SPHERICAL CONSTRUCTIVE GEOMETRY FOR THE FIVE PLATONIC SOLIDS

LANTERNLAND

A Rabelaisian World of Platonic Discoveries

By Pierre Beaudry

With model constructions and drawings by the author

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RABELAISIAN WISDOM OF LANTERNLAND

*"So, when you philosophers, with God's guidance and in the company of some clear Lantern, give yourselves up to that careful study and investigation which is the proper duty of man – and it is for this reason that men are called '**alphestes**,' that is to say searchers and discoverers, by Homer and Hesiod – they will find the truth of the sage Thales's reply to Amasis, King of the Egyptians. When asked wherein the greatest wisdom laid, Thales replied: "In Time." For it is time that has discovered, or in due course will discover all things which lie hidden; and that is the reason why the ancients called Saturn or Time the father of Truth, or Truth the Daughter of Time. They will also infallibly find that all men's knowledge, both theirs and their forefathers', is hardly an infinitesimal fraction of all that exists and that they do not know."*

François Rabelais [1]

[1] François Rabelais, *Gargantua and Pantagruel*, translated by J. M. Cohen, Penguin Books, New York, 1955, p. 710.

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INTRODUCTION

WHY ARE THERE ONLY FIVE PLATONIC SOLIDS?

The study of the Five Platonic Solids has led us to a very important question of physical geometry, a question that is usually discarded, because its significance is generally not perceived by those who indulge in merely academic studies of the Platonic solids. However, the problem is very simple but quite devastating in its implication. Why are there not more than Five Platonic Solids? Why are there five and only Five Platonic Solids? And, why are each and all of the Platonic solids derived from the dodecahedron by means of the golden section of divine proportion? I shall make three points with respect to these fundamental questions.

First and foremost, the question of the Platonic solids represents the highest challenge to sense perception. It not only represents the lawful limits of the domain of perception, but also engages the mind immediately into investigating the higher geometry that generated them from beyond the senses.

The physical conditions for a solid to be called Platonic are as follows:

Each solid must be composed of regular faces.
Each face must have an identical number of sides.
Each side must have corners touching a sphere.

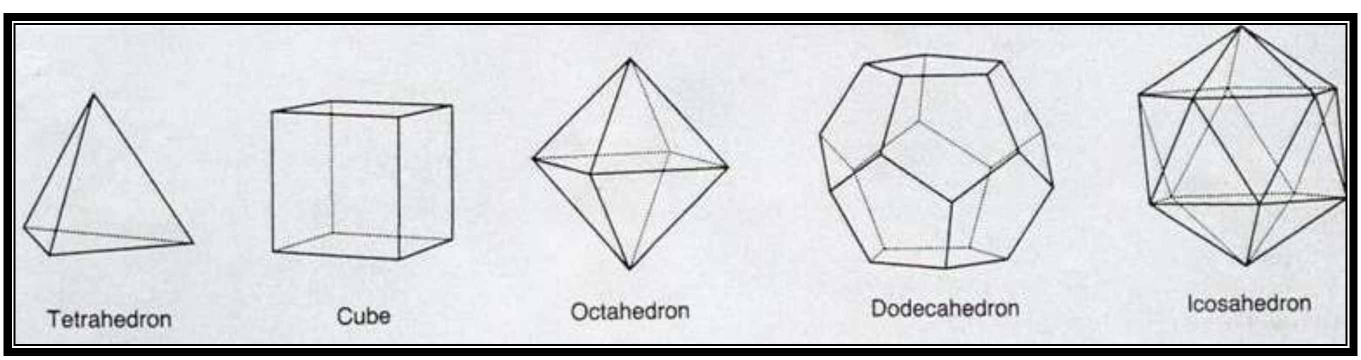


Figure 1

The last of these three physical conditions calls attention immediately to a higher manifold, and begs the question of how to generate the Platonic solids by some form of spherical action. Plato developed an initial insight into their construction in his *Timaeus* dialogue. Luca Pacioli and Leonardo da Vinci penetrated Plato's insight further, and left us the results in their famous book on the divine proportion.[2] Today, we must explore further along these same lines of investigation and bring new insights into the process.

Lanternland introduces three different methods of generating the Five Platonic Solids. Two are methods of folding, and one is a method of partitioning the sphere with great circles. All three methods require the mastery of different forms of multiply-connected spherical action in order to divide the sphere into the required number of parts.

The two methods of folding provide the required spherical action almost by search and find, where your hands "fingure out where to go," as Pantagruel put it. As for the method of great circles, it provides us with a more cognitive way to measure the significance of the question: why are there only Five Platonic Solids? The answer is not obvious. At first glance, it seems that only the octahedron can be generated directly from great circles of the sphere. In the case of the other Platonic solids, a spherical solution must be discovered through the generation of great circles that partition the spheres into equal parts. The result of such spherical partitioning yields a series of intermediary truncated spheres which generate two Platonic Solids at the same time. (**Figure 2**)

Figure 2 Two spherical duals and the spherical octahedron.

[2] Luca Pacioli, *Divine Proportion*, illustrated by Leonardo da Vinci, Publisher Paganino Paganini, Venice, 1509.

Thus, there are only three ways that a sphere can be divided into equal parts: a sphere of 3 great circles partitioning one another into 4 equal parts and yielding the octahedron; a sphere of 4 great circles partitioning one another into 6 equal parts and yielding the cuboctahedron; and a sphere of 6 great circles, partitioning one another into 10 equal parts yielding the icosidodecahedron. Note the progression, 3 into 4, 4 into 6, 6 into 10. Next, if you take the last sphere of 6 great circles partitioned into10 parts, and mix it with its inversion; that is, a sphere of 10 great circles which divide each other into 6 parts, the result will be a sphere of 16 great circles partitioning each other, everywhere, in the ratio of 3/5, the spherical golden section of the divine proportion. This is the chora sphere that integrates the intermediary Archimedean solids generating all of the Five Platonic Solids from a single sphere.

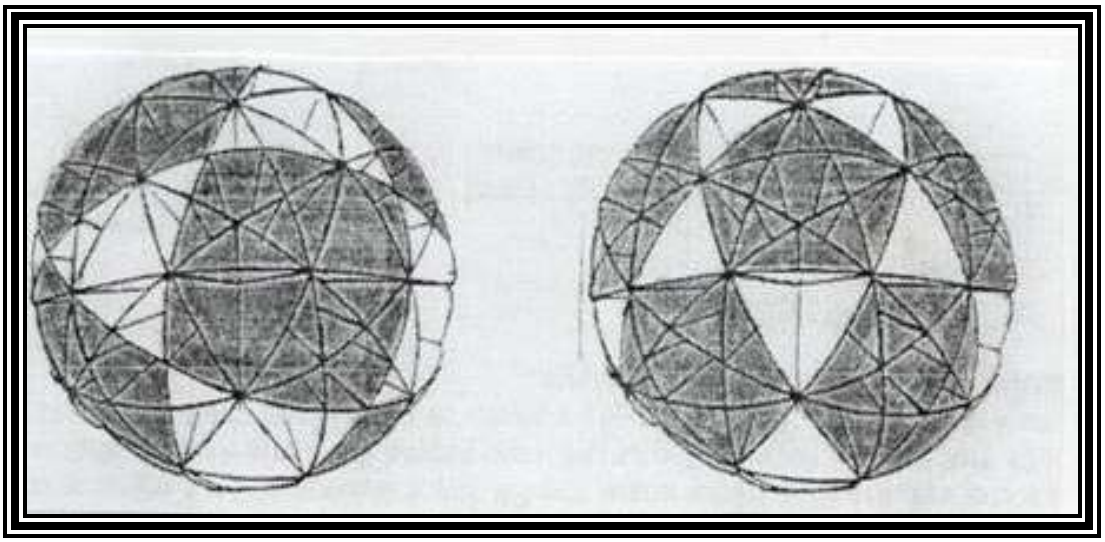


Figure 3 Chora sphere integrating both the cuboctahedron and the icosidodecahedron.

THE NECESSARY BOUNDARY CONDITION OF THE DIVINE PROPORTION

Secondly, the physical conditions for a solid to be called Platonic point to another crucial requirement, which is that of the boundedness of the Five Platonic Solids. This question must be raised because in all of the geometric studies that you will get to know during your entire lifetime there will not be a more important underlying question than the issue of boundary conditions. Why? Because, one cannot think properly if one doesn't establish the necessity of closure in the universe. This means that the universe, as a whole, is not unlimited, indefinite, neither in the apparently indefinitely large, or indefinitely small. The world is not an indefinitely extended bouillabaisse soup, no matter how many fish you think you can get to swim in it. The universe is finite yet unbounded. That is a sort of paradox. How do you solve that paradox? The issue is specifically and more acutely solved by answering the question: why is the divine proportion the spherical boundary condition of the Five Platonic Solids? What is it in the divine proportion that determines a boundary condition?

The golden section of the divine proportion brings into focus the axiomatic change that occurs between the non-living and the living manifolds. That is a discontinuity, a singularity. That is why Pacioli, Leonardo, and Kepler, have all identified the pentagon and the sublime triangle, within the dodecahedron, as the singularity expressing this axiomatic change, which they associate with the divine proportion. From this, Pacioli concluded that it is impossible to generate a sixth Platonic solid, because Five Platonic Solids are all that is necessary for the beauty of the universe. He argued that since there cannot exist a solid angle formed by figures of less than 3 sides (triangles), or formed by figures of more than 5 sides (pentagons), there cannot exist a solid formed by figures of six (hexagons), or more sides. In fact, if it were possible, then the universe would be so ugly, that it would be impossible to circulate in it without constantly bumping into all sorts of monsters. The point is that 3 (triangle) is a minimum, and 5 (pentagon) is a maximum. This is the reason why the golden section of a sphere also reflects a minimum of 3 and a maximum of 5; that is, a ratio of 3/5, as a boundary condition. This is also the reason why 6/10 is the spherical limitation for nesting the Five Platonic Solids into a single sphere.

Such a boundary condition separates the two domains of the non-living (sixsidedness) and the living (fivesidedness-tensidedness), and brings into prominence the idea that the living interacts with the non-living as the dodecahedron does with the other four Platonic solids, as a result of the golden section. In other words, it is the dodecahedron that generates the other four solids. Similarly, it is the living that generates the non-living, and not the non-living that generates the living! So too, proportionately, it is the cognitive which generates the living and the non-living, in the same way that it is the divine that generates the cognitive, as Vernadsky and LaRouche have demonstrated. Thus, the divine proportion: the divine is to the cognitive as the living is to the non-living.

$$\frac{\textbf{DIVINE}}{\textbf{COGNITIVE}} = \frac{\textbf{LIVING}}{\textbf{NON-LIVING}}$$

Now, if we extend Pacioli’s reasoning regarding the minimum solid angle of 3 figures (tetrahedron) and the maximum solid angle of 5 figures (icosahedron) to the spherical domain, the result will be a mixture of sixsidedness and of tensidedness since 6/10 = 3/5. Such are the boundary conditions of the Chora sphere of divine proportion, as the single generative sphere of all of the Five Platonic Solids.

THE DIVINE PROPORTION AS A BOUNDARY CONDITION FOR COGNITION

All of the construction exercises in *Lanternland* must ultimately be related to the divine proportion as the boundary condition for cognition, because this is the crucial precondition for being able to go from a lower dimension to a higher dimension. In other words, these constructive geometric proportions take their source in the higher proportionality of man created in the image of God the Creator. The point is that you won't be able to make crucial discoveries without fulfilling the requirement of such a boundary condition. The reason why this question is so important is twofold.

First, the question of the boundary condition forces the mind to accept the necessity of a limitation, which shows why Euclidean geometry is wrong. Internal to Euclidean geometry is a devastating paradox that can be easily demonstrated by the experiment of stereographic projection from the Riemannian sphere.

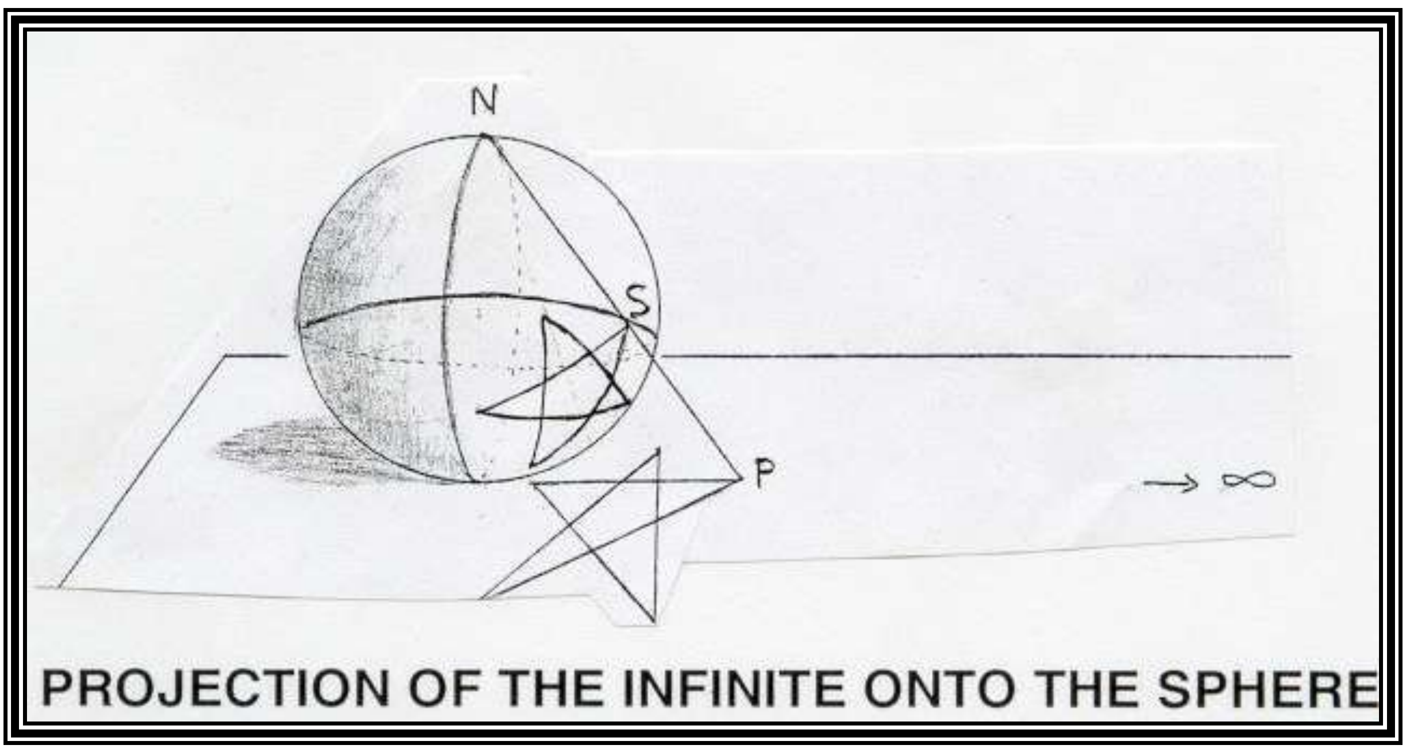


Figure 4

PROJECTION OF THE INFINITE ONTO THE SPHERE AND THE PROJECTION OF THE INDEFINITE ONTO THE PLANE

Note that by projecting the starred pentagon from the sphere onto the plane, the curved sides are deformed into straight lines. These plane shadows are straight, while the bounded sphere that casts them is curved. That is perplexing. Why? Because this transformation contains a paradox between a non-linear geometry and a linear geometry. This transformation includes a falsified notion of infinity. While you think you are getting closer to infinity as you are projecting outward into space in the plane, you are actually getting away from a true infinite. In fact, you are the true infinite projecting into the indefinite, otherwise known as a bad infinity. You are excluding yourself as an individual created in the image of God. Since the source of projection is at the north pole of the sphere, such a stereographic projection represents the geometric analogy of a metaphor for the projection of an image coming from a higher domain toward a lower domain. The paradox is such that, when you raise the line of projection closer and closer to the pole of the sphere, the shadows on the plane move further and further away from the pole, and will become more and more elongated and deformed, as they are stretched further away toward an indefinite horizon. The existing bounded infinite of the sphere will be represented by the North Pole, while a corresponding nonexistent indefinite, or “Cartesian infinite,” will be imagined at the other end of that projection on the plane, as if beyond some boundless horizon all around. That is the paradox.

How do you solve that paradox? The only useful mental image to have of that otherwise wrong bad infinity at the far end of the plane is to conceive of it as being bounded by the infinite great circle of an infinitely large sphere. That is a tough one, I know. But think it through. Consider that the closer your projection ray gets to the north pole of the sphere, the closer it is also getting to the infinite great circle on the plane. When you reach the North Pole, you also reach the infinite great circle. Thus, the North Pole corresponds to that infinite great circle! This is the Riemannian equivalent of Cusa’s solution to the paradox of squaring the circle.

Furthermore, such an infinite spherical projection is a useful imaginary sphere on which to map the stars of the night sky in your astronomy studies. Otherwise, there is nothing useful at that other end of the world; there are only the harmful imaginary fantasies of your extended sense perception. And, if you spend too much time in those warped regions of your imagination, you risk going weird like Harry Potter.

"GARDE FOU" AGAINST THE FOURTH DIMENSIONALITY OF TIME

Thirdly, the boundary condition establishes for the mind a safeguard against the madness of the lying magical views of the world. Pantagruel calls this a "garde fou," which is an appropriate French term for a "guard rail" against the mad Cartesian disease; that is, a safeguard against the tendency of those nut cases who extend everything "to infinity," to the so-called "Cartesian infinite;" either by small increments of indefinitely small linear extension, like the mathemagician, Augustin-Louis Cauchy had advocated for his bowdlerized version of the Leibniz calculus, or by large projective deformities as those introduced by David Hilbert to express the so-called fourth dimension. The following projections of Hilbert are said to depict such a fourth dimension. The shaded area in each diagram is said to represent a "region extending to infinity."

HHHEEELLLLLLOOOOOOOOOOO…..

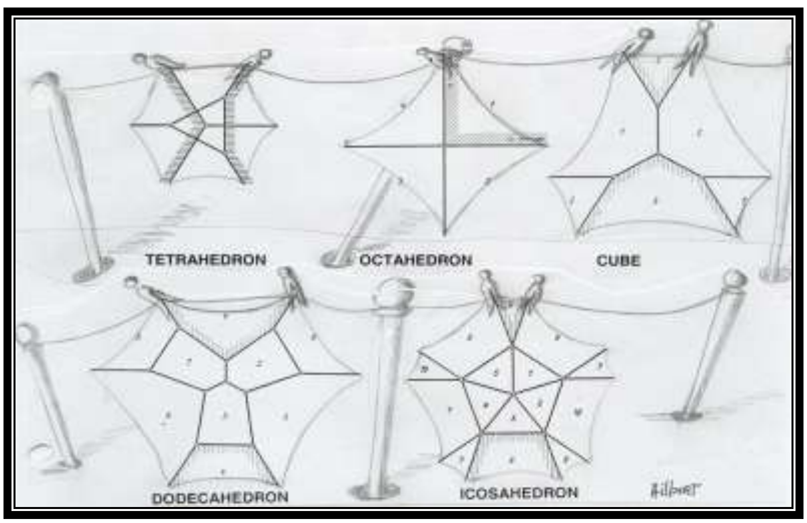


Figure 5

Is there anything more foolish than a grown-up attempting to deal with the Platonic Solids from the standpoint of Flatland; that is, by projecting the extension of his sense perception onto an infinite plane, and calling this, pompously, "the fourth dimension?" [3] The mistake that Hilbert made is a typical Aristotelian mistake. He projected the dimensionality of the solid onto a bad infinity. His false underlying assumption was that he was thinking linearly and he projected the higher solid dimensionality back to the lower dimensionality of the plane and called it the "forth dimension." As we shall see later, the

[3] **Figure 5** is an ironic arrangement composed by Pierre Beaudry and applied to the laundry of D. Hilbert and S. Cohn-Vossen, *Geometry and the Imagination*, Chelsea Publishing Company, New York, 1952, p. 148.

higher dimensionality of the solids is to be found in the spherics of Lanternland, not in the plane of Flatland.

Figure 6 Panurge discovering that an underlying assumption is always covered up.

WHAT IS AXIOM BUSTING? HOW DO GO FROM A LOWER DIMENSION TO A HIGHER DIMENSION?

Lanternland is the extraordinary Island where Francois Rabelais had his famous axiom buster giant, Pantagruel, visit in the last book of his histories of Gargantua and Pantagruel. In this modern interpretation, Lanternland becomes the site of Plato's Cave, the most important place in the entire noosphere where the power of cognition can be discovered by constructive geometrical means. This Rabelaisian island is inhabited by a wonderfully studious people called midnight oilers, whose role is to educate visitors that come from all over the world to find out how to discover the intention that lies behind things, and to study the noosphere in order to discover how to make bread for the mind. This is the reason why they also like to call themselves *alphestes*; that is to say, *industrious millers of ideas*. When you land there, pay attention to the intention because the very first question you will be asked may be a trap.

"Do you think that what you see with your eyes is the real world?" Well, if that is what you believe in, I have a surprise for you. The truth of the matter is that what you think is the real world is merely distorted shadows that are cast on the dimly lit wall of Plato's Cave. Like the images of a puppet show, those shadows have no existence in themselves; they are entirely dependent on the invisible source of light that casts them. And, the shadow you get depends on the intention of who controls the source of light.

You see, on the one hand, a shadow has a very thin existence, yet we must chase after its source if we want to discover what produced it. On the other hand, an object may cast a different shadow than what is expected. In other words, some shadows may be different from the objects that cast them. That is troublesome; that's what is called an anomaly. The task of the geometer is to make visible to your mind the source of that anomaly which is not visible to your senses; that is, make you discover what lies behind the visible world. Let me give you an example.

One of the most delightful papers on this subject is the short Snowflake paper written by the great astronomer, Johannes Kepler.[4] In that paper, Kepler wanted to find out why snowflakes always chose the hexagonal form, as opposed to the square form, or the round form. What shapes a snowflake into a six-cornered form? Is there someone in the heavens who is in charge of marking every drop of water with a freezing six-corner stamp, or is this caused by the triple folding of circular action, or more simply, is that form the result of a battle between hot vapor and cold air? Kepler finally brings his readers to discover that God not only has a reason and an intention for creating almost non-existing things, such as snowflakes, but also adorns them with beauty, maybe just for the pleasure of it, and for merely a passing moment.

