up along the perpendicular line, and mark that third vertex A, you can now fold each of the other two edges connecting the three vertices A,B,C, and create the equilateral triangle!

Figure 2

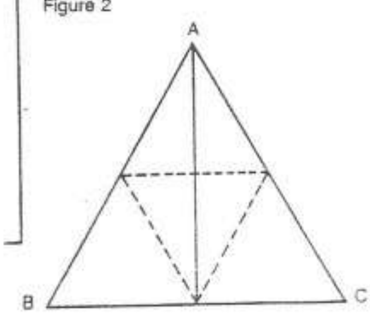
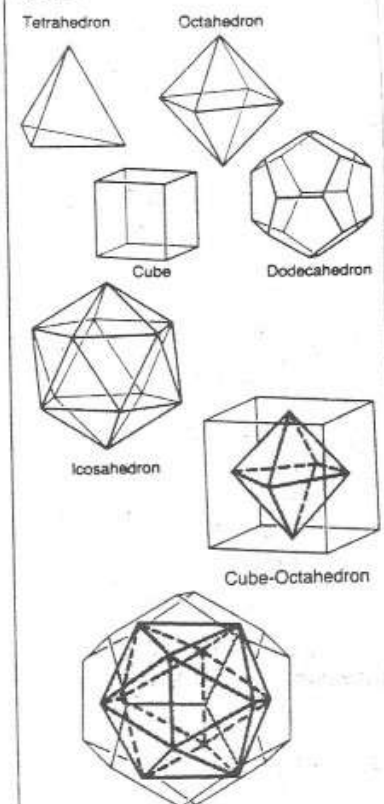


Figure 1



Singularities of the Five Platonic Solids

Solid	Faces	Edges	Vertices
Tetrahedron	4	6	4
Cube	6	12	8
Octahedron	8	12	6
Icosahedron	20	30	12
Dodecahedron	12	30	20

The Tetrahedron

This triangle is composed of 8 singularities, 3 vertices, 3 edges, 1 perpendicular, and 1 triangular face. Fold the triangle on itself such that vertex A is rotated over the midpoint of its opposite base. Then fold the other two vertices B, and C on the midpoint of their own respective opposite bases. By unfolding the triangle, observe that you have created four smaller, self-similar equilateral triangles. If you rotate these four faces together by connecting the three vertices A,B,C into one single point, you then have created the first Platonic solid, the tetrahedron! This tetrahedron has 15 singularities: 4 vertices, 6 edges, 4 faces, and 1 solid.

The Cube and the Octahedron

The cube is formed by rotating the tetrahedron on its center, as if within a sphere (**Figure 3**). With this rotational action you effect the doubling (power of two) of the tetrahedron and of the number of its vertices from 4 to 8 and the number of its edges from 6 to 12. Now, join together all of the 8 vertices by straight lines and you will see the cube appearing as a circumscribed solid. The cube will also have 12 diagonals (2 for each face) corresponding to the 12 edges of the rotated double tetrahedron.

The octahedron is formed simultaneously by the same rotation! You can see it as the inscribed solid inside the double tetrahedron. Because the tetrahedron acts as a sort of morphological mediation between the inscribed octahedron and circumscribed cube, the tetrahedron's 4 vertices and 6 edges will double to produce the cube (8 vertices, 12 edges, and 6 faces) as a sort of inverse image of the octahedron (6 vertices, 12 edges and 8 faces). However, this is not to say that the cube and the octahedron are produced by the tetrahedron; they are not. For the same reason as stated above, it would be wrong to consider the tetrahedron as a "building block." The Platonic solids are nothing but traces of circular action, and more specifically of equilateral perimetric action. However, because the tetrahedron is the absolute minimum polyhedron, it will act as a sort of mean between all of the five Platonic solids.

Such equilateral perimetric action compresses space on itself to the effect of producing a limited series of equilateral close packed singularities. In other words, the five Platonic solids are merely a limited number of forms by which equilateral circular action acts on itself—the capacity of circular action to give closure to Euclidian space, by close packing its singularities.