Well, in this paper, I have made a similar choice. I have taken the decision to play with some geometric problems, but not just for the pleasure of it. There is an intention behind my choice, just like there is one behind God's choice. And, I hope the problems I chose will last a little longer than the lifetime of a snowflake, and that you will enjoy them in a lasting way. These discoveries should last you a lifetime, and maybe longer. For example, there is a beautiful treasure to be found in Plato's Cave. There, you can find crystals that have the shapes of the Five Platonic solids. We are going to study the intention, or the formative faculty, that God put into them, and discover that their source is round while their shadows are straight. That is a sort of puzzle called a paradox: how can something that is flat and straight come out of what is spherical and curved? This is a special case where the shadow is different from the object that casts it.

There are other cases. Among other things, we will discuss a certain number of apparently impossible things, and also some apparently non-existing things. But, in all cases, we will attempt to solve the problem of what lies behind things. For example, try to solve the following problem: take 6 sticks of equal size and, without breaking them, make 4 equilateral triangles; that is, 4 triangles with 3 equal sides where each side is equal to one stick. The solution to this problem depends entirely on the difference of dimensionality between the source of light and the shadow cast on the wall. The problem is not solvable unless you are capable of changing dimensionality, or manifold. What is not solvable in a lower manifold becomes perfectly solvable from a higher manifold.

Well, let us see if we can shed some more light on this subject by using the transformative power of reason. In his view of the universe, Plato states that things are not self- evident, in and of themselves, because they are created by self-reflexive circular action. His crucial higher hypothesis is based on the fact that man is created in the image of God's Creative Power, which is best represented by the sphere, as Kepler demonstrated. From this higher hypothesis, Plato derived in his *Timaeus* dialogue, a proportionality that said:

[4] Johannes Kepler, *The Six-Cornered Snowflake, a New Year's Gift*, Paul Dry Books, Philadelphia, 2010.

"... God invented and gave us sight to the end that we might behold the courses of intelligence in the heavens, and apply them to the courses of our own intelligence which are akin to them…" [5]

In other words, your conception of geometry will be a reflection of your conception of God, man, and the universe as a whole which is an incommensurable sphere. It is that sense of proportionality, given as a potential by God, which enables man to change from a lower geometry to a higher geometry, anti-entropically, as Lyndon LaRouche has emphasized in so many of his writings. However, that cognitive capability requires a lot of work, as Plato himself indicated in his Letter VII to Dion. It is only after a long period of companionship committed to the good, that is, committed to bringing prisoners out of the cave, that one can acquire a true sense of proportionality.

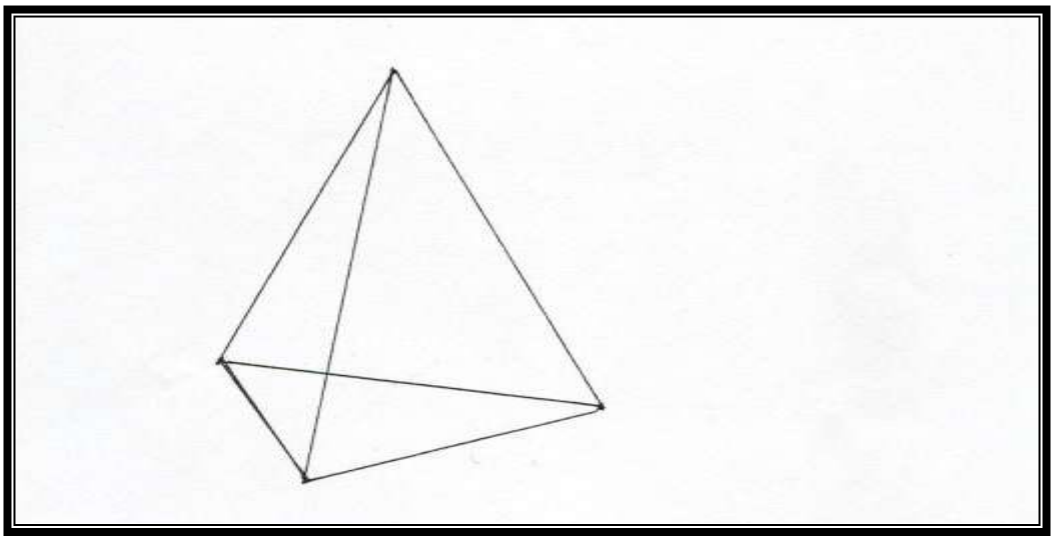


Figure 7 Solution to the Six-Stick Puzzle.

For example, if you try to solve this puzzle by laying the six sticks flatly onto the two-dimensional plane, the problem becomes unsolvable. However, if you raise the sticks into the solid dimension, you solve the problem by creating a tetrahedron that is made of four equilateral triangles.

Pierre Beaudry, Leesburg, November 9, 2017.

[5] *Plato Collected Dialogues*, translated by Benjamin Jowett, Princeton University Press, Princeton N. J., 1961, *Timaeus*, 47 b-c., p. 1175.

PART I
GEOMETRIC CONSTRUCTION BY SIMPLE CIRCULAR ACTION

Figure 8 Pantagruel arriving in Lanternland.

1- THE REVOLUTIONS OF LANTERNLAND ARE NOT ACCOMPLISHED BY SLAUGHTER, BUT BY LAUGHTER

During the summer of 2001, I was invited to give a series of classes in geometry to a group of children, aged 10 and 11. I thought that the best way to begin initiating the children to this important science was to introduce them to Plato’s Cave, and to the issue of the truth about what they see, and how they can discover what lies behind what they see. The challenge was very exciting and very new for me, since I had never taught children of that age. I spent quite a number of hours with the midnight-oilers, who are Francois Rabelais’ studious friends from the island of Lanternland, and I sought the wise council of Epistemon (aka Dehors Debonneheure) and of Pantagruel.

In Lanternland, the midnight oilers always start a discussion by posing a question like a paradox. They ask, for example: “How do you generate a straight line with circular action alone?” This sounds crazy because your eyes tell you differently. It seems impossible for your eyes, but not for your mind. The straight line and the circle line are two separate species of lines, yet, the same non-linear action produces the two of them. A first circular action produces a circle, not a straight line. But, what about a second circular action upon the first one? What does that do? What happens when you fold the circle on itself? You create a diameter, or a straight line! And, when you fold the circle a third time, on itself, you generate the center point! You see, the straight line and the center point are created by a double and a triple self-reflective circular action. In other words, points, lines, polygons, and solids, are sub-species of circular action and circular action comes from spherics. These things don’t exist as self-evident things, in and of themselves. They are all shadows born of circular action. Now, do you see the difference? If yes, then you see with your mind’s eye, not with your physical eyes.

Now, when you only believe in what you see, you become like the prisoners in the bottom of Plato’s Cave who are made to believe they cannot turn around and discover the light of reason which shines into the cave from the outside. They do not realize that the flat and linear shadows they see on the dimly lighted wall of the cave are mere illusions, deformations of the real world that lies outside of the cave. Our job, as human beings, is to bring those poor prisoners out of that cave for some fresh air, and a little bit of sunlight, and help them clean out the cobwebs they have in their attics. So, let us examine the first challenge that strikes you upon landing in Lanternland.

DO YOU ONLY BELIEVE WHAT YOUR SENSES TELL YOU?

In my first class, some of the children did not believe a word I said, when I told them that I had just returned from a long trip to the island of Lanternland, and had paid a visit there to Plato's Cave.

They thought I was making this up to fool them. So, I must repeat here what I told them then. I must tell you the whole truth. Lanternland and Plato's Cave are more real than the physical place they thought I did not go to. And, the reason it is more real is because that very special place is part of the noosphere, that is, the Vernadsky sphere of thinking humanity, which makes you think about the intention behind things. And, that is the most important thing in the world to discover. Again, as Epistemon put it: "Always pay attention to the intention!"

Let me explain. Sometimes, what is unbelievable is truer than what you hear with your own ears, and what you see with your own eyes. Do you only believe what your senses tell you? Well then, let me tell you what happened to me when our ship was approaching the island of Lanternland, and you will see what I mean.

After four days of voyage, the sea was very calm, the air was dry and clear, and the night was absolutely pitch-black. It was so dark that you could not even see your own nose in front of you. On the bow side, a sailor cried out: ahoy! Ahoy! Everybody rushed to see what was happening. The ship tilted, and we saw in front of it hundreds of tiny little flashing lights. Ping, ping, ping, ping, everywhere.

The sailor said these were flickering fish tongues that come out when the fish jump out of the water. Panurge chimed in and said: "no way, this is nonsense. These are bird eyes reflecting off of the light of the ship's lamp. We have just finished eating our meal, and they are coming over to eat the crumbs." My giant friend Pantagruel disagreed completely with the other two and said: "these are distant watch lights. We are approaching the island of Lanternland."

Then, somebody asked me: "What do you think this is?" "I don't know," I replied. I thought Pantagruel was right, because I thought he made more sense than the others. After all, we did expect to touch land soon.

A few minutes later, I saw what seemed to be a large fire on the water. “Land”, “yelled the sailor, “land!” Pantagruel had been absolutely right about the watch lights, I thought to myself. We had arrived in the port of Lanternland in the middle of the night. A group of midnight oilers were waiting for us there, and greeted us with their lanterns. We were asked to choose a lantern to be our guide.

Figure 9 Epistemon the Midnight Oiler, aka Dehors Debonneheure.

A tall and slim midnight oiler, who introduced himself as Epistemon, explained to us that these lanterns were all shaped in the form of spheres, Platonic solids, and Archimedean solids, etc. Their corners and edges were all lit up to show how points and lines are the footprints of spherical action that God used to create all of the elements of the universe. “As the stars in the heavenly sphere guide the sailor in the night,” he said, “so do these lanterns guide the moral and studious person through the obscurity of our time. They provide direction to divine reason and to the principle of spherical composition that everyone here is seeking to discover. However, none of those lanterns contain any truth about life, about justice, or about love of mankind,” he added. “They just contain enough oil in them to lead you to the edge of these important questions.”

“You see,” said Epistemon, “this means that Euclidean geometry does not give you the truth about the physical universe; only clues that you must seek to discover by studying shadows cast by those lanterns. That is why you would be wise to spend a few hours and burn a little bit of midnight oil with our studious people.”

After listening to a lot of these heavy ideas, I concluded that I could no longer trust my eyes to tell the truth. I guess that is the reason why God gave us two eyes; everyone must have a second chance. What everyone saw in the distance were, in fact, the false impressions of the lanterns of Lanternland. They were not merely watch lights marking the land for the safe piloting of sailors; there was a higher purpose to be discovered in the intention that lay hidden behind the spherical construction of the lanterns.

“That is absolutely right,” said Epistemon, “and what you don’t see with your physical eyes, you must now try to discover with your mind’s eye.”--“Bull’s eye,” cried out Panurge, “right on the nose of mind over matter.”

2- WHERE LINES AND POINTS COME FROM

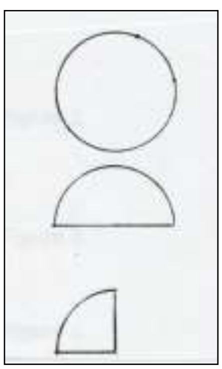
Aristotle's construction originated from Flatland.

A straight line is an infinite number of points joined together and going in the same direction. Like this: ....

Result

For Aristotle, lines and points are not created, but are given to you as the smallest self-evident things that can be perceived by your sense perception; that is, by your physical eyes. He called that: "learning."

Plato's construction originated from circular action in Lanternland.



1. Create a first circular action that closes perfectly on itself. This action creates a circle.

2. Create a second circular action that closes the circle perfectly on itself again. This creates a diameter.

3. Create a third circular action that closes the half circle perfectly on itself. This creates a point and two radii.

Figure 11

Result

Here you see how points and lines are actually created; that is, where they come from, because you have generated them by constructing them yourself! Now, you see with your mind's eye. Olato called that "cognition."

3- HOW THE LEAST ACTION PATHWAY OF LANTERNLAND WAS DISCOVERED BY WAY OF A CONTEST

Problem

On the island of Lanternland, there are three villages called Fig, Fa, and Fu. One fine day, the people living there decided to have a contest, and offered a prize to whoever would find the most economical road system that would link the three villages together; that is, finding the shortest interconnected pathway between them.

One man said that the best way would be to make the roads in the shape of a triangle, and he proposed to connect the three villages with three straight lines.

A second man said that was wrong, and that the solution was to make curved roads, connecting the three villages with circular arcs.

A third man said that the first two ideas were wrong, and he proposed to have straight lines between the three villages, but connected through their common center.

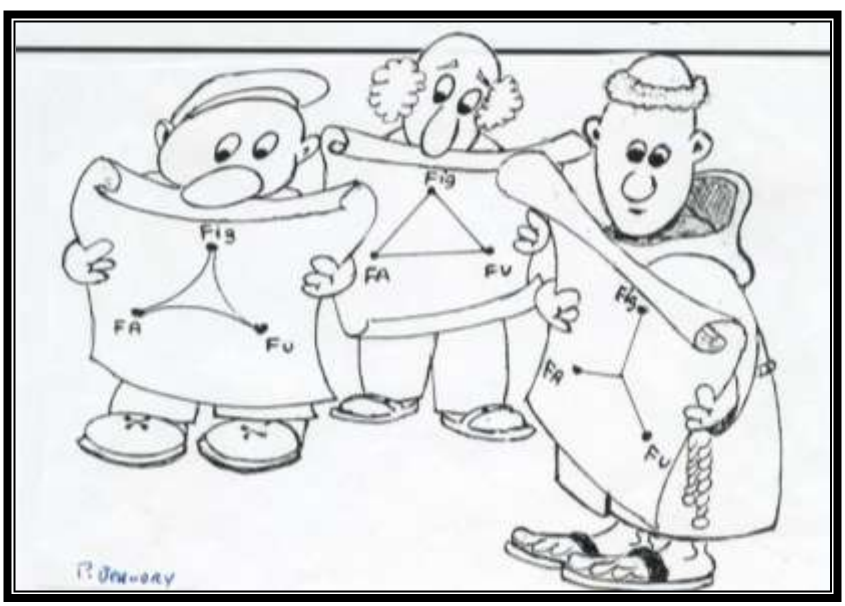


Figure 12 WHICH IS THE LEAST ACTION PATHWAY?

Figure 13 THE ANSWER OF PACIOLI

It was the third man who gave the correct answer. This is the great Italian Renaissance man, Friar Luca Pacioli, who showed that the most economical distance between the three villages must be determined by the natural angle of 120 degrees. Why? Because this is the favorite angle that nature chooses to produce what is called least action pathways of non-living processes. This is the angle chosen by soap bubbles when they stick together. The great astronomer Johannes Kepler discovered that this was also the favorite shape of the snowflake and of a lot of crystals, like quartz and garnets; and that bees construct their beehives in the same 120-degree angle. This is also known as 6 sided close packing; that is, when hexagons come together without leaving any space between them. All of these natural forms are created by a preordered intention of circular action, which God has chosen to determine as the basis for three of the Five Platonic Solids. Do you know which ones they are?

4- HOW TO USE YOUR COMPASS AND DIVIDER TO CONSTRUCT A HEXAGON INSIDE OF A CIRCLE

How to use your instruments

1. Use of the compass

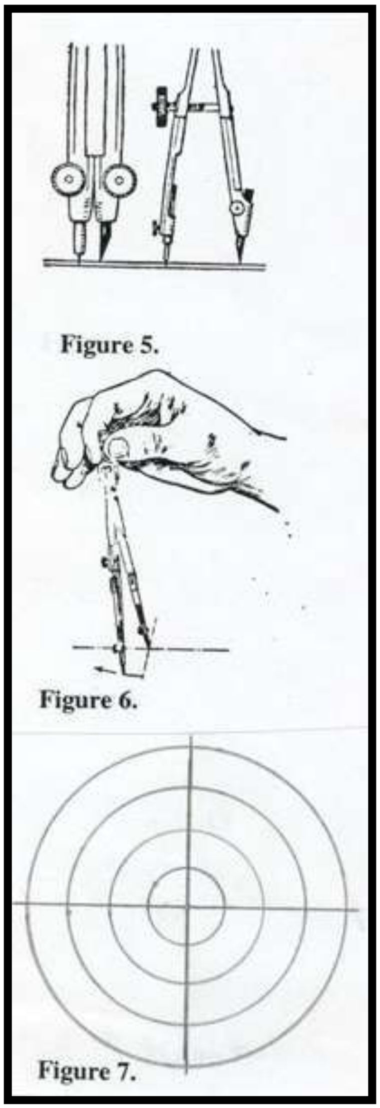


Figure 14[6]

a. The needle point of the compass must be firmly set.

b. Extend a sharp pencil lead a little less than the needle point.

c. To draw a circle, guide the position of the needle with your left hand, then raise the fingers of your right hand to the handle and draw the circle in one rotation.

d. Roll the compass with your forefinger and thumb only.

e. As the circles get larger, incline the compass slightly in the direction of the rotating motion.

[6] The illustrations 14, 15, and 16 are from Thomas E. French, *Engineering Drawing*, McGraw-Hill, New York, 1947.

Construction

1. Construct two diameters by folding a sheet of paper twice on itself. Trace the two lines with a pencil.

2. Place your compass at the intersection of the two diameters, and draw a series of 4 equally spaced concentric circles.

3. It is the radius of the first circle which determines the size of the other circles.

2. USE OF THE DIVIDER

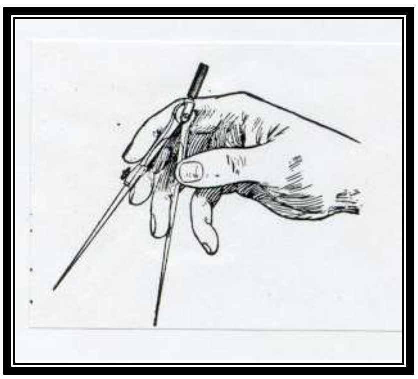


Figure 15

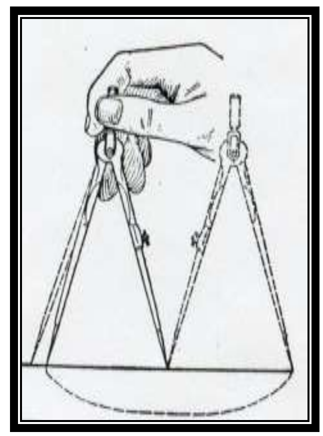


Figure 16

1. Only four fingers of your hand, not your pinky, are required to use the divider.
2. Open the divider with one hand only, holding it between your forefinger and your thumb.
3. Use your second and third fingers to open and close the divider into any position you want.
4. For small divisions, slip the second and third fingers gradually out of position.
5. The divider is used for dividing lines into equal parts, or to measure lines.
6. Knowing how to use the divider is essential for all sorts of constructions.
7. To measure lines, or to divide, hold the handle between your thumb and forefinger. Do not lean on the arms.
8. To divide a line in half, that is, to bisect a line, eyeball the mid-point, and set one arm in that position. Lift the other arm and rotate the divider to the other end. Reduce the excess, or the shortfall, by estimating the half or the remaining part one more time.

Problem

How do you construct your own instruments by circular action? Create an equilateral triangle and scalene triangles of 30, 60, and 90 degrees.

Construction

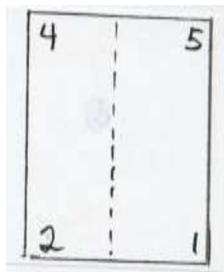


Figure 17

1. Fold a sheet of paper in half lengthwise and mark the corners1, 2, 4, and 5.

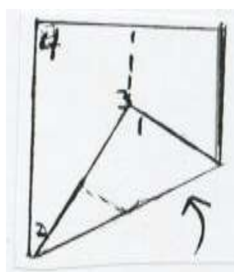


Figure 18

2. Rotate corner no. 1 to 2 onto the middle-fold and mark 3at that point.

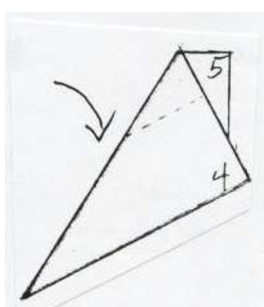


Figure 19

3. Fold no. 4 over the corner of no. 2 and no. 3.

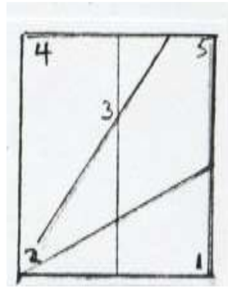


Figure 20

4. Open the sheet of paper and trace the folded lines 2-3 with a pencil.

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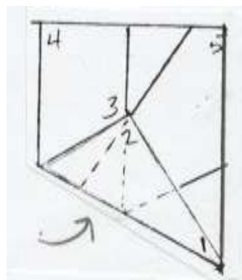


Figure 21

5. Rotate corner no.2 onto point no. 3, and lineup 1, 2, 3 to make the fold

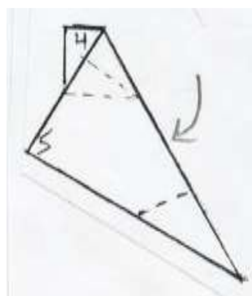


Figure 22

6. Fold no. 5 over the edge of no. 1, 2, and 3.

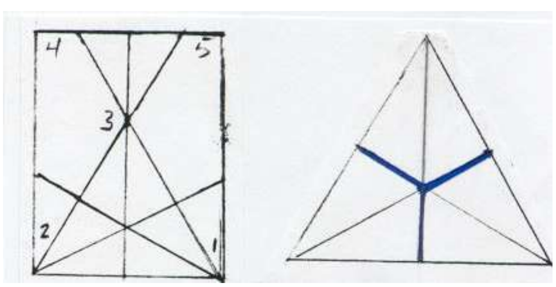


Figure 23

7. Open the sheet of paper and trace all of the folded lines with a pencil: you have created an equilateral triangle composed of 6 scalene triangles of 30, 60, and 90 degrees. Plato would like that because you have created all of those lines by circular action alone. Trace the remaining lines with a pencil, and cut off the excess paper. Trace this outline on a piece of cardboard, and cut it in half to create two scalene triangles.