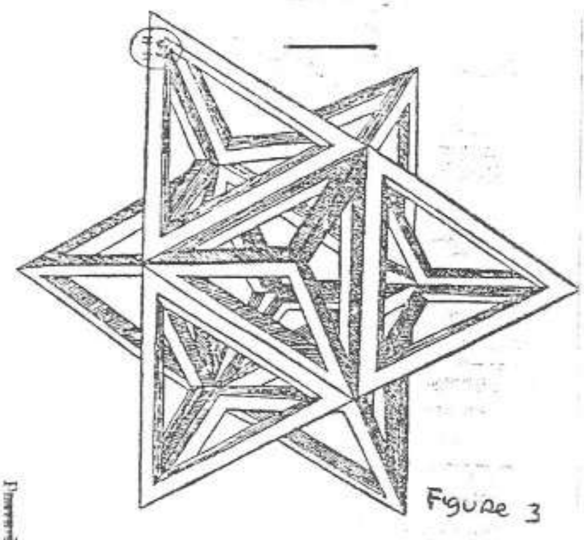


Figure 3

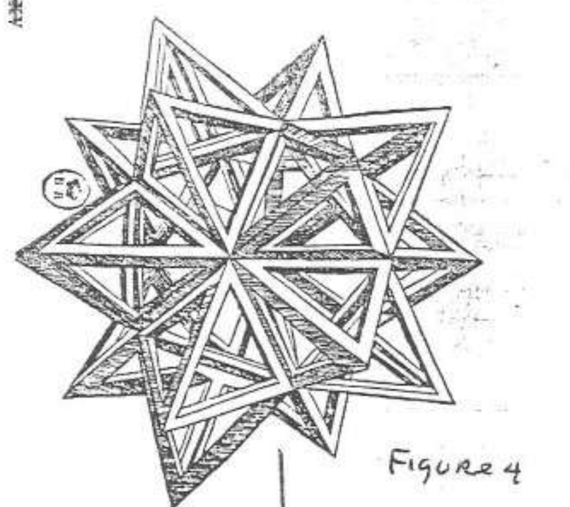


Figure 4

The Icosahedron and the Dodecahedron

It is important to recall at this point that circular action produced 15 singularities in the formation of the tetrahedron. We have used up one of those singularities (the solid) for the construction of the cube and of the octahedron simultaneously. We shall now use up all of the remaining 14 singularities (4 vertices, 6 edges, and 4 faces) in such a way that, by five rotations of all of the faces, edges and vertices, we shall form 20 close packed tetrahedrons whose bases form an inscribed icosahedron, and whose starred vertices form a circumscribed dodecahedron! (See **Figure 4**.) You can visualize the dodecahedron by joining together all of the outside vertices of the starred icosahedron! Note that as in the case of the cube and the octahedron, the icosahedron (12 vertices, 30 edges, and 20 faces) is a sort of inverted image of the dodecahedron (20 vertices, 30 edges, and 12 faces).

Furthermore, all of the five Platonic solids can be integrated into one another in such a way that the cube group can be inscribed in the dodecahedron group. Upon close examination, one can discover that five cubes can be inscribed in a dodecahedron, where two cubes can meet at every vertex of a dodecahedron.

From the standpoint of circular action, there are no fundamental differences between the five Platonic solids. The five solids are the working out of spherical, equilateral perimetric action. However, only in the dodecahedron group can you find the Golden Section, because only in the dodecahedron, from which all of the others can be derived, can you find the ratio of life. For this reason, the very structure of the dodecahedron is unique for a synthetic geometrical approach to biology.