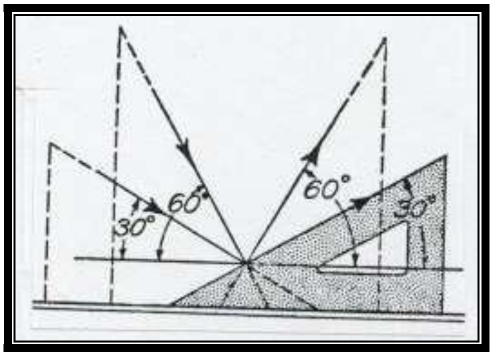


Figure 24

8. Use the two scalene triangles to create angles of 30, 60, 90, and 120 degrees.

CONSTRUCT THE HEXAGON INSIDE OF A CIRCLE

Instruments

1. A compass.
2. A set of two 30, 60, 90, and 120 degree scalene triangles.

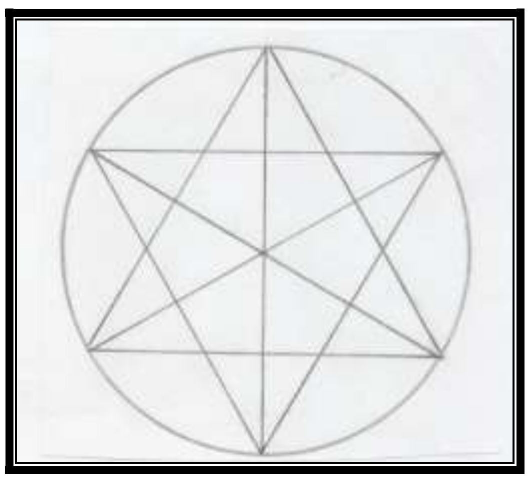


Figure 25

Construction

1. Draw a circle with a 4” diameter.

2. Draw a six-pointed Star of David inside of a circle by making six successive changes of position of the scalene triangles, one after another.

HOW NATURE FINDS THE SHORTEST PATHWAY

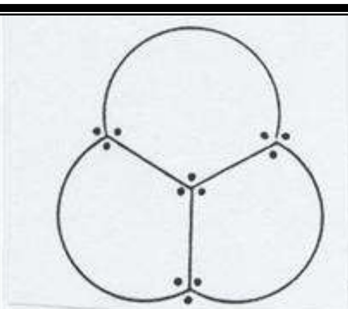


Figure 12
Soap bubbles

Figure 13
Crystals

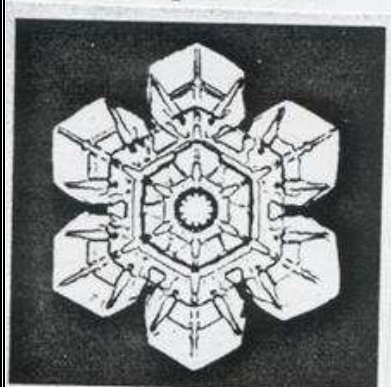


Figure 14
Snow flakes

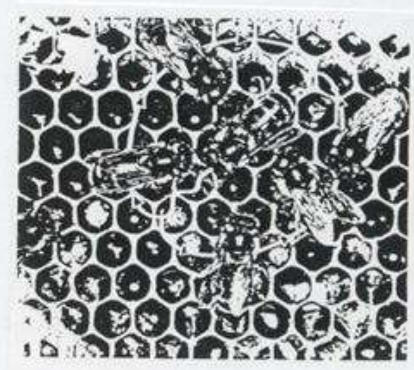


Figure 15
Beehive

Figure 26

5- CAN BEES MAKE CREATIVE DISCOVERIES?

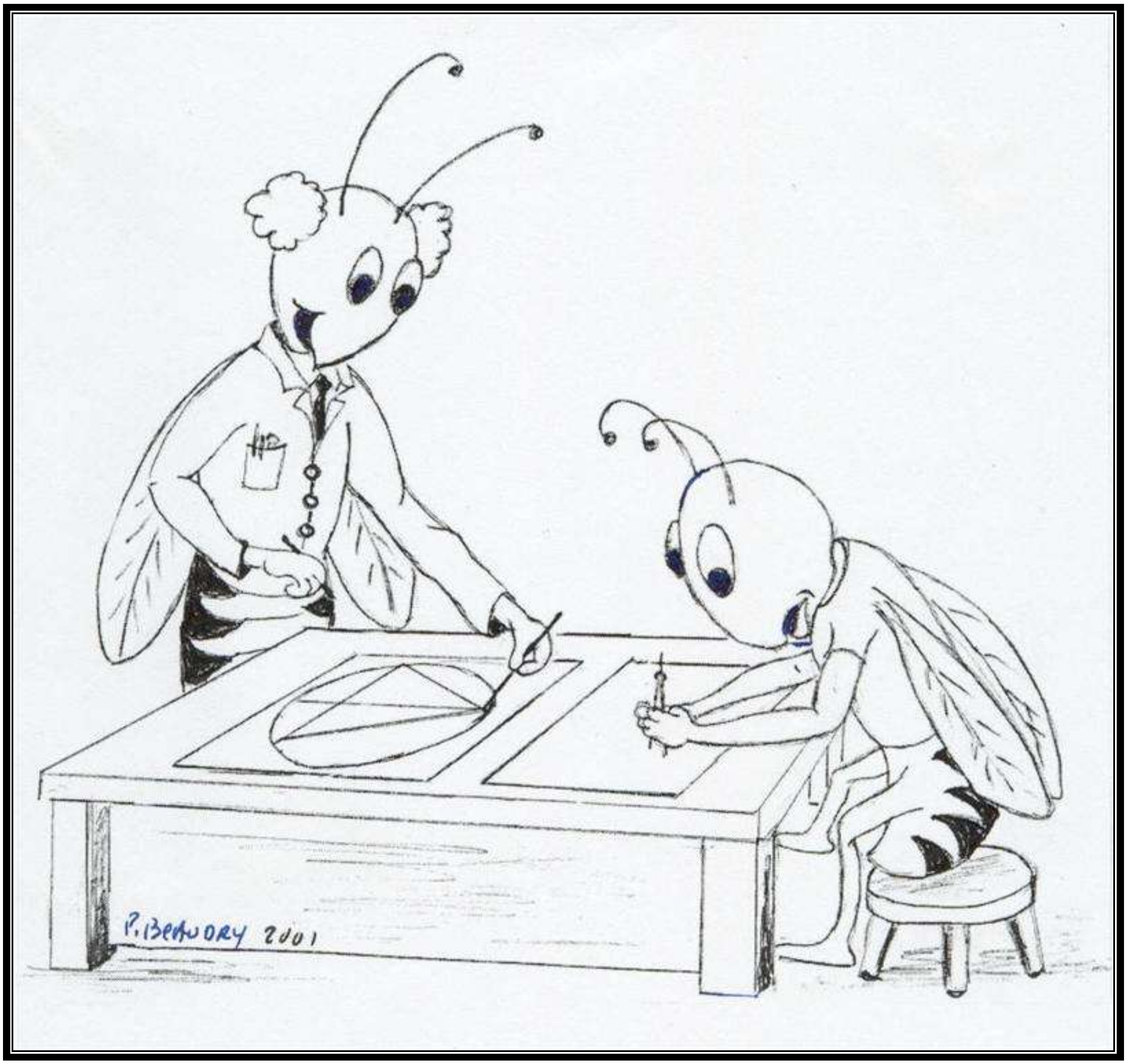


Figure 27

Some people say that honeybee workers are like Platonic thinkers: they use circular action patterns to announce where their ideas are located, and how to discover them. If you have a chance to observe honeybees closely, you will discover that in order to communicate with other bees, one of them performs two kinds of circular dances.

When flowers are discovered to be closer than about 100 yards from the beehive, the dancer bee will perform a circular dance. The change of direction of the dance gives the orientation for the newly found discovery.

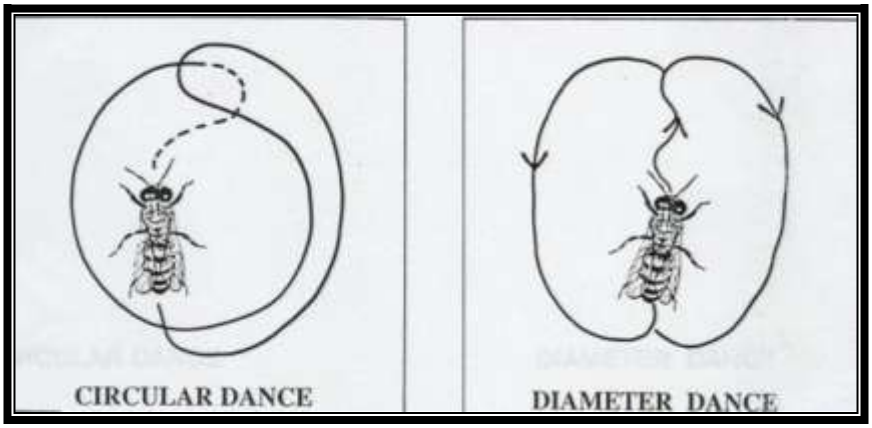


Figure 28[7]

If flowers are discovered further away than 100 yards from the honeycomb, the dancer bee will communicate the location of the find by doubling the dancing within the circle. It will divide the circle in half, and do a diameter dance, which will double the circular action. The diameter tells the other bees at what angle the food source is located in the biosphere. This method can be very accurate up to several miles.

Such an amazing circular action language is a beautiful example of what Plato called *Hylozoic Monism*; that is, how the universe as a whole is alive, and is determined by a unique law of underlying multiply connected circular action. The communication between honeybees, and the orientation of their intention, is a clear reflection of the existence of a single harmonic ordering principle in the universe as a whole, and provides us with a special case of cognition located at the frontier of the non-living, the living, and the cognitive.

Is this, in the minds of the bees, the reenactment of a discovery, and the communication of a universal principle that is transmitted to the other bees? Does this

[7] Illustrations 28, 29, 30, and 32 are from H. & A. Fisher-Nagel, *Life of the Honeybee*, Carol Rhoda books Inc., Minneapolis, 1986.

mean that honeybees can think like you; that is, can make cognitive discoveries? What is a cognitive discovery?

CAN BEES MAKE AXIOMATIC CHANGES?

Think about this. Can a bee change its mind and decide to do something else besides making honey and hexagons? Can a human being change his mind and decide to break with his pattern of habits? If you can understand the difference between those two questions, you have made a cognitive discovery.

Think again. Can a bee make such a distinction? No. Why not? Because, bees are not capable of making profound changes in their lives; that is, they cannot make axiomatic changes. You see, an axiomatic change occurs when you take an axe, and chop off the bad habits in your life. Now, a bee cannot do that because it does not have bad habits. Besides, have you ever seen a bee with an axe to grind? ∗

Bees will always follow the same laws of the universe, because God gave them only good habits. Bees cannot change their patterns of good behavior. You will never hear a bee saying: “well, bzzzzzzzzz, I am tired of making honey. From now on, Bzzzz bzzzz bzzzz, I think I am going to make strawberry jam.” In fact, if it said that, it would be told to buzz off immediately, and it would be forced to leave the beehive. It simply could not do it. And besides, its jam would taste absolutely awful.

On the other hand, human beings are capable of making fundamental changes in their behavior. For example, you can say to yourself: “instead of being distracted and noisy in class today, I am going to be quiet and pay attention.” Now, I grant you, that might represent a miraculous change for some of you, but you could do it, if you wanted to! That is my point. That would be an axiomatic change. And, that is what makes human beings uniquely superior to all other creatures. Such is a cognitive discovery. With such discoveries, human beings are capable of changing the whole universe; that is, if they want to!

∗ The Lanternland axiom of truth says in Lanternspeak:
• ┌ ◂ • ⫽ • ✔ ? ✔ ● ✔ ⑊ ┌ ▲ ⫽ ■ ✖ • ● ⎯┘□□□. Translated into good American English, this means: “He who has an axe to grind has got to change his mind.”

WHAT LIES BEHIND THE FAMILIAR FACES OF THE HONEYCOMB

THE BEEHIVE AND THE ART OF SIX-CORNERED-PACKING

What is in the intimate recess of the bee's soul that makes it prefer the six-corner packing? What purpose did God have in providing the honeycomb with a hexagonal architecture? In his investigation, Kepler found four main reasons.

Only three plane surfaces can cover a surface without gaps: the triangle, the square and the hexagon. Among those three, it is the hexagon that is the roomiest, and can contain the maximum amount of honey.

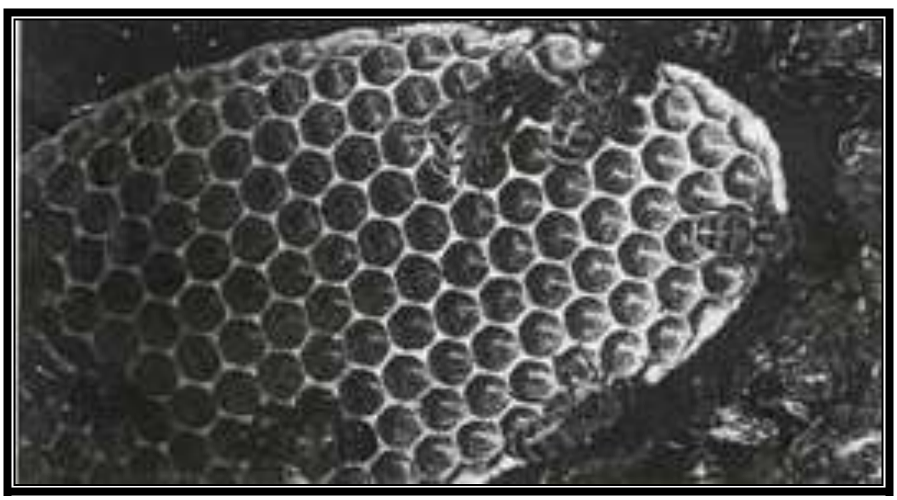


Figure 29

The tender bodies of the bees are more comfortable with more obtuse angles. The square has deeper corner recesses. With its acute angles, the triangle is even more uncomfortable, and the toes of the bees would get squished. That is not ideal, that is an ordeal. Note that the hexagon seems to be closer to the circle than the triangle, and the square.

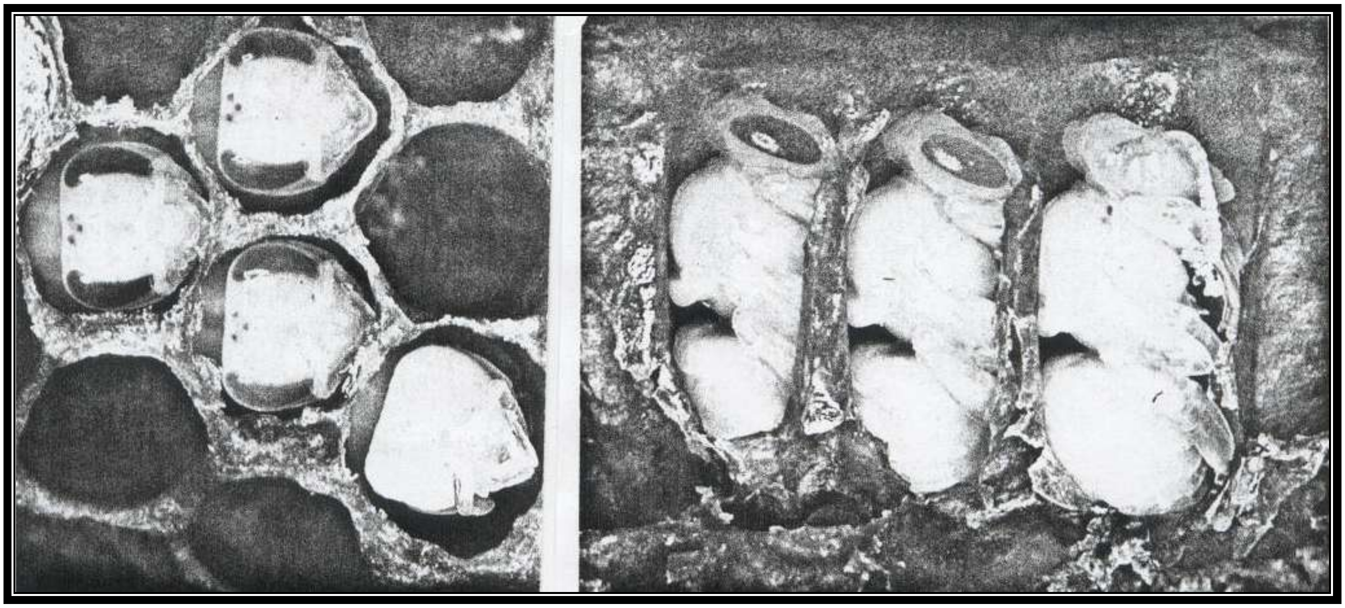


Figure 30 Top and side views of bees being hatched.

The hexagonal shape saves labor and area. There is no room to spare, and the stability of the frame is stronger by the introduction of the rhomboid shape ◊ of 120 degrees. Round shapes would leave gaps through which cold air would seep through in between the cells. The hexagonal shape is self-insulating.

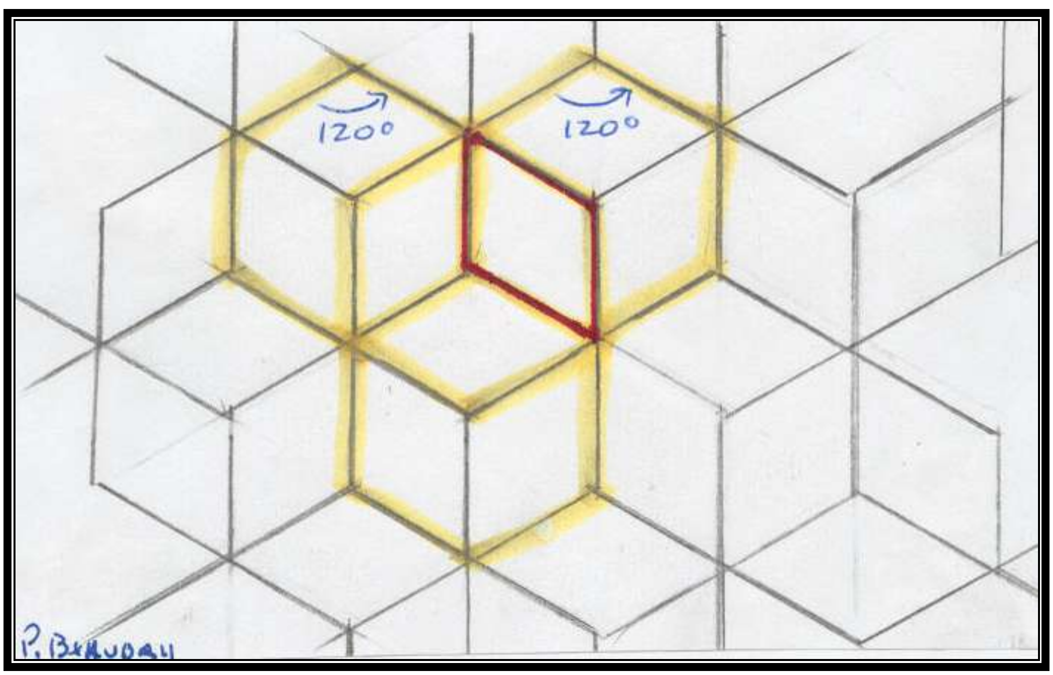


Figure 31 The rhomboid shape figure

Perfection, beauty, and community represent the qualities of soul that orient the bees to prefer the physical geometry of hexagonal and rhomboid shapes. Bees are very social, and like human beings, they love to have a lot of neighbors. Can you figure out why a bee manages to get 9 neighbors? Between you and me, it is a good thing that bees all go to bed at the same time at night. Don't you think?

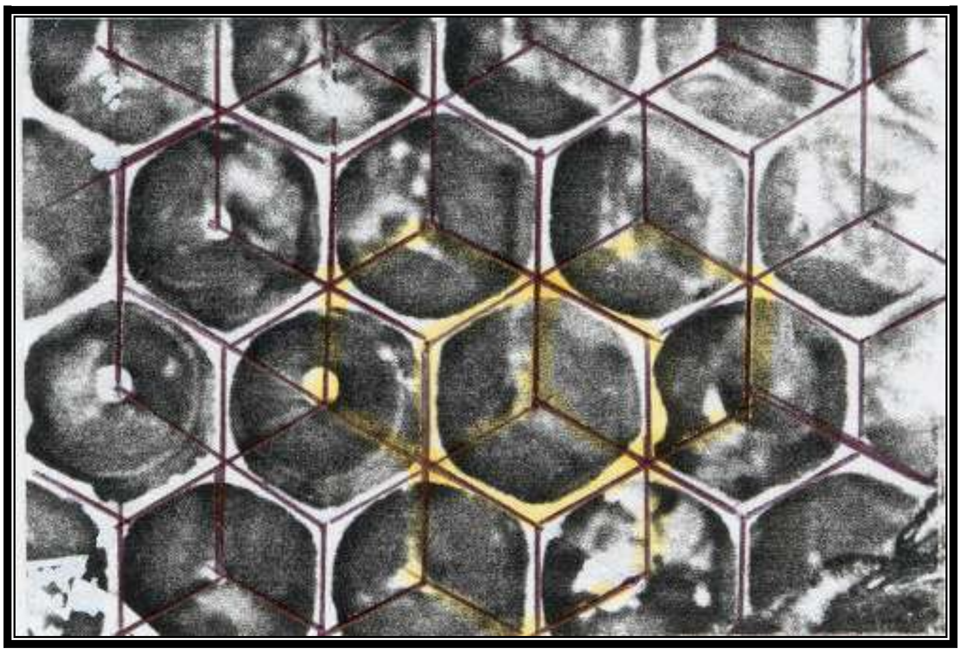


Figure 32

THE SPHERICAL PRINCIPLE BEHIND HEXAGONAL CLOSE-PACKING

Problem

Inside of a honeycomb, each bee shares not only 6 walls with 6 other bees in the same row, but also shares 3 planes in the bottom of the cell, with 3 other bees in the second row. Thus, each bee has 9 neighbors. If the cells were covered all around, each bee would have 12 neighbors! Do you know which Platonic solids are formed by close-packing of 12 balls of the same size?

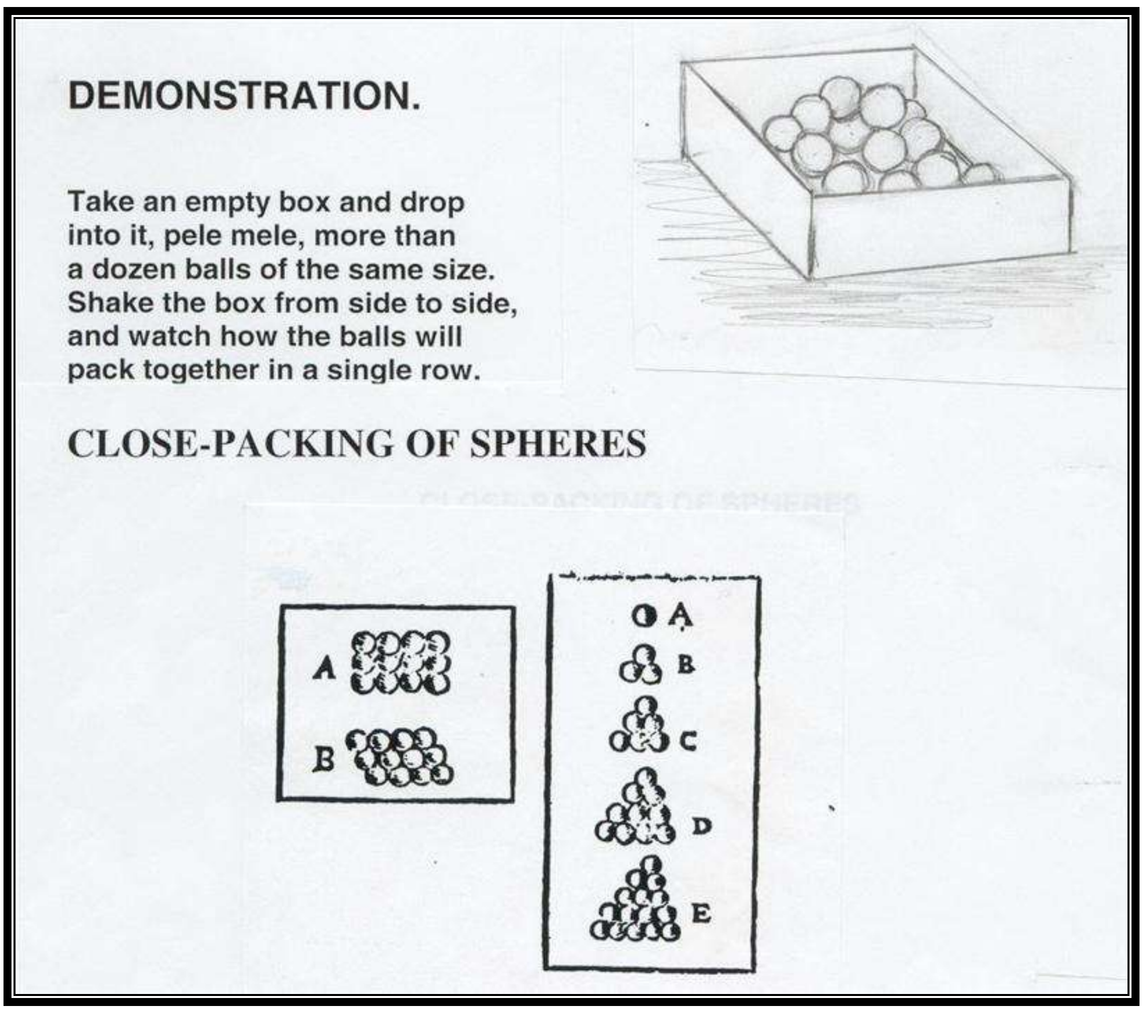


Figure 33

PART II

HOW THE AXIOMS OF FLATLAND WERE CHANGED

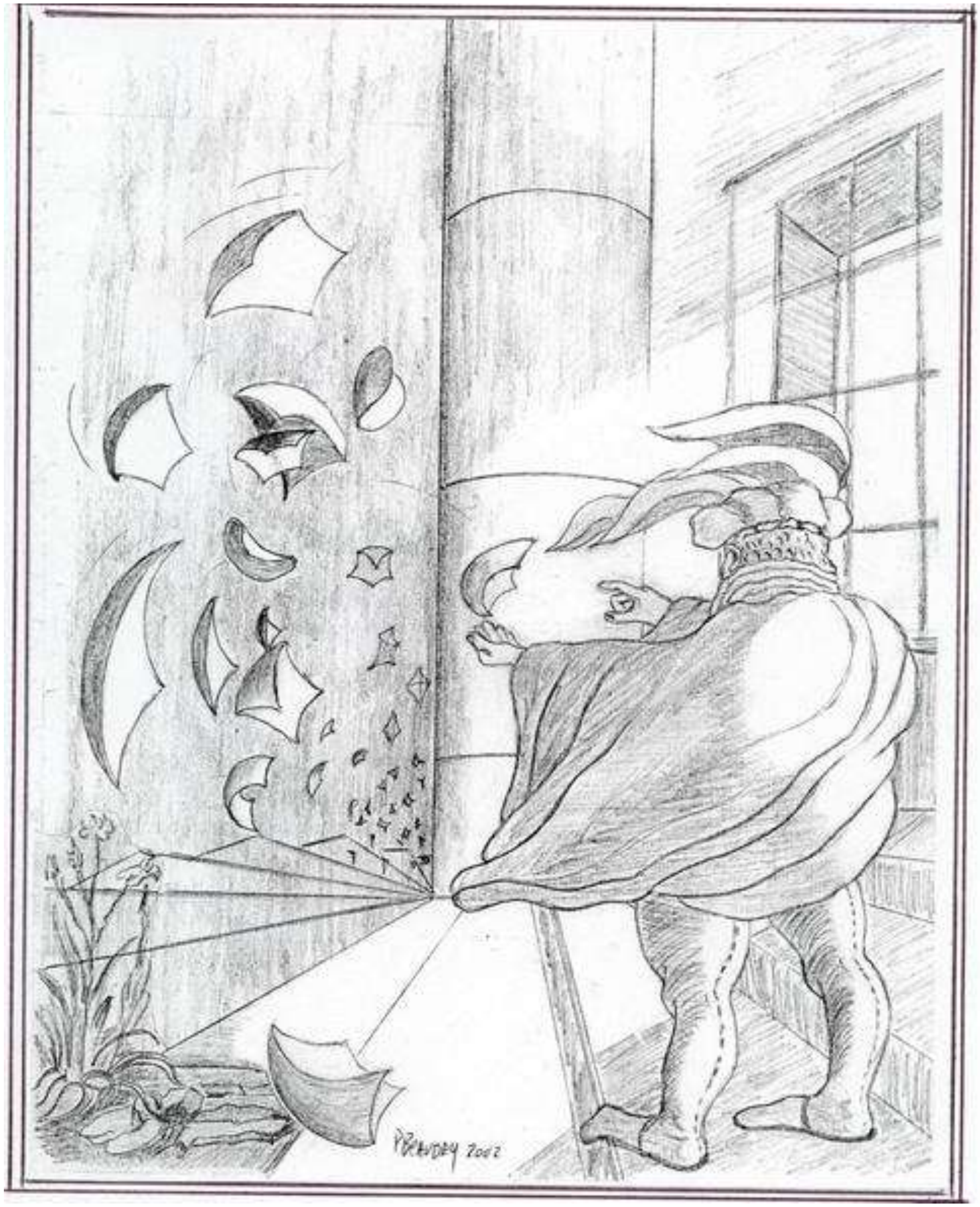


Figure 34

The squares had obviously won their revolution since the polygons and the circles were all very flexible, and were standing upright on their edges making all sorts of somersaults and circular gestures as they were coming out of the cave, ready to fold and unfold.

6- HOW PANTAGRUEL BUSTED THE AXIOMS OF THE FLATLAND SQUARE AND BROUGHT HIM SAFELY OUT OF PLATO'S CAVE

The day after we had arrived in Lanternland – and I swear to you that the midnight oilers are the most extraordinary people in the whole world – our guests invited us to dinner. They live only on ideas. They drink nothing and they eat nothing but ideas. And the ideas they eat and drink are not just any old idea that you can pick up along the sidewalk. Oh no. They eat axiom busting ideas; that is, they crush old ideas and mill them into new and better tasting ones. And that takes place mainly in Plato's Cave, which is located there. So, by late morning, I came across Pantagruel and we were brought to visit the cave and to meet with its inhabitants. At the entrance of the cave there was an impressive inscription engraved in capital letters:

"DESTINY LEADS THE WILLING, BUT THE UNWILLING DRAGS"[8]

That sounded ominous. We entered and made our way down through a lot of steps, and through a long and wide landing space. In the deepest recess of the cave we found the Flatlanders all staring at the back wall and attempting to make sense of the deformed images that were projected there from the source of light that came from outside of the cave. They were like a bunch of kids watching TV and believing that what was on the screen represented the real world.

"What are you looking at," asked Pantagruel?

"We are studying the lines, "replied the square. "We are trying to make sense of what we see. Imagine an immense sheet of wall paper on which straight lines, triangles, squares, pentagons, hexagons, and other figures moved freely about on the surface instead of remaining fixed in their places, but without the power of rising above that surface or sinking below it. They cannot project any shadows because they cannot stand-out. This sort of universe can only be flat like a two dimensional universe. How can a two-dimensional mind think of higher things?"

---

[8] Latin inscription on the Temple of the Bottle in Lanternland: ***DUCUNT VOLENTEM FATA, NOLENTEM TRAHUNT***. Rabelais, Op Cit., p. 689.

"Right," responded Pantagruel, "how can they think of anything that passes over their heads? You are only seeing illusions on this wall. If you want to have 'a higher view of things,' as you say, you must break your chains and come outside of the cave with me to discover the source of your illusions."

"Oh no," replied the square, "in such a country you will immediately see that it is impossible for a solid to exist. All we can see are triangles, squares, and other figures, moving about as I showed you. Nothing can be recognized except straight lines and flat surfaces; and I will tell you the reason in a jiffy."[9]

"But wait," said Pantagruel, "why do you insist that you can only distinguish things with your physical eyes? Is your mind only capable of apprehending what is given to your senses and nothing else?"

"Absolutely," replied the square. "As my great ancestor, the great square Aristotle once said: 'There is nothing in your mind that has not first been registered through your senses.'

"But if this is true," said Pantagruel, "what about ideas?"

"All ideas come from your senses," replied the square. "There is nothing else in your mind."

"Boy! This guy is in real trouble," thought Pantagruel to himself. "A real brainwashing job has been done on him. I've got to help him out." As quickly as he could say "come on," Pantagruel grabbed the square by the right angle, and peeled him off from the flat plane.

"Hey!" screamed the square, "you can't do that. I am only a two-dimensional being. I can't go flying into the third dimension like that!"

"Oh no?" said Pantagruel, "you just watch! How do you think you were created in the first place?" Then, Pantagruel swung the square round and round gently in the air until he became flexible enough to go outside of the cave on his own.

"Now," said Pantagruel, "bend in half!"

"I can't do that. That is impossible," said the square.

"Yes you can do it," said Pantagruel, "you can bend in half because this is how you were created in the first place, but you don't remember that."

"But I always thought I had been created by two points at the end of a moving straight line," replied the square in a state of perplexity.

"That is nonsense," said Pantagruel, you were created by circular action."

---

[9] Several parts of the dialogue in this section were inspired by Edwin A. Abbott, *Flatland, a Romance of Many Dimensions*, Dover Thrift Edition, New York, 1992. **http://www.geom.uiuc.edu/~banchoff/Flatland/**

After many attempts during which the square was fighting his backward linear tendencies, all of a sudden, as if with no effort at all, the square bent completely on himself from corner to corner.

"Hey, I have done it!" said the square in wonder. "This is very exciting. I never thought I could do gymnastics like this before."

"This is not gymnastics," replied Pantagruel, "this is thinking."

The square was so happy he could hardly stay in one place. "This is fantastic," he said. "Let me do the other two corners." And then, he bent down on the other side of himself at a right angle to himself, and he folded the two corners flat on each other without any help. "What a discovery," said the square. "I can make right angles and 45 degree angles everywhere, in all directions, backward and forward, just by folding on myself. And that generates lines and points." This was almost too much for the square to endure.

"You see," said Pantagruel laughing, "you have discovered the idea of circular action. You don't see it with your physical eyes, but you see with your mind's eye that the results are beginning to appear in the shadow that you are now projecting by folding and unfolding yourself." You can now reflect on yourself. The square was so ecstatic that he never wanted to go back into Flatland again. After a moment, he calmed down and smiled.

"Wow!" said the square, "this feels real good."

"You are not 'feeling' good about this," said Pantagruel, "you are 'thinking' good about it. You have now discovered your first true idea, a Platonic idea that did not come from your senses. And the idea is that all squares are made in the image of their creator; that is, they are all proportional to circular action, and to the sphere! That is not just an idea, this is a revolutionary idea. At some later date, I will show you how you could even transform yourself into a cube."

Figure 35 Condition of the square in Lanternland.

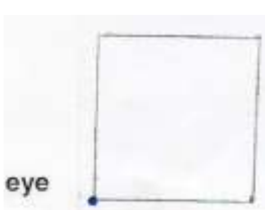


Figure 36 Condition of the square in Flatland.

"You see," said Pantagruel, "you have been told lies all of your life by the priesthood circles in Flatland who had a class interest in keeping you edgy, flat and laying low; and that is why you were never told you could fold by circular action. You have been treated like a mushroom. Do you know how to grow mushrooms," asked Pantagruel?

"No." said the square.

"By keeping them in the dark and feeding them bull shit," replied Pantagruel "That is the condition that you and your people have been in for too long in your country. My dear square, you must now take leadership and go back to free your people."

"But what if they don't want to follow me," asked the square? "I know my people very well; they are stubborn, very stubborn."

"I know," replied Pantagruel, "I just discovered that myself. If you were able to discover the truth, so can they, and I will be there to help you."

Later that evening, everybody celebrated the square's victory, by sharing the agapes in a great banquet; a feast of ideas at the Lantern Inn.

7- HOW THE SQUARE FROM FLATLAND DISCOVERED THE WAY TO DOUBLE HIS SURFACE AREA IN SIZE WITH THE USE OF THE SOCRATIC METHOD

After we had celebrated the victory of the square for his successful escape out of Flatland, we returned to Plato's Cave, and Pantagruel brought up for discussion the discovery of Plato's famous problem of how to double the area of the square.[10]

"I would surely like to know how to do that," said the square, but I am afraid that this is totally beyond my capabilities. You see, us polygons have been limited for a long time while we lived in Flatland, and we were not even allowed to investigate where we came from. We were told that we could not change, and that we had to spend our entire life with the same predetermined size. That was the fixed rule, and it could never be changed. Also, we've gotten so used to the flatness of our figures, that we never even thought of exercising our minds and discover that we came from circular action."

"That's quite all right," replied Panurge, "I know a lot of people who never use their minds either. But, you can change that. Like Yogi Berra said: "You don't have to be an intellectual to use your head."

"Very well then," said Pantagruel, "listen carefully and I will teach you the Socratic method of constructing your knowledge by yourself, and not with magic either. In Plato's *Meno* dialogue, Socrates demonstrated how a slave boy was able to make the discovery of doubling the area of a given square with only the help of a few questions.

"Wow," said the square, "you mean to say I can double my size."

"Yes," replied Pantagruel, "this is how the problem was introduced."

"Socrates drew in the sand a square **A, B, C, D,** whose area is 4 square feet. This means that the side of the square is 2 feet.

[10] Plato, *The Meno*, in *The Collected Dialogues of Plato*, Edited by Edith Hamilton and Huntington Cairns, Princeton University Press, 1961, p. 353.

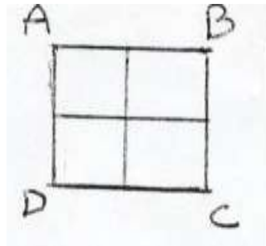


Figure 37

“Then, Socrates asked the slave boy to determine another figure that would be double the size of the first one; that is, a square with an area of 8 square feet. What do you think the slave boy did?” asked Pantagruel

“I don’t know,” said the square, “what did he do?”

“The first thing the slave boy did,” replied Pantagruel “was to think that in order for the second square to be double the size of the first, he had to double the sides of the first square. What do you think of that?” asked Pantagruel

“That sounds reasonable to me,” said Panurge, “I don’t see anything wrong with that. Besides I don’t see anything else that can be doubled inside of **Figure 37**.”

“I don’t know,” said the square, “maybe there is something else that can be doubled but which is not visible to your physical eyes. Maybe we can only see it with our mind’s eye, like the case of circular action. Remember, it was only a few hours ago that I discovered I could fold in half.”

“That is a very important thought,” said Pantagruel, “you are now paying attention to the intention. Hold on to your thought and we will get back to it in due course. Meanwhile, let’s first examine if the slave boy’s idea was right. The slave boy thought and also you, Panurge, that if you double the sides of the square all around, the new square will be such that each of the 4 sides would be made up of 4 feet. But then, the square will have an area of 16 square feet, and that is much too big. Don’t you think the square of 8 will have to be smaller that the square of 16, and also larger that the square of 4? Then, Socrates drew the following square in the sand, with the following divisions.

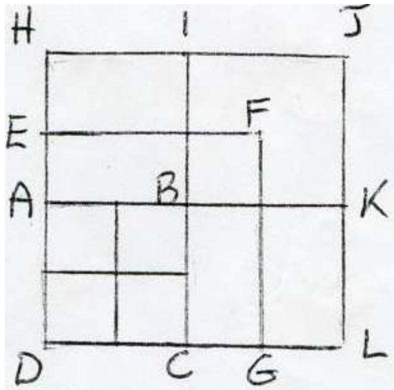


Figure 38

"The slave boy realized his mistake," said Pantagruel, "and he began to think that since the side of the square of 8 had to be longer than 2 feet, and less than 4 feet, he would try the side of 3 feet. But, then," said Pantagruel, "if the side is 3 feet, the area will be 9 square feet, which is still more than 8, as the square **D, E, F, G,** shows. How do you solve that dilemma," asked Pantagruel?

"I know how to do it," said Panurge boldly. "Just divide the whole area of 16 into two equal parts. That will give you 8 smaller squares of 1 square feet each."

"Yes, but how will you make my "square" figure out of those 8 little squares," interrupted the square? "You don't know what you are talking about."

"Just make a rectangle out of them," replied Panurge. "It's the same thing."

"No way!" replied the square. "You've got some nerve. You can't just have any old rectangle replace a square like that. Squares have four equal sides," said the square, "that is our uniqueness, our characteristic distinction you know. And we are proud of that."

"Oh! I am sorry," said Panurge, "I didn't mean to pull on your edges in the wrong way."

"Well then, how do you solve the problem," asked Pantagruel. "One can see that the square of 8 would need to have a side a little less than 3 feet, and a little longer than 2 feet. In other words, somewhere between 3 and 2, there must lie the side of a square whose area is 8, but it doesn't look like we are going to find it with this method of approximation. It seems like we might never find that side even by infinitely dividing it. There would always be a little gap left over, no matter how small it may be, between the lesser than and the longer than. So, how do you go about finding the exact side of the square of 8," asked again Pantagruel?

"I just don't know," replied the square. "I have no idea," replied Panurge.

“Then,” said Pantagruel, “just when the slave boy was getting perplexed and was beginning to give up on finding the solution, just like you too, Panurge, Socrates observed that, at the beginning of this process, the slave boy had been somewhat bold in saying that he knew how to solve the problem, but then he started to doubt himself and became perplexed. Now you’re also like the slave boy. Not only do you not know the answer, but you don’t even think you can find the answer.”

“Quite true,” replied the square, “I have to admit that this is the only certainty that I have. I know, for an absolute fact, that I don’t know how to solve this problem. That’s for sure.”

“And so do I,” admitted Panurge grudgingly. “I have to admit that all I know is my own ignorance.”

“Well, that is precisely the point that Socrates was making,” replied Pantagruel. As Nicholas of Cusa said about the Socratic Method: “The important thing to acquire is *learned ignorance*.”

“Do you think that now the slave boy knows more than he knew before,” asked Pantagruel?

“Well, at least he knows he doesn’t know,” said the square.

“That is right,” said Pantagruel, “and that is why his becoming perplexed made him a better person. Do you suppose the slave boy could have made the discovery of how to double the area of the square before he experienced this perplexity, and before he recognized his ignorance,” asked Pantagruel?

“I don’t think so,” said the square. “That is, I don’t know, I mean, I think I understand, but then, what do I know? Learning ignorance seems like such a strange thing to learn. How can I know more by knowing less?”

“You’re telling me,” said Panurge. “How much ignorance can a guy learn in one day? But then again, the advantage is that, if you forget that kind of knowledge, it’s no big loss, no matter how much you didn’t know.”

“Shush,” said the square. “I think I am beginning to understand something. The point that Socrates is making means that you cannot make any discovery unless you humble yourself and become perplexed, because you discover that the previous knowledge you had was wrong. Is that what Cusa meant by learned ignorance?”

“That is absolutely right,” replied Pantagruel “it is only after you have recognized your own ignorance that you can begin to know something. That is the wisdom of Lanternland.” Then, said Pantagruel, “Socrates erased the two squares he had drawn in the sand, and drew again a new square of area 4, like this **A, B, C, D**, with a diagonal line across it, from **D** to **B**, and added to it a second square **B, C, E, F,** like the first one, and then a third one, whose corners are **C, E, G, H**, and a last one **D, C, H, J**. Add a diagonal for each new square as below.

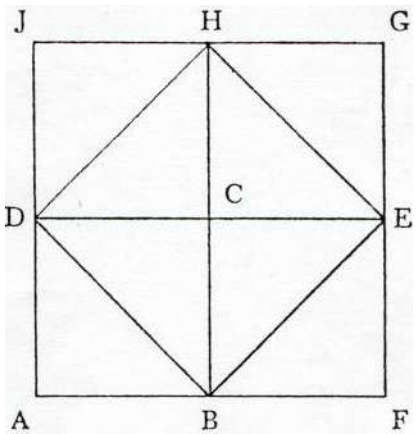


Figure 39

Pantagruel asked: “What did Socrates do this time that he did not do before?” Both Panurge and the square were silent and stared at the figure for a good two and a half minutes. Then the square replied: “He made the squares bend over and fold on themselves on the diagonal, like you had me do, Pantagruel, when I discovered circular action by folding.”

“It is also the same area as the previous square of 16,” said Panurge, “but divided differently.”

“That is correct,” said Pantagruel, “Socrates did not divide the sides of the square like the slave boy did, but he divided the area of the same squares with diagonals by folding. And each of these diagonals divided each square into two equal parts. Then, it became clear that the solution of the problem of doubling the square was staring us in the face.”

“What do you mean?” asked Panurge. “I don’t see that.”

“Don’t you see, said Pantagruel, “that these diagonals, which divide each of those four squares into two parts, also divide the square of 16 into two equal parts? It is the same in the parts as in the whole.”

“I suppose so,” replied the Panurge: “So what?”

"Then, do you realize that four folds of such squares produce another square **B, D, H, E** which is half of the large square of 16**, A, J, G, F**," asked Pantagruel?

"Of course," exclaimed the square, with total excitement and wonder. "**THAT IS THE SQUARE OF 8.** We have doubled the area of the square of 4!"