Knowing the Universe

Dodecahedrons & Living Processes

by Pierre Beaudry

By simple, triply connected rotational actions, as we discussed last week, the five Platonic solids are constructed in a continuous series generating the tetrahedron, the cube, the octahedron, the icosahedron, and the dodecahedron. Today, I shall demonstrate the uniqueness of the dodecahedron in relation to the golden proportion, as the dominating harmonic feature of living processes.

There is abundant evidence in nature showing that inorganic matter is dominated by the geometric ordering of four of the five Platonic solids. For instance, iron ores relate directly to the cube, boron compounds, at the molecular level, have the structure of the tetrahedron, octahedron and icosahedron, and the snowflakes of shechtmanite, an aluminum alloy or manganese, show the geometric ordering of the icosahedral pentagon. However, the dodecahedral pentagon is the exclusive geometric ordering of living processes.

The Dodecahedron

Although Plato and his associates understood implicitly the relationship of the dodecahedron to living processes, it was not until Leonardo da Vinci that the unique topology of the dodecahedron was made explicitly congruent with the harmonic pattern of life that is consistent with the Golden Section. The first literary evidence of this can be found in Fra Luca Pacioli's book, "The Divine Proportion," inspired by his teacher, Piero della Francesca, and illustrated by Leonardo da Vinci.

Leonardo's contribution to this effort was to establish, from the standpoint of his discoveries in hydrodynamics (wave characteristics of water, light, air, and sound), that the geometric organization of the inorganic universe, as expressed by four of the five Platonic solids, is uniquely generated by the principle of least action, congruent with the geometric construction of the dodecahedron as the unique form of synthetic geometry defining living processes.

The simple experiment of dropping a pebble in water and generating equally spaced circular waves will reveal a first approximation of the golden proportion! However, if the reader scrutinizes Leonardo's drawings of water flows (see **Figure 1**) he will be able to discover a very interesting form of circular action, which is the key to constructing the Golden Section, the pentagon, and the dodecahedron by one continuous motion! Figure 1 shows Leonardo's studies of three-dimensional vortex filaments in water, describing helical and spiral transformations. They show what Leonardo called "onde colonnale" (columnar waves) which are especially intense vortices formed by a rapid water flow hitting an obstacle. Such columnar vortices, that can also be observed in the formation of twin tornado filaments, show a direct similarity with the structure of the DNA molecule, whose geometric ordering is generated by the dodecahedron! How so?

Focus your attention on the upper right drawing in Figure 1. This shows the formation of a twisted wave which, if it were to recoil on itself, would form a Moebius band (see **Figure 2**).

Triply Twisted Circular Action

Take a paper strip approximately 16 in. long by 2½ in. wide, and bring the two ends together. Before the two ends meet, twist the strip on itself making a full rotation. You have now created a surface where the inside connects with the outside, continuously. You can prove that this is the case by tracing lengthwise a continuous line along the entire strip without ever lifting your pencil! This twisted circular action introduces a special kind of transparency of space, from within which circular action is no longer bounded by the inside/outside limit of simple rotational action.

When rotational action is at once twisted three times on itself, and connected as in the previous case, the compacting and flattening of that space produces immediately the Golden Section and the pentagon! (See **Figure 3.**) You can obtain the same result by simply making a knot! Behold! every time you make a knot in your shoelaces, you construct a Golden Section and a pentagon!

Now, take a long band of calculating machine tape and, starting at one end, make 20 interconnected pentagonal knots in a continuous fashion. Once this is done, leave two single pentagons at each extremity and fold, two-by-two, the remaining 18 pentagons in between. This will produce 12 interconnected pentagons as in **Figure 4.** Rotate these 12 pentagons by connecting all of their vertices and edges together to create the dodecahedron!

The Harmonic Golden Proportion

This triply twisted circular action enables us to create the simplest and most exciting mean, producing the harmonic Golden Section. As Figure 3 shows clearly, the proportion of one of the diagonals to the side of the pentagon is a perfect Golden Section. Furthermore, the kind of "transparency" afforded by such twists is indicative of the way in which this acting harmonic principle operates in distinguishing non-living processes from living processes.