"Isn't that beautiful?" added Pantagruel enthusiastically? "The slave boy discovered that he could double any square by squaring its diagonal for the benefit of three other squares. There is no longer an approximation of greater than and lesser than, that was sought for by dividing the side of the square. You have now relived one of the most important discoveries of ancient Greece, a discovery that you should not forget to transmit to future generations, just as you have relived it yourself today. However, there is more. This discovery implies that the rule of the game has also been changed. Indeed, the slave boy discovered a way to transform the surface area by *changing the internal boundary condition*, and such a discovery takes place in three steps: **PERPLEXITY, AWE**, and **LAUGHTER.**

Figure 40 The three faces of discovery: **PERPLEXITY, AWE**, and **LAUGHTER.**

8- THE ORIGIN OF THE PYTHAGOREAN THEOREM, AND HOW EPISTEMON DISCOVERED A CONSTRUCTIVE GEOMETRICAL PROOF FOR ITS FORMULA: $A^2 + B^2 = C^2$

When Panurge met Epistemon on the way back from Plato's Cave, he told him about the slave boy's discovery of doubling the square. Epistemon then took the opportunity to show Panurge how the Pythagorean Theorem was derived from the idea of doubling the square; that is, by *changing internal boundary conditions.*

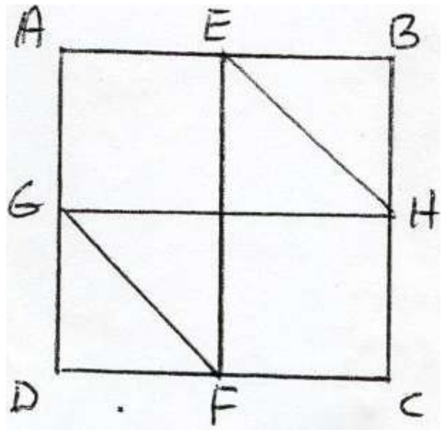
"The constructive proof for doubling the area of the square," said Epistemon, "can serve to demonstrate how you can change the axioms of the present school system in two fundamental ways. One, it consists in demonstrating that algebra is a derivative of constructive geometry, and that constructive geometry is not a derivative of algebra. Secondly, it demonstrates that the Pythagorean Theorem is not, as mathematics teachers often teach wrongly, a way to discover the third side of a right angle triangle. The Pythagorean Theorem is actually a transformation function whose purpose is to accomplish surface transformations by changing internal boundary conditions; that is, a discovery of principle."

"That seems to be very interesting," said Panurge, "but what does it all mean?"

"It's simple," replied Epistemon, "do you know where the well known elementary formula $\mathbf{(A + B)^2 = A^2 + 2AB + B^2}$ comes from?"

"I haven't the faintest idea," replied Panurge. "I always have the impression that these formulas are concocted by some magician in a cave somewhere for the purpose of torturing children. I beg of you, please light up my lantern and tell me where it comes from."

"I will show you how to construct this formula geometrically," said Epistemon. "Take the following square composition which is a variation of the Meno's slave boy problem that you just did.



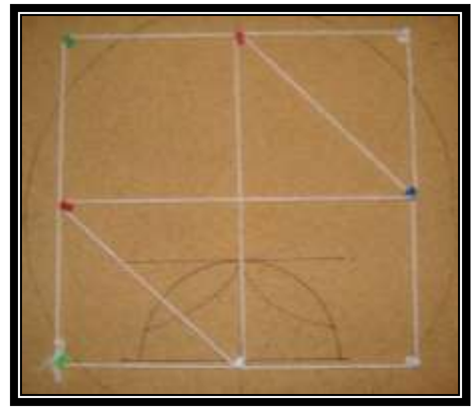
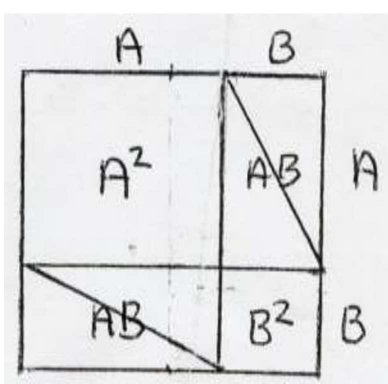
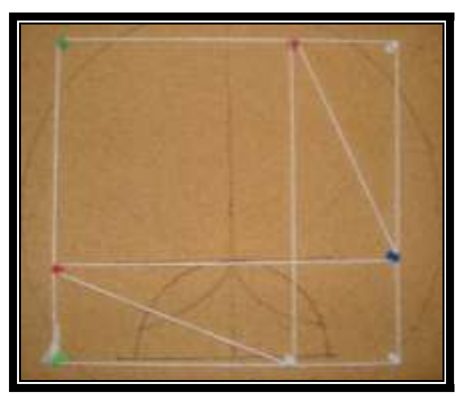


Figure 41

"Construct this figure on a cork-board with 8 pushpins and add a long and continuous piece of string that you will tie around a fixed square **A, B, C, D.** Take a long enough string to also go through the flexible diagonal sections **E-H** and **G-F**, and the cross sections **E-F** and **G-H.** You can rotate the string around the pushpins in 9 steps. Start at **GH,** and go continuously through **HC, CD, DA, AB, BH, HE, EF, FG.**

"Next, move this closed movable string in a position such that you can generate a large internal square with area $\mathbf{A^2}$, and a smaller square with area $\mathbf{B^2}$. You will succeed in doing that by moving line **EF** of **Figure 41** to the right toward **BC**, and by moving line **GH** down toward **DC**. Then, identify the rectangles as the two areas of **AB** (or **2AB**). Note that we have just constructed the geometrical formula: $\mathbf{(A + B)^2 = A^2 + 2AB + B^2}$**.**





$$\mathbf{(A + B)^2 = A^2 + 2AB + B^2}$$

Figure 42

"That's right," exclaimed Panurge, "how about that. And, I can change the internal area of the two squares to any position I wish, and that will always give me a variation of values corresponding to that same formula. I can even stretch those squares and rectangles proportionately without insulting my friend the square."

"Perfectly right," said Epistemon, "since **$(A + B)^2$** corresponds to the area of the total square, you can subtract from it the two internal squares **A^2** and **B^2**, and leave instead an empty space that is bounded by the remaining two rectangles **2AB**.

"If you dispose these four triangles in the following configuration (see **Figure 43**), you have changed the internal boundary conditions, while maintaining the outer boundary in its original condition. Then, behold this beautiful discovery: we have created a new square that did not exist before, which we will call **C^2**. For Panurge's sake, we will call this one: ***the square that is not there***. Moreover, follow how this formula is derived from a synthetic geometric construction and generates the famous Pythagorean Formula. Since **Figure 42** shows that **$(A + B)^2 - 2AB = A^2 + B^2$,** then it must be the case that **$(A + B)^2 - 2AB = C^2$**, and therefore, **$C^2 = A^2 + B^2$**

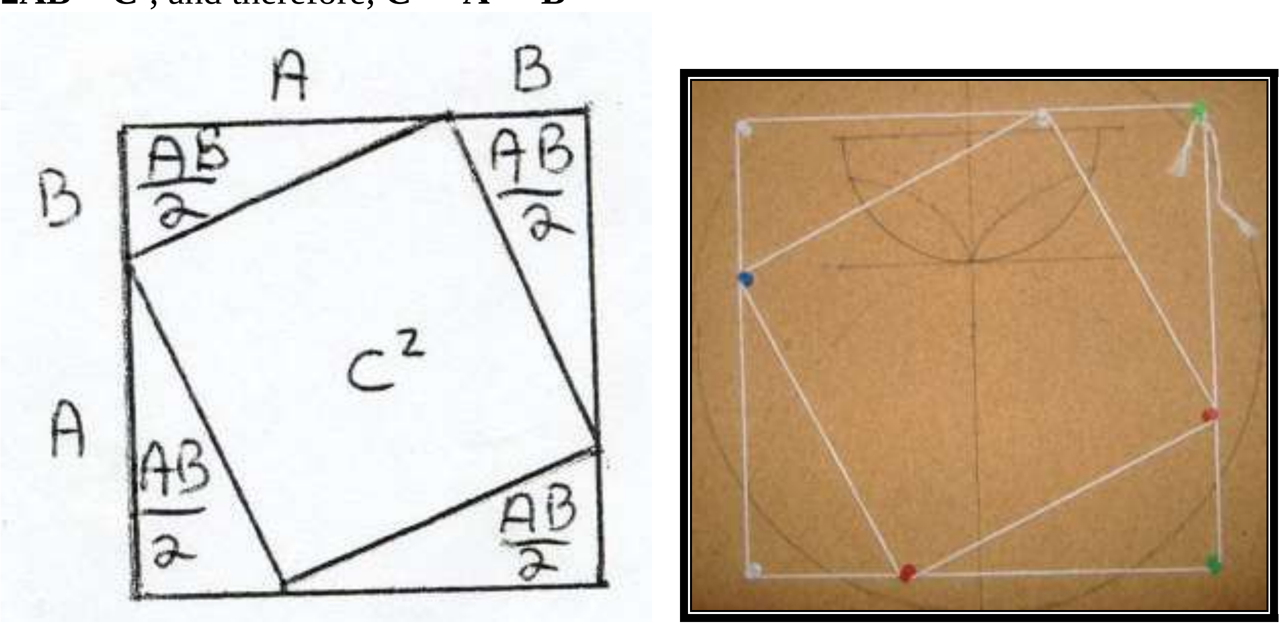


Figure 43

"You have now discovered the generating principle of the beautiful Pythagorean Theorem," said Epistemon, "but, we have also fallen into a new degree of perplexity. Everyone can see, with their mind's eye, that the algebraic notation is linear, and curiously tautological. The formula simply says in a boring deductive way: "If **$A - B = X$**, and **$A - B = Z$**, then **$X = Z$."** However, the synthetic transformation illustrated in **Figure 43** is non-logical, and non-deductive, and demands a non-linear leap of faith to be experienced. By discovering the Panurge ***square that is not there***, that is **C^2**, you have made a leap of faith into the domain of learned ignorance!

“If that is the way you acquire some ignorance,” said Panurge, “then I must have gotten a whole bunch of it because I am completely lost. I am back into some really deep and profound depth of that cave you’ve been talking about.”

“Follow me, step by step, on this one,” said Epistemon, “and you won’t get lost. Ask yourself: ‘What prompted me to subtract the two internal squares, $\mathbf{A^2}$ and $\mathbf{B^2}$ from the larger one?’”

“I don’t know,” replied Panurge. “You just have weird ideas like that sometimes.”

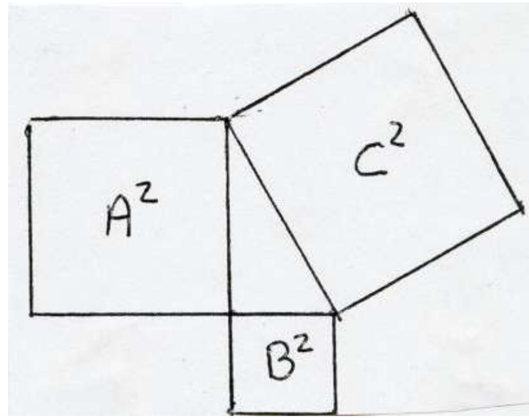
“No sir!” replied Epistemon. “I am not Harry Potter. I don’t fool around with magic. It was not a weird idea that prompted me. I was simply looking for ***what was not there***. That’s the crucial point.

“So again, *pay attention to the intention*. I simply was interested in finding out what area would be left over, once the two squares were taken out. I was actually creating this area that did not exist. This is not a magical trick; I was creating it for real. But how did I do it?

“I figured that if I were to change the internal boundary conditions and leave the outside boundaries intact, the area would be the same quantity, but the new figure would have to be different, as it turned out to be. The two areas are the same, and they are different, at the same time. I have transformed the areas of two squares into the area of a third square through diagonals and not through the sides, like the slave boy had done in the *Timaeus*. And that is what makes the whole difference. ***That is the crux of the idea of the Pythagorean Theorem; that is, finding a square which is the sum of two other squares through diagonals.***

“Wow! Just the same as in the doubling of the square,” realized Panurge.

“You are right,” replied Epistemon, “this incredible jump is lost when you only learn the algebraic formula. So you see, if you don’t take such lawful liberties and jump into the unknown, into *learned ignorance*, you can never be creative. This triply-connected Pythagorean Theorem Principle is exactly the same as the Principle of the Peace of Westphalia, but we can go into that question later. Now you can put the three squares together like in **Figure 44** and really know where the idea came from, because you didn’t just learn a formula; you have constructed it yourself! ”



Thus: $\mathbf{A^2 + B^2 = C^2}$

Figure 44

"In other words," said Epistemon, "we can now organize the three squares $\mathbf{A^2}$**,** $\mathbf{B^2}$**,** and $\mathbf{C^2}$ together, as in **Figure 44**; and in this way, you can generate the famous triple squares of the Pythagorean Formula that you have learned in school.

"You can easily see how, by this diagonal transformation, the principle of discovery of the Pythagorean Theorem is actually derived from the principle of doubling the area of the square. That is how you acquire true scientific knowledge, by constantly looking for new ways of changing boundary conditions," concluded Epistemon.

As LaRouche said after he had made the same discovery in Lanternland:

"***Believe nothing that for which you cannot give, yourself, a constructive proof.***"

9- CONSTRUCTING THE SQUARE AND THE CUBE WITH CIRCULAR ACTION

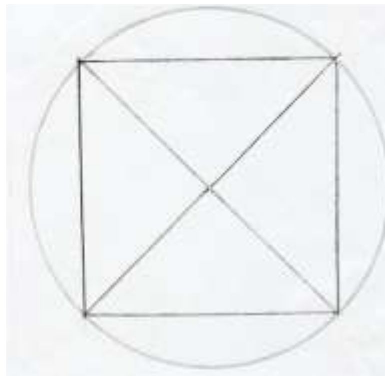


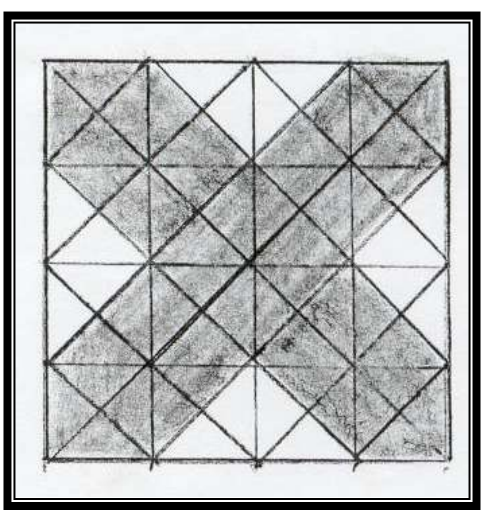
Figure 45

1. Take a large paper circle and fold it on itself twice to generate two diameters forming a right angle. **Figure 45**

2. Fold the four circular arcs from the four points around the circumference.

3. Fold the square on itself four times to get 4 squares each including 4 squares to form 16 squares in all. Color the surface of 6 squares as in **Figure 46**. The folded diagonals of 12 little squares form a cube.

4. Fold the non-coloured parts inwardly on each other and the 6 coloured squares will form the cube. **Figure 47**



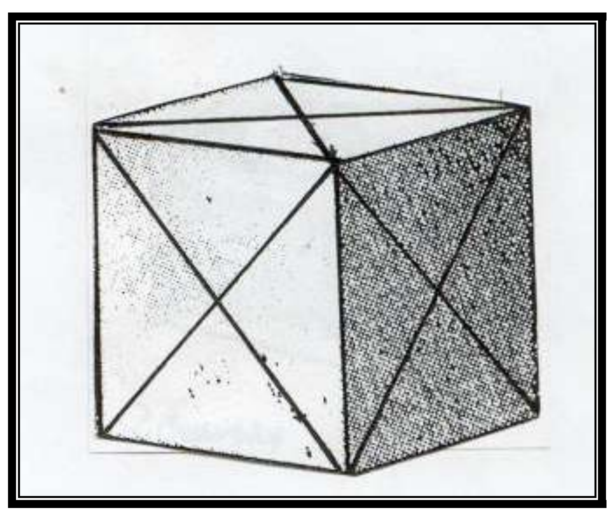


Figure 46 **Figure 47**

10- HOW EPISTEMON BROUGHT US TO PLATO'S CAVE AND SHOWED THE METHOD THAT NICHOLAS OF CUSA USED TO SOLVE THE PARADOX OF SQUARING THE CIRCLE

After a good midnight banquet in the Lantern Inn, we retired to rest, and the next morning, Pantagruel went back to Plato's Cave early with the square to help him with his revolution. Our friend Epistemon led the way with his lantern. He said that his lantern would never fail to bring us anywhere we wished to go, within reasonable distances. "Our lanterns have always been our best guides," he said, "because they were constructed with the principle of reason built into them."

After a pleasant walk of about two hours, we approached Plato's Cave. The entrance was wide open and a group of squares was having a loud discussion with hexagons and pentagons. They had obviously won their revolution since the polygons and the circles were all very flexible, jumping from their edges and making all sorts of somersaults and spherical gestures as they were coming out of the cave. "Pantagruel did a great job," said Epistemon, "but what is all this commotion?"

"We are in a quandary," replied General Square, the victorious leader of the revolution, "we can't seem to agree on a sticky question."

"What seems to be the problem," asked Epistemon, "maybe I can help."

"The pentagons say they are of a higher rank than the hexagons because they are built according to the divine proportion," answered General Square. "However, the hexagons claim to have a higher rank because they have one more side which makes them closer to the circle. You see, in Flatland, everybody agrees that the greater number of sides a polygon has, the higher the rank and the closer you are to becoming a circle. As the author of Flatland, Edwin Abbott, reported to us, circles just pretend they are superior to everyone else. As he said:

> ***"Although popularly everyone called a Circle is deemed a Circle, yet among the better educated Classes it is known that no Circle is really a Circle, but only a Polygon with a very large number of very small sides. As the number of the sides increases, a polygon approximates to a Circle; and, when the number is very great indeed, say for example three or four hundred, it is extremely difficult for the most delicate touch to feel any polygonal angles."*** [11]

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[11] Edwin A. Abbott, *Flatland, a Romance of Many Dimensions*, Dover Thrift Edition, New York, 1992, section 11. Concerning our Priests. **http://www.geom.uiuc.edu/~banchoff/Flatland/**

“Do you realize,” said Epistemon abruptly, “how everything you have just said about circles is completely absurd. You don’t seem to realize that what you have just said implies that only false circles exist in Flatland?”

“What do you mean,” replied the shocked square.

“My dear square,” replied Epistemon, “you don’t seem to understand that a ten thousand sided polygon is still a polygon, even if it looks like a circle. Looking like a circle does not make you a circle,” replied Epistemon, in a loud voice so that everyone could hear.

“But, I thought it did,” replied the square *sotto voce*.

“You have been completely misled,” said Epistemon more calmly. “It is circular action of spherics that produces circles and polygons, not polygons that produce circles. You see, none of you polygons could ever become circles, because polygons and circles are two different species. Circles are like angels, they belong to a higher world. You have been fooled, just like those who think that it is the amount of money that you have which makes you rich. That is silly. What makes you rich is what you can change and improve with your mind.

“The revolution that you have just fought and won makes precisely that point: ‘All polygons are generated by circular action; that’s why you were able to fold on yourselves and see your shadows. I am afraid that some of you are still clinging to old habits that you were brainwashed with back in Flatland. What you have been told in Flatland is simply a lie and a deception invented by a bunch of polygons suffering from illusions of grandeur calling themselves a circularchy.

“When during the Italian Renaissance, Nicholas of Cusa reworked the construction of Archimedes on the question of squaring the circle,” continued Epistemon, “he proceeded to show how the area of the square could be approximated with the area of the circle. Let me show you how Cusa resolved this apparent paradox that he called *squaring the circle*.”

“You are pulling my right angle,” said the square, in a moment of exaltation, “Cusa really said that my area is equal to that of a circle?”

“Don’t get me wrong,” responded Epistemon, “it was not meant to be a compliment. Just think through the following consideration:

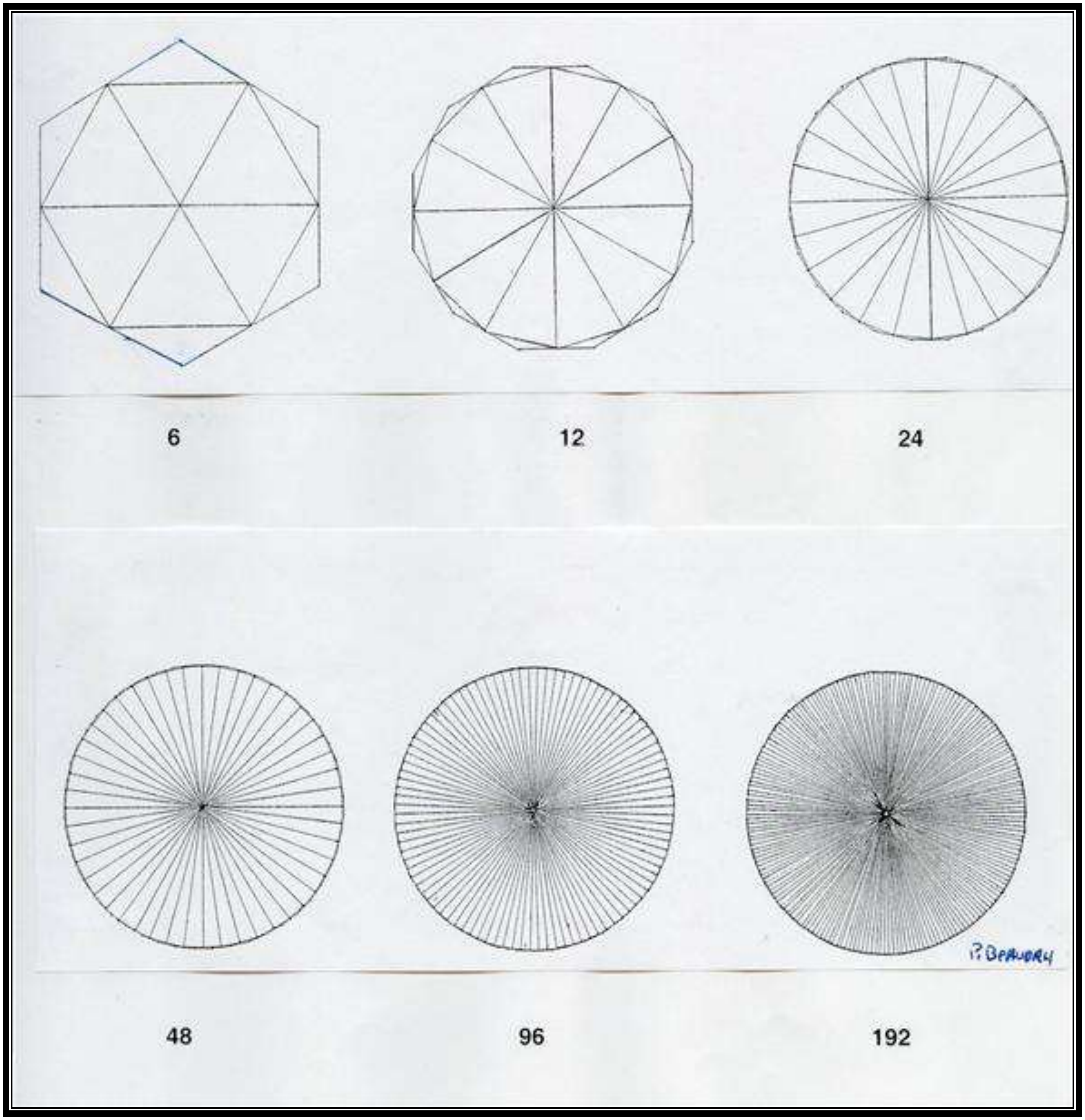


Figure 48

“Firstly, it is simply wrong to believe that there exists a perimetric equivalence between a polygon and a circle. Follow my reasoning. Let’s say you choose an invisible circle with a diameter of **2r = 1**. Inscribe a hexagon inside of that circle. Now, double the number of sides of regular polygons following the series **6, 12, 24, 48, 96, 192**, etc. So, the relative value of **π** for these inscribed polygons is as follows:

$\pi6$ = 3.0,
$\pi12$ = 3.1058,
$\pi24$ = 3.1326
$\pi48$ = 3.1393
$\pi96$ = 3.1410
$\pi192$ = 3.1414

If you circumscribe the same invisible circle with a similar series of regular polygons, their corresponding values for **π** will be as follows:

$\pi6$ = 3.4641
$\pi12$ = 3.2154
$\pi24$ = 3.1596
$\pi48$ = 3.1460
$\pi96$ = 3.1427
$\pi192$ = 3.1418

"Now, take the average between the two polygons of 192 sides. Do you see the result?" asked Epistemon.

"Yes absolutely," replied the square, "I can see how you have proven that the average value of your inscribed and circumscribed polygons is **$\pi = 3.1416$** the value of the ratio between the radius and the circumference of a circle. And that is why the circumference of the circle, whose diameter is **1**, has been made to be **$\pi = 3.1416$**. This is what I have always believed in Flatland."

"Precisely," replied Epistemon. "The circumference of the circle and the average perimeter of the polygon of **192** sides are convincingly the same. This means that their areas are convincingly the same as well."

"Well, does that mean that the perimeter of a polygon of **192** sides is a transcendental number," asked the square?

"No," replied Epistemon, "it simply means that, in every school in the world, mathematics teachers cheat and lie, as in Flatland, and tell their pupils that **$\pi = 3.1416$** is the ratio of the circle's perimeter to its diameter; while, in reality, it is the ratio of the perimeter of circumscribed and inscribed polygons of **192** sides to their diameter."

"To be frank with you, I am tempted to say that I find that wonderful," said the square, "but I suspect there is an intention behind your argument, and, I think what you mean to say is that we never got a chance to discover what a real circle was in Flatland."

“That is precisely my point,” replied Epistemon. “But let me take this a step further if I may.”

“By all means,” said the square.

“Secondly,” continued Epistemon, “while the mathemagicians appear to have succeeded in squaring the circle, and have created an apparent equivalence of area between the polygon and the circle, they have fallen into a devastating paradox.”

“This sounds ominous. Why is that,” asked the square?

“The polygon has **192** angles while the circle has zero angles,” replied Epistemon. “Don’t you think that the triangle which has only 3 angles is closer to zero angles than the polygon of **192** angles?”

“So it seems,” replied the square.

“So,” said Epistemon, “the more angles you create for your polygon, the further you are going away from the circle, and the less number of angles, the closer you are getting to the circle. Thus, it is demonstrated that of all polygons the triangle is the closest to the circle!!”

“You‘ve got me over a tile floor on that one,” replied the stunned square.

“Such is the nature of this paradox,” concluded Epistemon. “The closer you are to approximate the area of the polygon and the area of the circle, the further away the perimeter of that polygon is from the circumference of the circle.”

“You are shaking me up in all of my four axiomatic right angles,” said the square. “I must admit that you are absolutely right.”

“So you see, my fellow polygons,” said Epistemon, turning to the assembly of polygons, “nature prescribes a single direction for circular propagation, and that is why polygons are generated by circular action and not circles by polygons. So, this whole discussion about who is closest to the circle is a false problem. There are no distinctions of classes. You are all children of circular action. The revolution that you have just made proves the point in spades. And the means by which circular action produces polygons is with the idea of folding. Your folds are all the result of folding the plane onto itself.” At that point, all the polygons, each in his proper angle, clapped their edges together and saluted Epistemon by bending forward.

PART III

SPHERICAL AND SPIRAL ACTION: GEOMETRY OF SIXSIDEDNESS

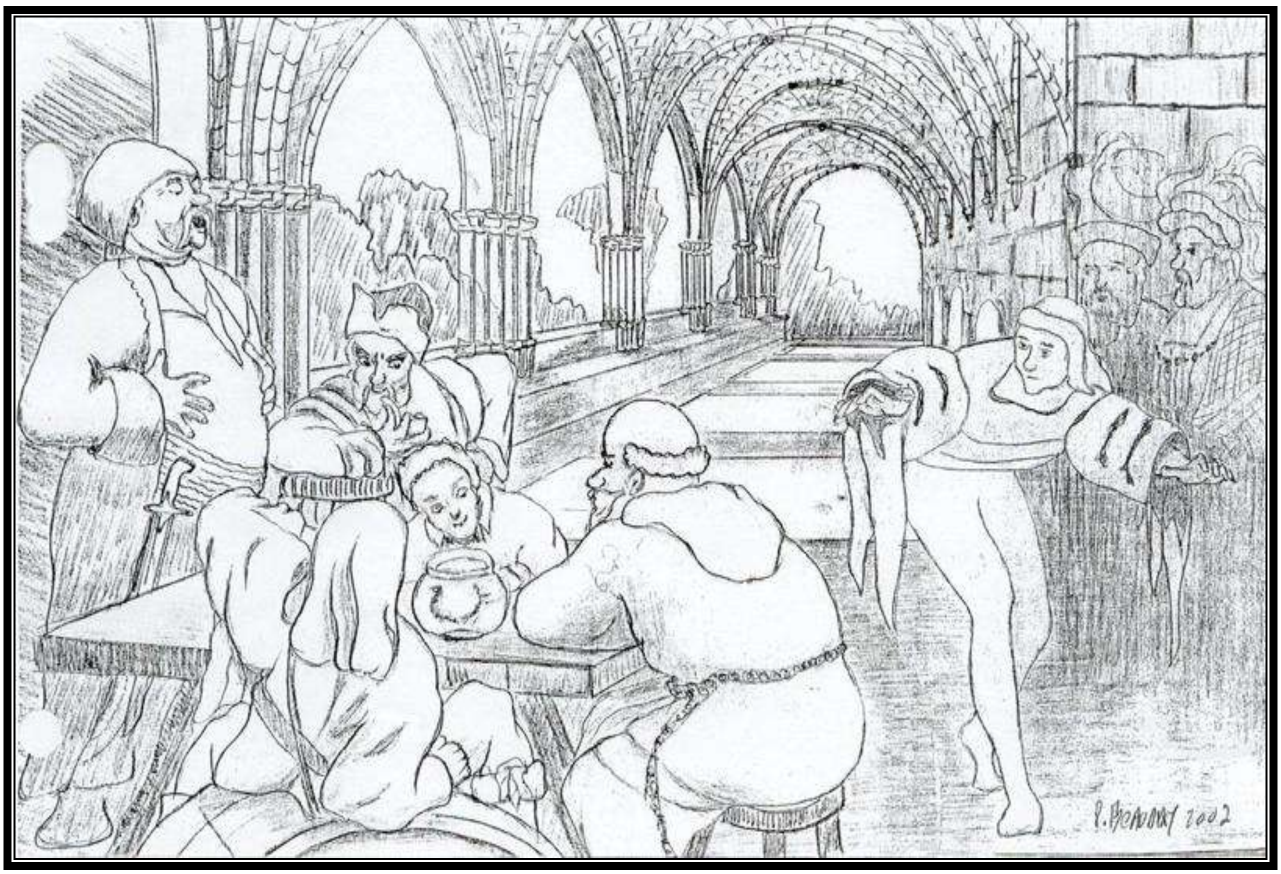


Figure 49 "It is just your axiomatic assumptions that are being shocked, not you," replied Bacbuc, laughing in the company of Ilia Repin's Cossacks.

11- PANURGE'S DISCOVERY OF THE CURVATURE OF HUMAN VISION IN A SPHERE OF WATER THAT PROJECTS AN INVISIBLE IMAGE

Only a few steps inside the cloister of the Abbey of Theleme, a beautiful spherical lantern stood on a table but did not seem to be illuminating anything. I said "seem" because it was not clear whether it was a source of light or not, since it was filled with water and projected strange images inside and outside of itself. At any rate, this was a very special lantern, because, depending on how it was rotated, or depending on how you rotated yourself around it, the curious object projected different images that fascinated Panurge to no end. While we stood in curious admiration before this sphere of water, we were greeted by the venerable Bacbuc and a group of Cossacks who welcomed us, and asked us to sit around the table.

"What do you see in that sphere," asked Bacbuc?

"I see the arched colonnade," replied Panurge.

"Do you notice anything special about this arched colonnade," she asked?

"Yeah. Ha, ha! All of the arches are upside down," laughed Panurge, with a mixture of amusement and perplexity. "How can the water turn everything upside down like that," he asked?

"Are you sure it is the water that turns everything upside down," asked Bacbuc? "Now, take a tall glass of water, and look through it. Is everything upside down," she asked?

"No," said Panurge, "everything is right side up. I guess you must have put a different kind of water inside of the sphere."

"Don't be silly;" said Bacbuc, "things will be seen right side up, or upside down, depending on the curvature of the boundary condition. Furthermore," she said, "note that in both the glass and the sphere, everything that is on the right is seen on the left, and everything on the left is seen on the right. This demonstrates that it is the convexity of the glass and of the sphere that changes the image, not the water. If you do the same experiment with a jar filled with water, and you rotate it, you will see what I mean. Always pay great attention to inversions."

“How about that!” replied Panurge.

“All right then,” said Bacbuc, “let us pay attention to the intention of that sphere. This is a very special sphere, because it shows how the crystalline of human vision works. That is, whatever is represented in the field of your vision will enter your eyes in the same manner that light goes through a sphere of water. Both Leonardo da Vinci and Kepler did a lot of experiments in this field of study that they called optics.”

“I hope this is not the same thing as crystal ball gazing,” exclaimed Panurge, “because I don’t believe in any of that magic stuff.”

“No, no,” replied Bacbuc, who was amused by Panurge’s remark, “we don’t indulge in such nonsense in Lanternland. We are not here in Newton’s Great Britain, ya kneow! The experiment that we are about to conduct is called experimental science. However, everything that deals with the science of vision, that is, optics, is not entirely visible to your physical eyes. Or, rather, it is visible, but you don’t see it. The constructive geometrical principle will only be visible to your mind’s eye. In that sense, this spherical experiment is the clearest example of not only how your eyes see, but also how your mind’s eye sees as well. And those are two very different things.”

“This is very strange,” said Panurge, who always felt uneasy whenever he could not perceive something with his senses. “What would be the reason for a lantern to exist, if it could not shed any light? I don’t appreciate at all your turning off the lights in the middle of the day like this.”

“But, that is precisely the point,” said Bacbuc, “you must turn off the lights of your senses in order for the light of your mind to be turned on. Underneath everything that you see, which is merely an illusion, there exists a true reality that can be grasped by a higher understanding, a higher hypothesis, and without having an image of it that would identify objects as this or that, but rather, as in the state of their being changed into this or that. Plato called this principle his Nurse Chora, the receptacle of all transformations, which you cannot perceive by your senses, but of which you can have a wonderful apprehension only with your mind’s eye.”

“First and foremost,” said Bacbuc, “you must understand what Kepler said about the sphere, and how it was meant to reflect the intention of the Creator.” She then opened her large book and read what Kepler wrote:

> ***“First, it was fitting that the nature of all things imitates God the founder, to the extent possible in accord with the foundation of each thing’s own essence. for when the most wise founder strove to make everything as good, as well adorned and as excellent as possible, he found nothing better and more well adorned, nothing more excellent, than himself. For that reason, when he took the corporeal world into consideration, he settled upon a form for it as like as possible to himself. Hence arose the entire category of quantities, and within it, the distinctions between the curved and the straight, and the most excellent figure of all, the spherical surface. For in forming it, the most-wise founder played out the image of his reverend trinity. Hence the point of the center is, in a way, the origin of the spherical solid, the surface is the image of the inmost point, and the pathway to discovering it. The surface is understood as coming to be through an infinite outward movement of the point, out of its own self, until it arrives at a certain equality of all outward movements. The point communicates itself into this extension, in such a way that the point and the surface, in a commuted proportion of density with extension, are equals. Hence, between the point and the surface there is everywhere an utterly absolute equality, a most compact union, a most beautiful conspiring, connection, relation, proportion, and commensurateness. And since these are clearly three – the center, the surface, and the interval, they are nonetheless one, inasmuch as none of them, even in thought, can be absent without destroying the whole.”***[12]

“You see,” added Bacbuc, “because of this triple quality of the sphere, the ‘outward motion of the point,’ which generates its projection for the benefit to all living beings, is also a divine quality pertaining to light, which takes its characteristic from the natural projection of the divine mind in such a way that the divine proportionality of interplay between light and mind through physical spacetime causes one and the other to move backward and forward by least action pathways to help everything else develop; that is, in ways that can only be understood as the intention of sufficient reason, otherwise known as the light of reason.

[12] Johannes Kepler, *Optics, Paralipomena to Witelo & optical part of Astronomy*, translated by William H. Donahue, Green Lion Press, Santa Fe, New Mexico, 2000, p. 19.

"So, because this intention was so beneficial to light, and this game was so pleasurable to his mind, God made the sphere the common playground between light and mind. But, you must understand how light plays a game of hide and seek with the human mind inside of the sphere. Kepler confirmed that when he said: 'As God the Creator played, so He also taught nature, as His image, to play the very game which He had played before her.'[13] So, let us see how the sphere of water is the archetype of this light and mind playfulness."

Figure 50 Sphere of water projecting both an upright and an inverse image.

[13] Quoted by Max Casper, Clarisse Doris Hellman, *Kepler*, Dover Publications, Inc., New York, 1993, p. 378.

“Put a sphere filled with water on a table next to a window, and put a piece of white cardboard vertically behind that sphere at a distance equal to the radius of that sphere. The entire scene enclosed within the frame of the window will appear to your eyes with perfect straight lines, perfect clarity, and perfect color upon the cardboard, but in an inverted position. Now, here is the paradox: if you were to place one eye where the cardboard is located, your vision would become entirely blurred and you would see nothing but a confusion of objects refracted through the glass sphere which would have become either entirely bright, or entirely dark. If your single eye gets closer to the globe, the objects on the opposite side will appear larger and erect. If you increase the distance of your eye away, by the distance of the radius, the same objects will be perceived distinctly, but in an inverted position, and smaller. If you then put the cardboard in this last position, everything will be blurred, and the picture will have vanished.”

“This is fascinating,” said Panurge. “You mean to say that there is always that invisible image at the radial distance of a sphere of water, and each time I try to see it with my own eyes, it goes into hiding.”

“That is correct,” replied Bacbuc, “and yet, the image doesn’t hide from the piece of cardboard in that position. You just don’t see it”

“This is absolutely extraordinary,” added Panurge.”It is as if there was an image of something in the middle of nowhere, which did not exist for anyone to see.”

“That is like the experiment of Plato’s Cave,” added Bacbuc. “You know it but you don’t see it. Something similar goes on with the refraction of the light through water,” continued Bacbuc, “because the convex shape of your eyes is similar to the convex globe of water. If you replace your eye with another sphere of water, behind the first one, the image would also be blurred and would disappear. Now, pay attention to this illustration of Kepler’s, and you can discover why the invisible image behaves like it does,” said Bacbuc. “Kepler is showing us the result of the intention of the entire process; that is, what your eyes could not see in that position, but that your mind was able to discover.”

“Lastly,” said Bacbuc, “do the following experiment with direct sunlight. Place the same sphere of water in direct sunlight, and position the white cardboard behind the globe, as before, at the distance equal to the radius of the sphere. [Do not do the same experiment with your eyes, because you might hurt your eyes.] Now, observe how the density of the sun’s rays forms a high concentration of light on the piece of cardboard. This is called a caustic. Kepler showed that when the cardboard comes to the point of the invisible cone of light at ψ, ***‘the illumination is strongest, so much so that gunpowder in cold water is ignited when the sun is intensely hot.’***[14]

[14] Kepler, *Ibidem*, p. 211.

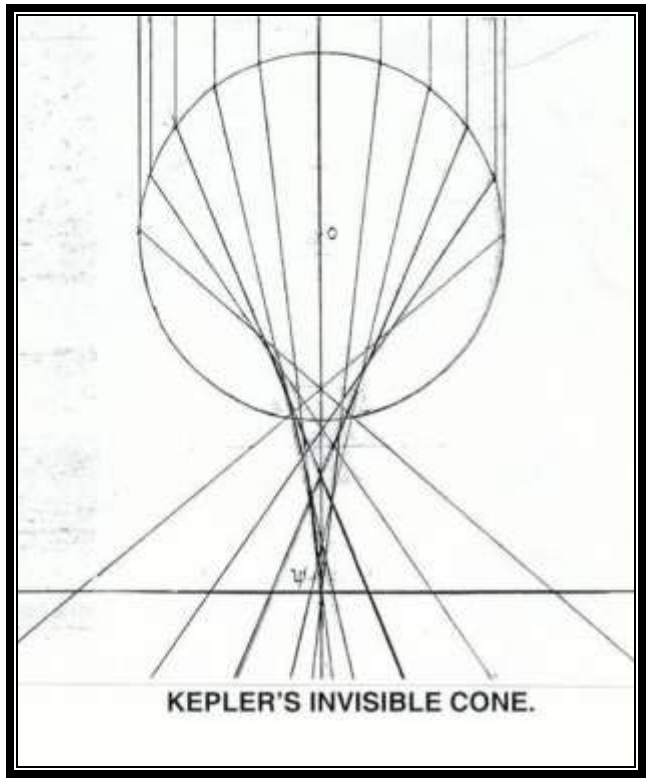


Figure 51 The invisible image of a cone of negative curvature inside of the sphere.

“This sphere of water is surely a powerful weapon,” added Panurge, half-joking. “I would not leave this sphere hanging around in the sun too long if I were you.”

“You don’t have to worry about that,” added Bacbuc, “but metaphorically speaking, yes, it is a powerful weapon to teach how light and mind can playfully resolve a number of paradoxes.

“How is that,” asked Panurge?

“Did you not notice that light traveled differently when it goes through the air than when it goes through water?” asked Bacbuc.

“Yes,” replied Panurge. “I noticed that the rays seem to go upside down when they go through the sphere, and they change their angles of direction when they form that invisible cone inside of the sphere.”

“Right,” said Bacbuc. “Those changes reflect the paradox of vision.”

“What do you mean,” asked Panurge?

“I mean that when you look at something with your physical eyes, you are wrong in thinking that what you see is actual reality. What you see is not the real world. It is merely an impression of something that changed inside of your eyes.

“Hey! Wait a minute here,” replied Panurge. “Aren’t you exaggerating just a little bit?”

“Not at all,” said Bacbuc. “Just follow the crucial transformation that is going on here.

1) The sun projects parallel rays through the air.
2) The sunrays are refracted through the sphere of water like through your eyeball.
3) Then, the sunrays are inverted through an invisible cone inside the sphere.
4) The image appears upside down onto the cardboard of your brain.
5) Then your brain reconstructs all of these changes into an upright visible image.

“That is quite extraordinary,” added Panurge, “especially when you realize that this whole process is happening just by playing around in water. You mean to say that the image of my own vision is like that invisible process inside the sphere, and that my eyes have to do all of this work in order to see anything. Is that correct,” asked Panurge?

"Yes Panurge," said Bacbuc. "But of this whole process, all you see is step number 5. You have now discovered the crucial intention of Kepler's experiment. You have discovered that what your eyes see is not reality, but merely a transformed projection of reality that your eyes project upside down onto your brain and that your brain restores right side up through a cone of negative curvature. Furthermore, this also means that your eyes cannot see the change and that only your mind's eye can see the process of transformation that is going on. That leads us to another paradox."

"Which one is that," asked Panurge? "I'm not sure I can take any more shocks like this. This is a bit much in one day."

"It is just your axiomatic assumptions that are being shocked, not you," replied Bacbuc laughing with the Cossacks. "Now, let's look at the paradox of Pierre Fermat. That is the paradox by means of which a ray of light always follows the most economical pathway. This means that the intention of light is to follow the pathway of least action, a pathway of least distance in reflection, and a pathway of least time in refraction: this is a crucial universal physical principle!"

"Wow! Hold on there. Not so fast," said Panurge. "Back up a little bit. What do you mean by the 'intention of light?' Do you mean to say that the rays of light know exactly when, and by how much, they can change direction in and out of that sphere of water," asked Panurge?

"That is precisely what I mean," replied Bacbuc, with a smile. "And you should have seen the freakout that this created in France around René Descartes and his sycophants when Fermat said that. A leading spokesman for Descartes, a caustic character by the name of Claude Clerselier, even went as far as writing a letter to Fermat telling him that his principle of least action was absurd. He said to him:

> "***First, the principle you [Fermat] take as a basis for your proof, to wit, that nature always acts by the shortest and simplest path, is only a moral principle, not a physical one - it is not and it cannot be the cause of any effect in nature" […] "And this principle cannot be the cause, for otherwise we would be attributing knowledge to nature: and here, by nature, we understand only that order and lawfulness in the world, such as it is, which acts without foreknowledge, without choice, but by a necessary determination" […] "This same principle must make nature irresolute, not knowing which way to go when it makes a ray of light pass from a less dense to a more dense medium…***"[15]

[15] Letter of Claude Clerselier to Pierre Fermat, May 6, 1662, in *Oeuvres de Fermat*, Tome Deuxieme, Gauthier-Villars et Fils, Paris, 1894, p. 464. (Translated by Pierre Beaudry)

“If I understand this correctly,” said Panurge, “this poor Clerselier guy did not pay attention to the intention, right?”

“That is absolutely true,” replied Bacbuc. “He forgot that light, just like mind, is a phenomenon of the noosphere which subsumes all of the phenomena of the biosphere, and so, he discarded the principle of sufficient reason, and God’s intention when He created light. God had definitely inscribed such a playful intention in the sphere of water clearly with the purpose of having light and mind play together and figure out the purpose of the whole game. But, the Cartesians were a little bit slow in catching up with Fermat. They understood quickly, but it took them a long time to figure it out.

“So, Clerselier made the mistake of considering that there could not exist any intention which would cause a ray of light to take one course rather than another, because he reduced the intention of the ray to his reductionist way of thinking, and concluded that because a ray cannot think like he does [and God forbid that anything else could either], it cannot have any built-in foreknowledge of where it was going.”