Non-living processes are formed by simple addition of layers of simple circular actions (multiple sets of triply self-reflexive circular actions upon triple circular actions). This can be observed in the formation of crystals, for example. And, the tetrahedron, the cube, the octahedron, and the icosahedron are exclusively interconnected by their singularities (vertices and edges), as a result of simple circular action. Such a division of space also integrates the dodecahedron as the "outer limit" of this form of circular action.

On the other hand, a living process has the unique feature of being able

Figure 1

Leonardo da Vinci's drawings of three-dimensional vortices on a water surface, as a transformation of water waves at high speed.

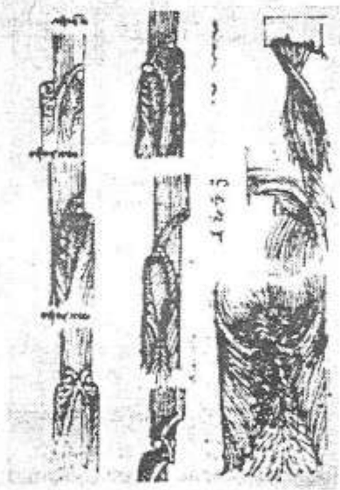


Figure 2

A Moebius strip has one continuous side.

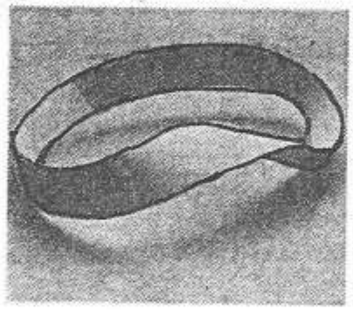


Figure 3

Relation of Golden Mean to pentagon. The ratio of the diagonal to the side is a Golden Section.

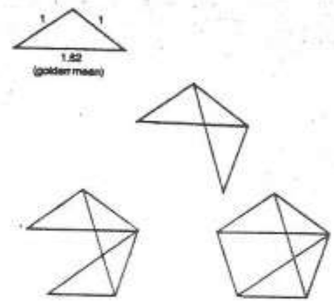
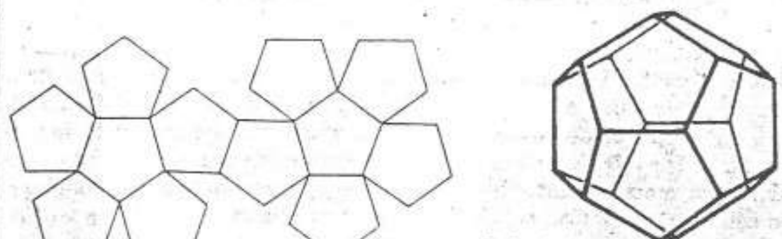


Figure 4

12 regular pentagons folded together form a dodecahedron.



to transgress boundary layers and act on itself throughout itself. The smallest part of a living organism applies constant action on the whole of the being. Each part is ubiquitous.

This also characterizes the dodecahedron. This becomes evident when the triple twisted circular action produces the pentagon. This special sort of twisted circular action does not produce boundary singularities, as in the case of simple circular action. The "transparency" is such that there is no inside/outside in any direction of space. The entirety of space is covered in all directions without any discontinuity, any singularity. Again, you can prove this by tracing a continuous line, as we did before with the Moebius strip, but this time, trace all along the length of the 20-triply-twisted band before you form the pentagon, and close the dodecahedron on itself. Of course, this is physically impossible, but not conceptually. In other words, the space covered is, in all directions, without singularities. Only when the pentagon is formed, and subsequently the dodecahedron, can you see emerging singularities whose only function is to connect the dodecahedron with the lower species of Platonic solids.

From this standpoint, the dodecahedron belongs to a higher domain of circular action derived from the general form of conical self-similar spiral action, while at the same time acting as the intermediary between living and non-living orders.

FIN