“That is really too bad,” added Panurge, because, if you think like Clerselier, you might never get anywhere. As Yogi Berra used to say: ‘If you don’t know where you are going, you might not get there.’ So, it seems clear to me that since a ray of light always reaches its destination, it must know, in some way, how to get there. And besides, when I fart, the intention is very clear: the gas being released is not directed everywhere randomly and without intention. It clearly intends to reach your nose by the shortest possible pathway, unless your feet decide to leave the premises in the shortest possible time. Now, any old fart knows that!”

12- CONSTRUCTING THE TETRAHEDRON, THE OCTAHEDRON, AND THE ICOSAHEDRON BY FOLDING

Take the equilateral triangle as we have constructed it earlier by multiply-connected circular action and generate three sides called edges and three corners called vertices. If you fold each vertex onto the midpoints of the three opposite sides, you will produce four smaller equilateral triangles. The tetrahedron is generated by triply-folding all of those singularities on themselves, edge-to-edge and vertex-to-vertex.

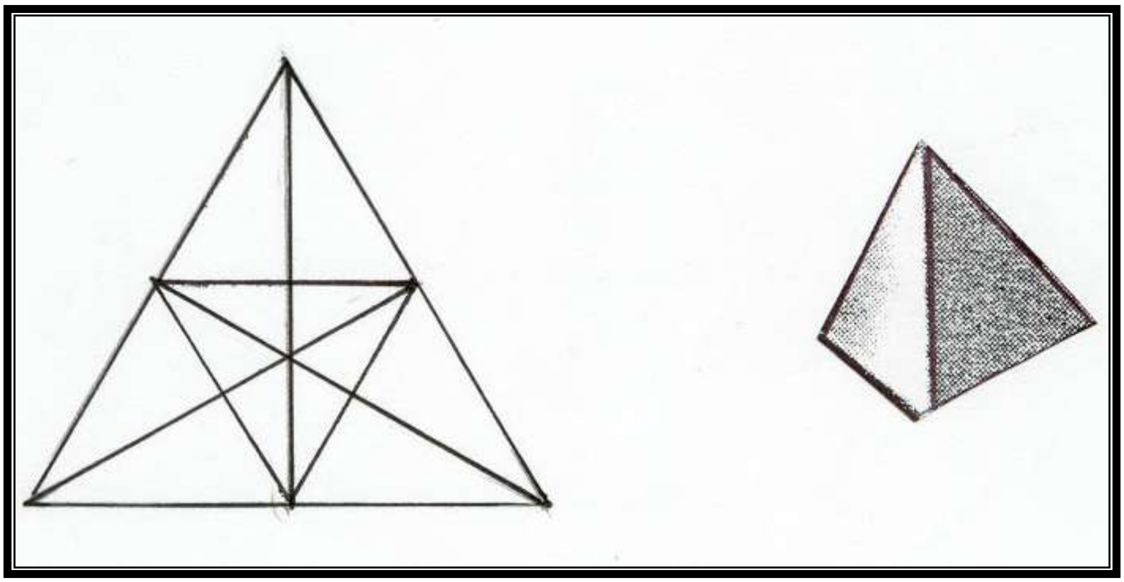


Figure 52 THE TETRAHEDRON

Next, take a large equilateral triangle formed by 4 smaller equilateral triangles, as generated for the previous tetrahedron, and fold each of the small equilateral triangles into 4 smaller equilateral triangles to produce a total of 16 equilateral triangles inside of one large equilateral triangle. This is where numbers and their powers come into existence, by means of a simple geometrical construction. In this case, the growth of the triangles follows the multiples of the power of 2:

1- The first triple action generates the equilateral triangle (1 triangle).
2- The second triple action generates the tetrahedron (4 triangles).
3- The third triple action generates the octahedron (8 triangles).

The best way to avoid reducing numbers to linear magnitudes is to count them in this way, as expressions of circular action. Think of numbers as intervals of action. Now, construct the octahedron.

Since by folding the total number of triangles is 16, and you require only 8 for the octahedron, the next step is to try to cover the octahedron twice with the same form of circular action. To accomplish that, fold 2 of the triangular tips of the large equilateral triangle, one over the other, tip to tip. This will give you the shape of an octahedron with one open side. Then, cover the whole octahedron, a second time, by rotating the remaining triangles with an inverted twist such that the third triangular tip closes the solid with a single closing lid. All of the sides of the octahedron are formed by multiple triangles, with the exception of the top and the bottom.

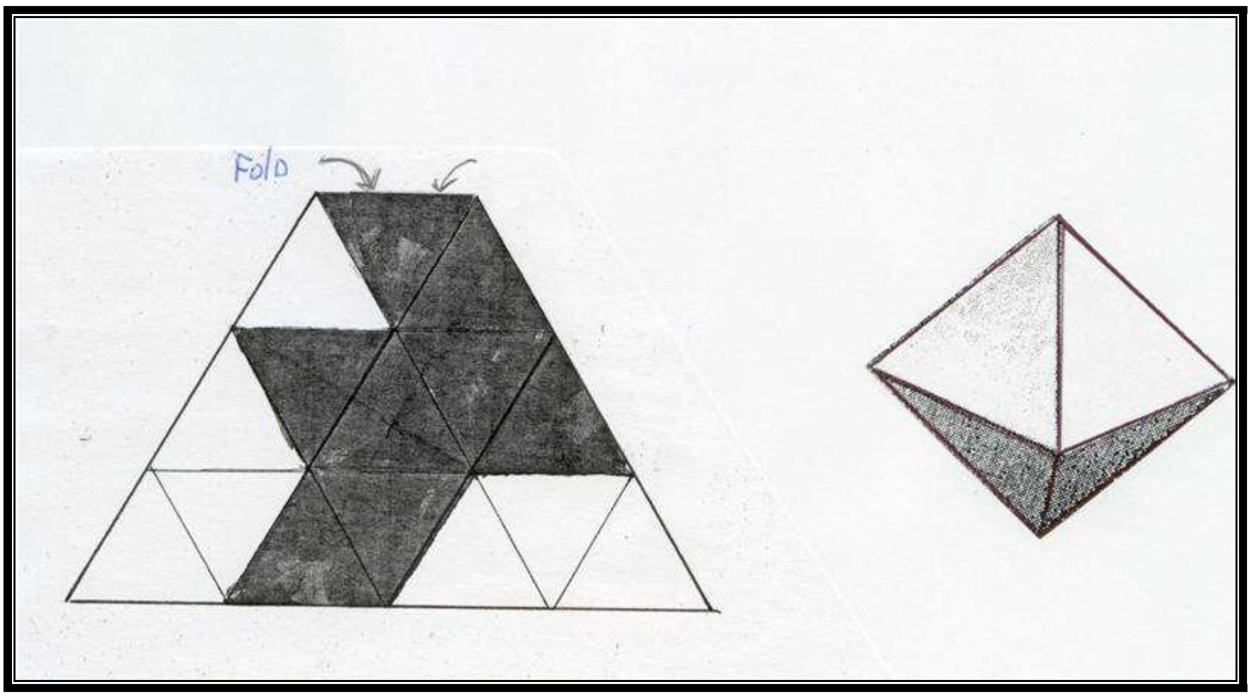


Figure 53 THE OCTAHEDRON

To construct the icosahedron, begin with two large equilateral triangles. Divide each into 16 smaller equilateral triangles. Each large triangle will serve as a half-shell, two of which are required to complete the icosahedron.

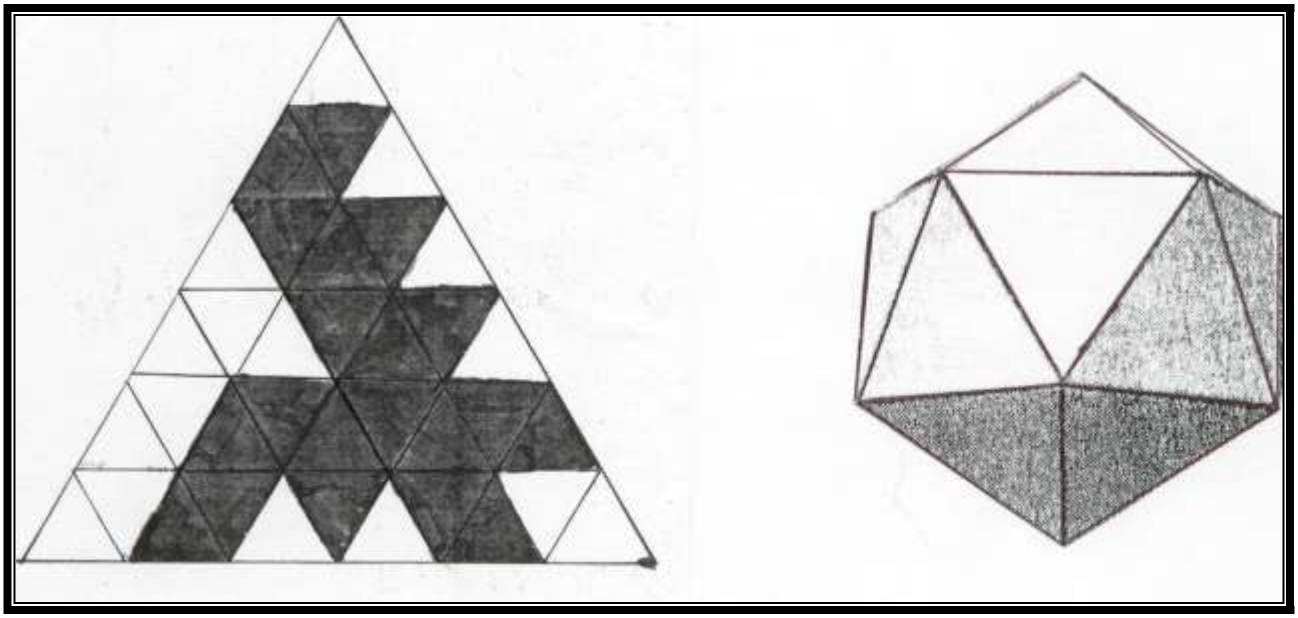


Figure 54 THE ICOSAHEDRON

Take one large equilateral triangle divided into 36 small equilateral triangles and eliminate 16 small equilateral triangles. You eliminate those 16 triangles by folding them on themselves. The remaining 20 small equilateral triangles will automatically form the complete icosahedron. You will obtain that result by following these steps:

1- Take a large equilateral triangle and fold the 3 vertices onto their common center. This will form 3 triangles folded on a hexagon.
2- Fold the hexagon into 6 equilateral triangles. Make sure the folds are very flexible, front and back.
3- Fold each of those 9 equilateral triangles into 4 smaller triangles. This will generate 36 small equilateral triangles. Colour twenty triangles as shown above and fold them together into a complete icosahedron.
4- Fold the remaining triangles on themselves, and pinch them inside of the solid.

13- CONSTRUCTING THE SPHERICAL CUBOCTAHEDRON

Material

Heavy cardboard, compass and divider, scalene triangles, pencil and eraser, scissors or board cutter, glue, and a lot of patience.

Construction

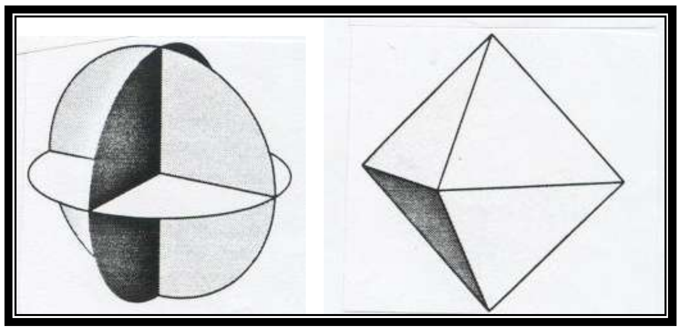
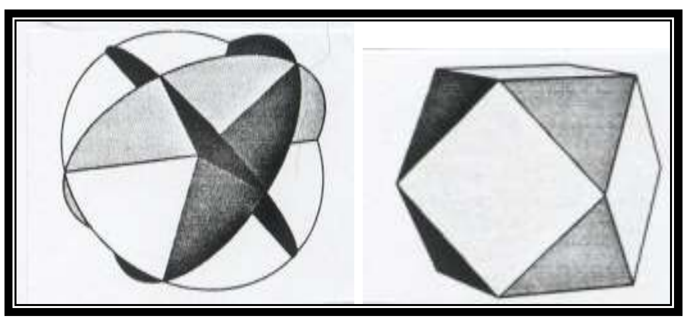


Figure 55 THE OCTAHEDRON



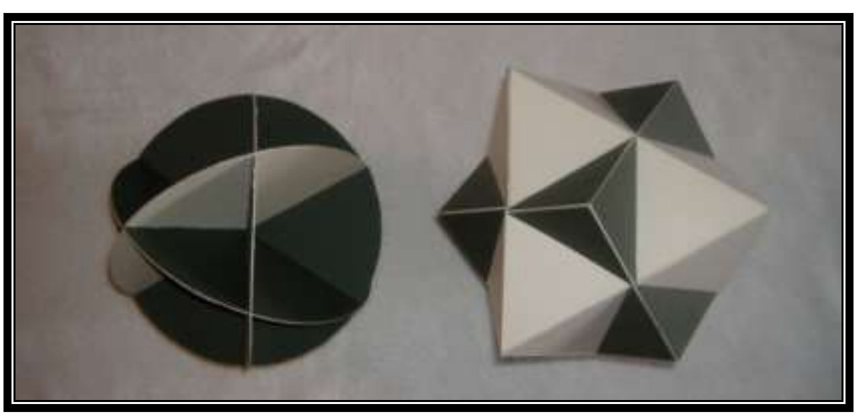


Figure 56 THE CUBOCTAHEDRON

1. Take 3 great circles of 8 inches in diameter and divide each into four equal arcs.

2. The surface of the octahedral sphere is made up of 8 equal spherical triangles.

3. Take 4 great circles with diameters of 8 inches and divide each into six equal parts.

4. The surface of the Cuboctahedral sphere will be made up of 6 regular spherical squares and 4 regular spherical triangles.

The two platonic solids formed by close packing of 12 balls of equal size produce the combined cube and octahedron.

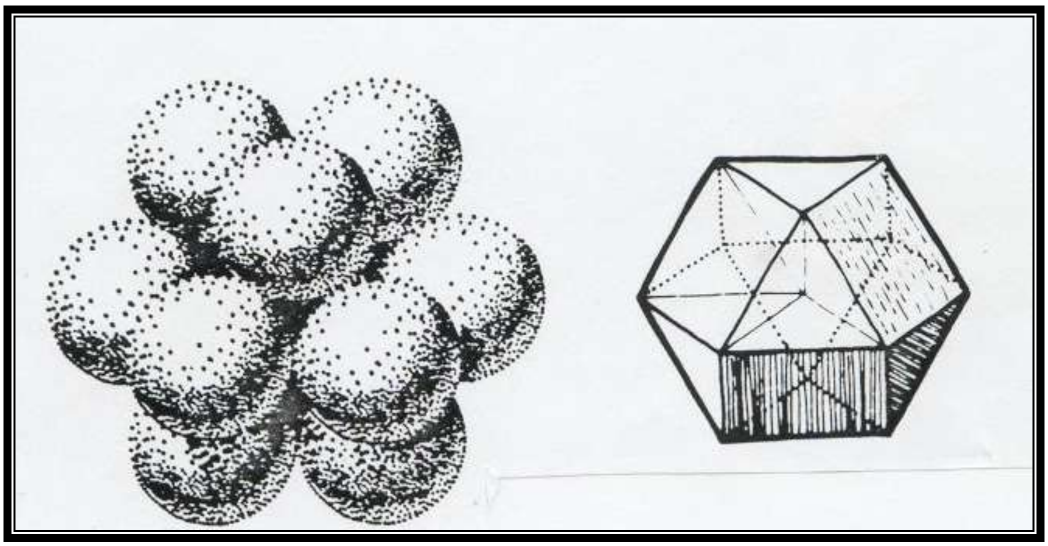


Figure 57[16]

Why do you see only 9 spheres? Can you locate the three "ghost spheres" that you don't see? Mark their positions on the cuboctahedron.

[16] Illustration 57 is from Robert Williams, *The Geometrical Foundation of Natural Structure*, Dover Publications, New York, 1972.

14- WHY DID EPISTEMON PROMOTE THE METHOD OF SYNTHETIC GEOMETRY INSTEAD OF ANALYTICAL GEOMETRY

I must say, Epistemon was quite a genius, because he was always thinking, and never fell into the trap of popular opinion. He always claimed that because the method of synthetic geometry was based on discovery, then, everything in it had to be constructive. He considered that if you did not construct your knowledge yourself, somebody else would do it for you. This is why Epistemon was given his name, which means knowledge in Greek. He did not at all like the idea of having someone else do his thinking for him. He claimed that everyone should become, as early as possible, a self-made independent thinker, as Lazare Carnot recommended in his *geometry of position and transposition*.

In the constructive method, what you discover is always something new that you did not expect, a sort of surprise, like solving a puzzle that seems impossible to solve. The reason for this is because constructive geometry is a geometry of motion; that is, of transformation. This kind of geometry starts with a human experiment of cognition, and not with an apriori abstraction. Epistemon thinks that constructing puzzles and paradoxes is the best method to establish an experiment, and to discover the cognitive power of your mind. He says that this is the best way to discover where things come from, how they are created, and what sort of process generates them. When you investigate these questions, you discover contradictions that you try to resolve. You discover things that exist, and do not exist at the same time. For example, the paradox of Nicholas of Cusa who talks about an infinite straight line that is also an infinite circle. The question is: How do you go from one to the other? Now, try to imagine that. The constructive method makes you discover that behind things, there exist proportionality, self-similarity, and circular action. The mastery of this method does not require numbers or algebraic formulas; it requires motion. And, you are even allowed to make mistakes. In fact, if you make the same mistake more than once, it is no longer a mistake. Since it keeps coming back, it becomes a boomewrong.

Epistemon banned algebraic formulas on principle. He said that unless you can demonstrate how it is constructed, you should never use an algebraic formula. Ultimately, for the midnight oilers, the only form of measurement allowed in constructive synthetic geometry is proportionality and multiply-connected-circular-action. He said there was a long list of synthetic geometers that we should get acquainted with by reliving their discoveries. They are Thales, Plato, Archimedes, Nicholas of Cusa, Pacioli, Leonardo da Vinci, Kepler, Pascal, Leibniz, Monge, Carnot, Poncelet, Steiner, Gauss, Riemann, and LaRouche. They have all practiced this method, each in their own way. The LaRouche motto for the synthetic method is:

“BELIEVE NOTHING THAT FOR WHICH YOU CANNOT GIVE YOURSELF A CONSTRUCTIVE PROOF.”

Epistemon considered that the analytical method, otherwise known as the algebraic method, was an insidious idea based on learning formulas. He rejected it because it only gives the appearance of having knowledge. “The formulas do the thinking for you,” he claimed, “and are based on the simplistic logic of induction and deduction.” Just like on television. In the analytical method, what you discover is always already included in the formula, and all you have to do is learn and memorize. Don’t ask where an algebraic formula is generated from; no one is able to tell you. All that a mathematician can say is that the formula works, and it impresses people. That is why Epistemon calls them mathemagicians. He says that after a prolonged use of algebraic formulas, your mind becomes mushy and lazy, because you have taken the habit of letting the formula do the work for you. The purpose of the algebraic formula is precisely to eliminate and destroy your power of cognition. Ask a formula anything. It cannot answer you. It can only repeat the same thing over and over again.

“A formula is like a recipe, “concluded Epistemon, “it makes you forget how to cook. Throughout history, Aristotle, Euclid, Galileo, Newton, Descartes, Kant, Euler, Laplace, Cauchy, Russell, von Newman and others have all been very bad cooks, because the motto of their analytical method of learning has been:

TRUST ONLY MATHEMATICAL FORMULAS; THEY WILL GIVE YOU THE IMPRESSION OF HAVING POWER.

“However,” continued Epistemon, “if you follow the constructive geometrical method of Plato, Cusa, Leonardo, Raphael, Kepler, Leibniz, Monge, Carnot, Poncelet, Steiner, Gauss, and Riemann, their motto for their transformative constructive geometry will be:

IF YOU CONSTRUCT ANYTHING FROM UNIVERSAL PRINCIPLES, THAT WILL GIVE YOU A TASTE OF WISDOM.

“The proof of the matter, here, will be made by discovering the constructive principle by means of which you can go beyond the spherical method of composition of dual solids to a universal spherical transformation of multiple solids. The proof that the soup is good is when you taste it while you are cooking it. Are you ready for that?”

“I’m game if you are,” retorted Panurge.

“Good,” said Epistemon, “Then, let’s mix six-sidedness with ten-sidedness.”

PART IV
SPHERICAL AND SPIRAL ACTION: GEOMETRY OF TENSIDEDNESS

Figure 58 The ten-circle Egyptian sphere.

15- THE MIXING OF SIXSIDEDNESS AND TENSIDEDNESS IN A SINGLE SPHERE GENERATES THE GOLDEN SECTION

This golden section of divine proportion is the favorite proportion that God used to create the universe as a whole. That is why he applied it to everything that is living and grows, including the spiral galaxies, vegetables, animals, and human beings. The maple leaf is the simplest example of a golden section, because it is in the form of a pentagon. The natural angle that God chose for living processes is called the sublime angle, which is the 36-degree angle of the decagon and of the pentagon. It is interesting to note that the number of degrees of the circle is 360 degrees; that is, 10 times 36 degrees. Geometrically speaking, the best way to generate the sublime angle of the pentagon and the golden section of the divine proportion is in the form of close packing of 10 great circles into a sphere containing all of the 5 platonic solids, plus the angular determination for the slope of the great pyramid of Egypt, which is 52 degrees.

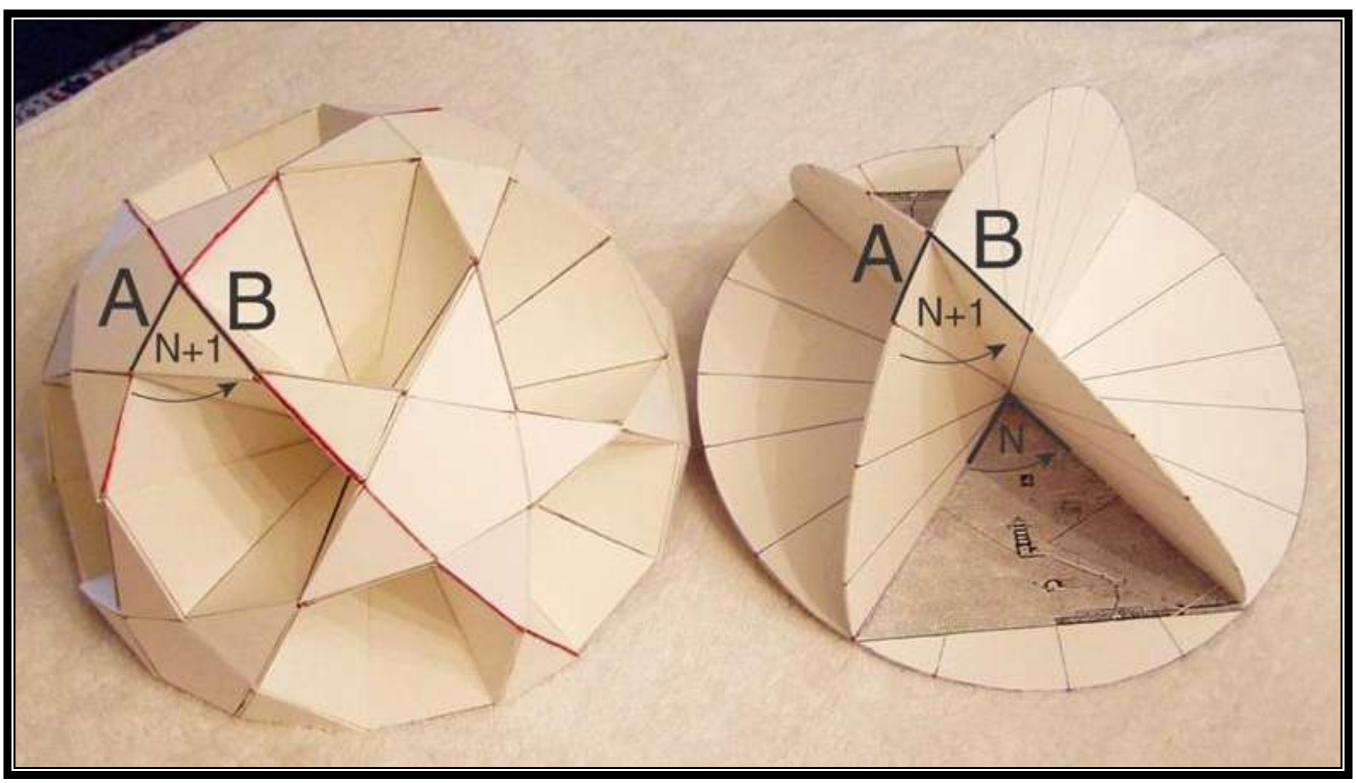


Figure 59 The great pyramid star-dodecahedron**.**

16- HOW THE ANCIENT GREEKS CONSTRUCTED THE PENTAGON INSIDE OF A CIRCLE, AND HOW EPISTEMON DISCOVERED AN EASIER WAY BY FOLDING A STRIP OF PAPER

HOW DO YOU INSCRIBE A REGULAR PENTAGON INSIDE OF A CIRCLE?

CONSTRUCTION

1. Draw a circle with a 4 inch diameter.
2. Draw a diameter **AB** and a radius **CO**.

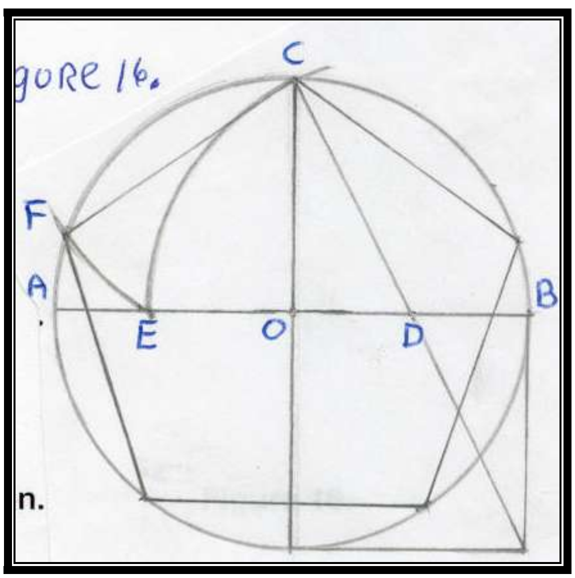


Figure 60

3. Bisect line **OB** at point **D**.
4. With **D** as a center, and radius **DC**, draw arc **CE**.
5. With **C** as a center, and radius **CE**, draw arc **EF. CF** is the side of the pentagon.

HOW EPISTEMON BUILT THE PENTAGON AND THE SUBLIME TRIANGLE WITH A STRIP OF PAPER

1. Take a strip from a narrow roll of paper and make a knot with it.

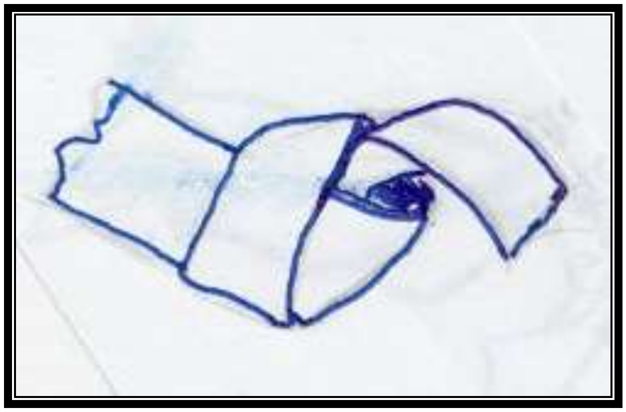


Figure 61 KNOT

2. Pull in the corners snugly and pinch the edges to make the sublime angle. The result will be a pentagon.

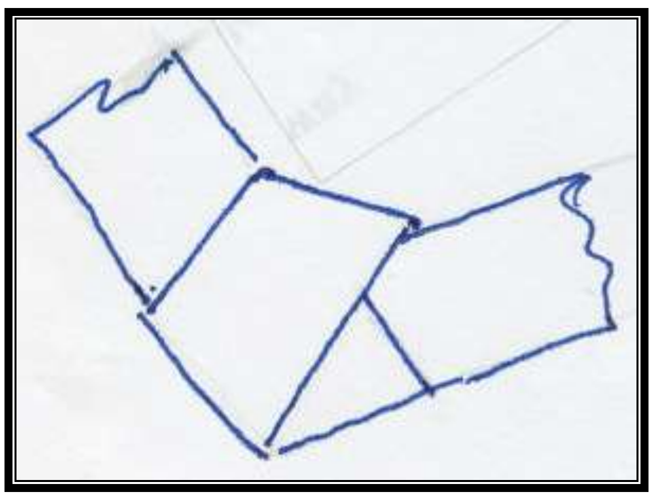


Figure 62 PENTAGON

PROBLEM

Do you know what the golden section is? Did you know that everything in the universe grows according to the divine proportion of the golden section? It was Luca Pacioli and Leonardo da Vinci who rediscovered the Greek construction of the golden section of divine proportion during the Italian Renaissance.

EXAMPLES

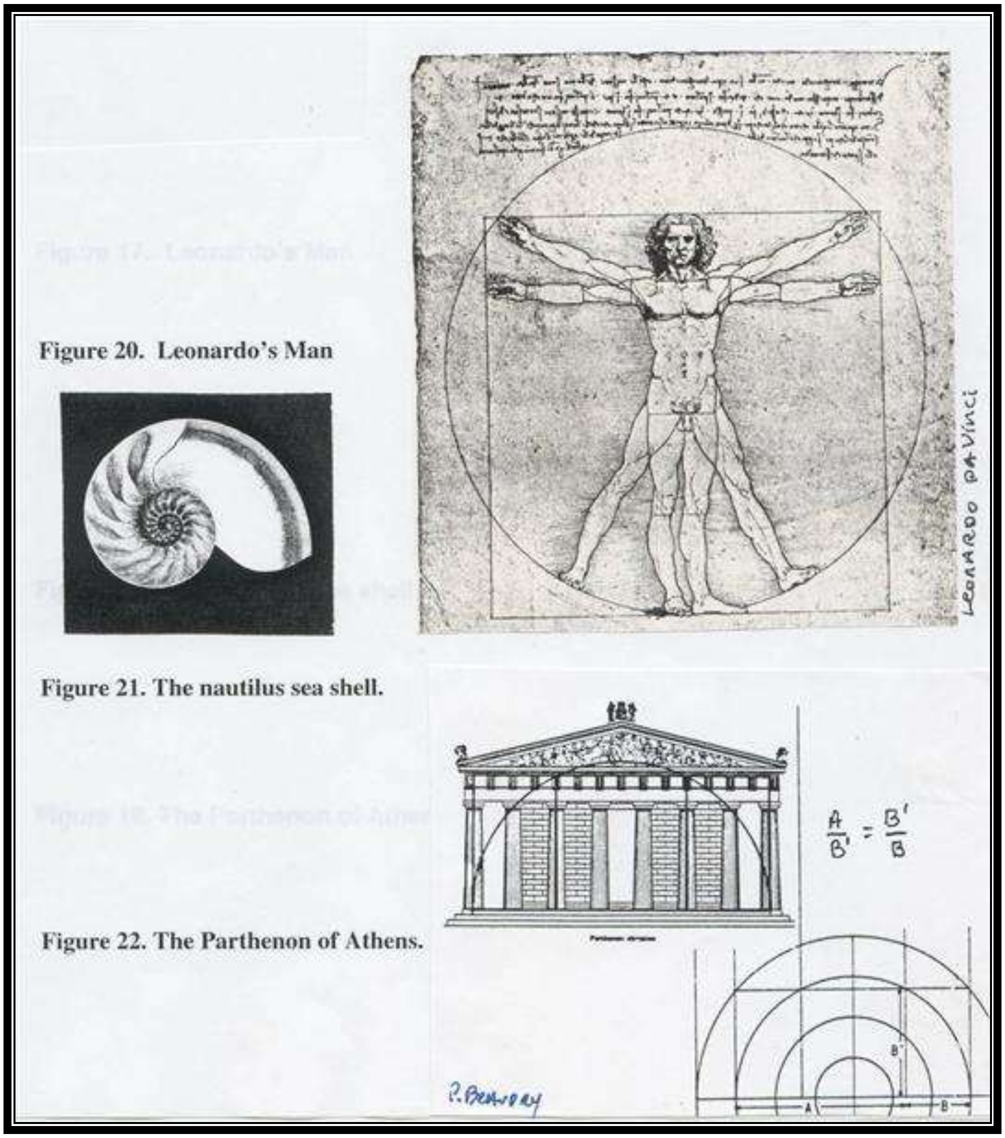


Figure 20. Leonardo's Man

Figure 21. The nautilus sea shell.

Figure 22. The Parthenon of Athens.

Figure 63

17- PANTAGRUEL'S CONSTRUCTION OF A FOLDED DODECAHEDRON USING A SINGLE STRIP OF PAPER

This construction of the dodecahedron requires a long strip from a roll of paper to be folded into a continuous series of 20 pentagonal knots. The construction is based on the pattern discovered by Pantagruel. He called his method of folding, spiral action. In fact, all he had to do was to fold, twist, rotate, make knots, invert, squeeze, flatten, pinch, overlap and scotch-tape a strip of paper for the construction of platonic solids. It is both an unusual, and a controversial form of spherical spiral action; unusual, because the use of twisting, rotating, spiralling, knot making, inverting, squeezing, flattening, pinching, folding, overlapping, and scotch-taping is not customary in Euclidean geometry. It is controversial because it introduces a number of anomalies and paradoxes, which cannot be explained by Euclidean geometry!

Figure 64 Pantagruel with the pentagonal series.

To make the construction simple, don't make the strip too long. Make 2 or 3 pentagons, at the most, with a single strip, and attach them firmly and tightly together as you go along. The completed dodecahedral series of 20 pentagons will look like a long **DNA** strand in a continuous spiral pattern.

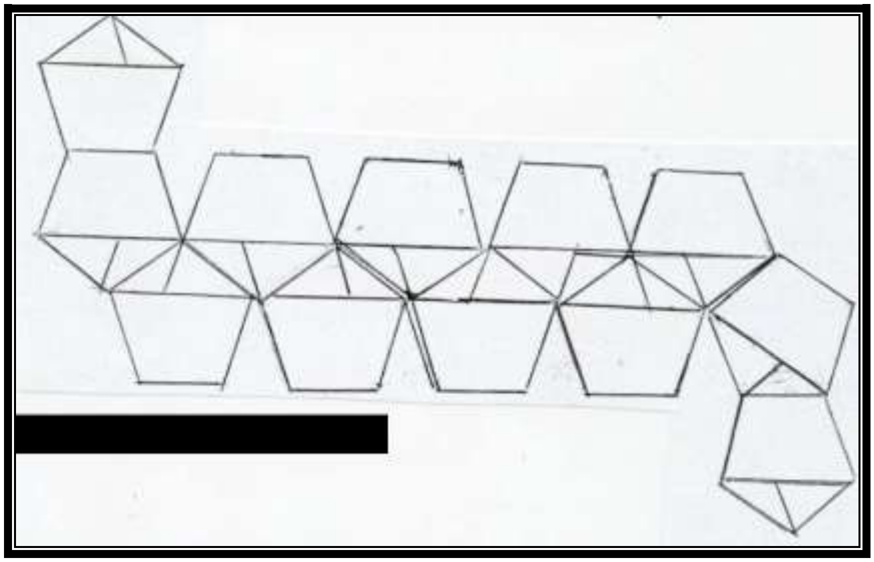


Figure 65 Twenty pentagons folded 2 by 2.

Take that long strand of 20 pentagons and fold them, 2 by 2, to get 8 doubled pentagons and two singles at each end. Unfold the first and the last in such a way that the 2 first and the 2 last pentagons are folded into singles. Attach them firmly together as shown in **Figure 65.** You can construct all of the Five Platonic Solids with the same folding method.

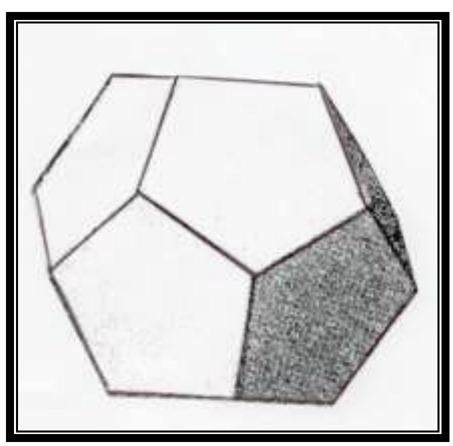


Figure 66 Dodecahedron

Figure 67 Crystal pyrithohedron or "fool's gold". **Cochise College**

Figure 68 Platonic solid dual transformations

18- CONSTRUCTING THE SPHERICAL ICOSADODECAHEDRON

1. Make 6 circles with diameters of 8 inches, and divide each into 10 equal parts.

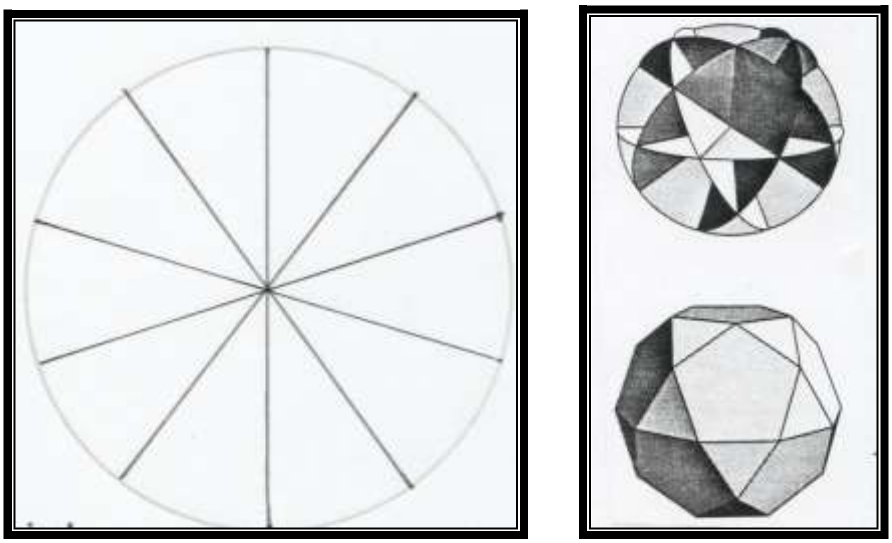


Figure 69 ICOSIDODECAHEDRON

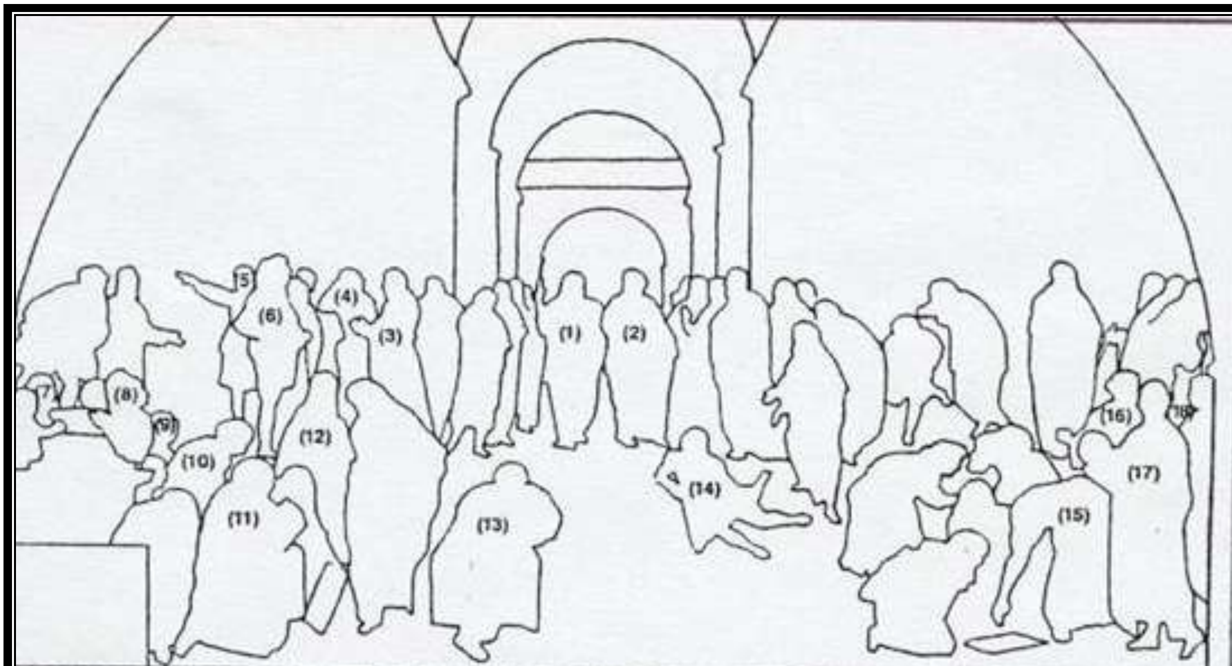
2. Glue all of the parts of the 6 circles together and the surface of the sphere will be made up of 12 regular spherical pentagons and 20 regular spherical triangles.

PART V

THE SPHERICAL GOLDEN SECTION: THE MIXTURE OF SIXSIDEDNESS WITH TENSIDEDNESS

Figure 70 *The School of Athens* by Raphael Sanzio (1483-1520). From **http://hrsbstaff.ednet.ns.ca/darcysr/Grade%2011/School%20of%20Athens.htm**

19- HOW RAPHAEL PAINTED THE IDEA OF SIMULTANEITY OF TEMPORAL ETERNITY AND BROUGHT PAST DISCOVERIES TOGETHER INTO A SINGLE SPACETIME



Above: Personalities gathered together by Raphael. (1) Plato, (2) Aristotle, (3) Socrates, (4) Xenophon, (5) Æschines, (6) Alcibiades, (7) Zeno, (8) Epicurus, (9) Federico Gonzaga, (10) Averroes, (11) Pythagoras, (12) Francesco Maria Della Rovere, (13) Heracleitus, (14) Diogenes, (15) Archimedes, (16) Zoroaster, (17) Ptolemy, (18) Raphael's self-portrait.

Figure 71 Some key personalities of *The School of Athens*.[17]

[17] Diagram courtesy of Vatican Museums. **https://www.thinglink.com/scene/856087596575490048**

“Since you are all eager to hear, let me tell you the story of Raphael’s discovery,” said Epistemon, “I will tell you what I know about it. “Every person in the world should get a chance, at least once in a lifetime, to stand before this painting, which is in the Vatican, and relive the discovery of principle that Raphael made in *The School of Athens*. There, you shall find one of the greatest fresco masterpieces of all times, showing a large assembly of past discoverers, and also a self-portrait of Raphael himself (in the lower right corner), bringing to your attention the method of pure synthetic constructive geometry by which he brought all of these people together, including himself, into a unique location of *creative mental spacetime*.

“This unique event, in the history of art, is nothing but a pictorial representation of the location of the *simultaneity of temporal eternity* that Raphael developed, as a reflection of his own mind. That *spacetime* location is very special, because it is the most intimate place where all of your true friends come together, and relate to each other and to you in a more intimate way than do the children in your classroom. But, he did more than bring them together for you to meet; he is also showing you how he did it.

“These friends,” said Epistemon, “who come from different times and places, and that Raphael never met in real life, are closer to him than the people of his own town, and even of his own family. This assembly of discoverers is not a fantasy. It is the meeting place of the creative activity of the mind. It is the workshop of effective creative change caused by these discoverers, a *phasespacetime* which is more real in this absolute timelessness of centuries than the space and time of Raphael’s ordinary daily life. That is the *eternal phasespacetime of change* that Plato had identified in the *Timaeus* as Chora, the receptacle for the generation of everything that exists.”

Then, Epistemon indicated how the principle of this *phasespacetime* worked geometrically with the group of Archimedes (15), Zoroaster (16), Ptolemy (17), and Raphael (18) located to the lower right side of the painting. (**Figure 71** and **72**)

“Concentrate on the student looking up at the sphere of Zoroaster and pointing to the geometric drawing of Archimedes down on the floor. What is he discovering,” asked Epistemon? “What he is bringing your attention to is the principle of change of the entire fresco.

“Archimedes has drawn a sort of Star of David, which is the sketch of the entire architecture of the fresco out of which can be generated all of the platonic solids, as if each and all of them were generated from a sphere. This sketch illustrates the paradigm and the very boundary condition of Raphael’s mind,” added Epistemon.

Figure 72 Detail of a group of four students with Archimedes, one of whom is looking up to the celestial sphere of Zoroaster, who is asking Raphael how the Star of David exemplar relates to Spherics. Raphael is looking back at you, the viewer, to see if you can answer that.

From **http://hrsbstaff.ednet.ns.ca/darcysr/Grade%2011/School%20of%20Athens.htm**

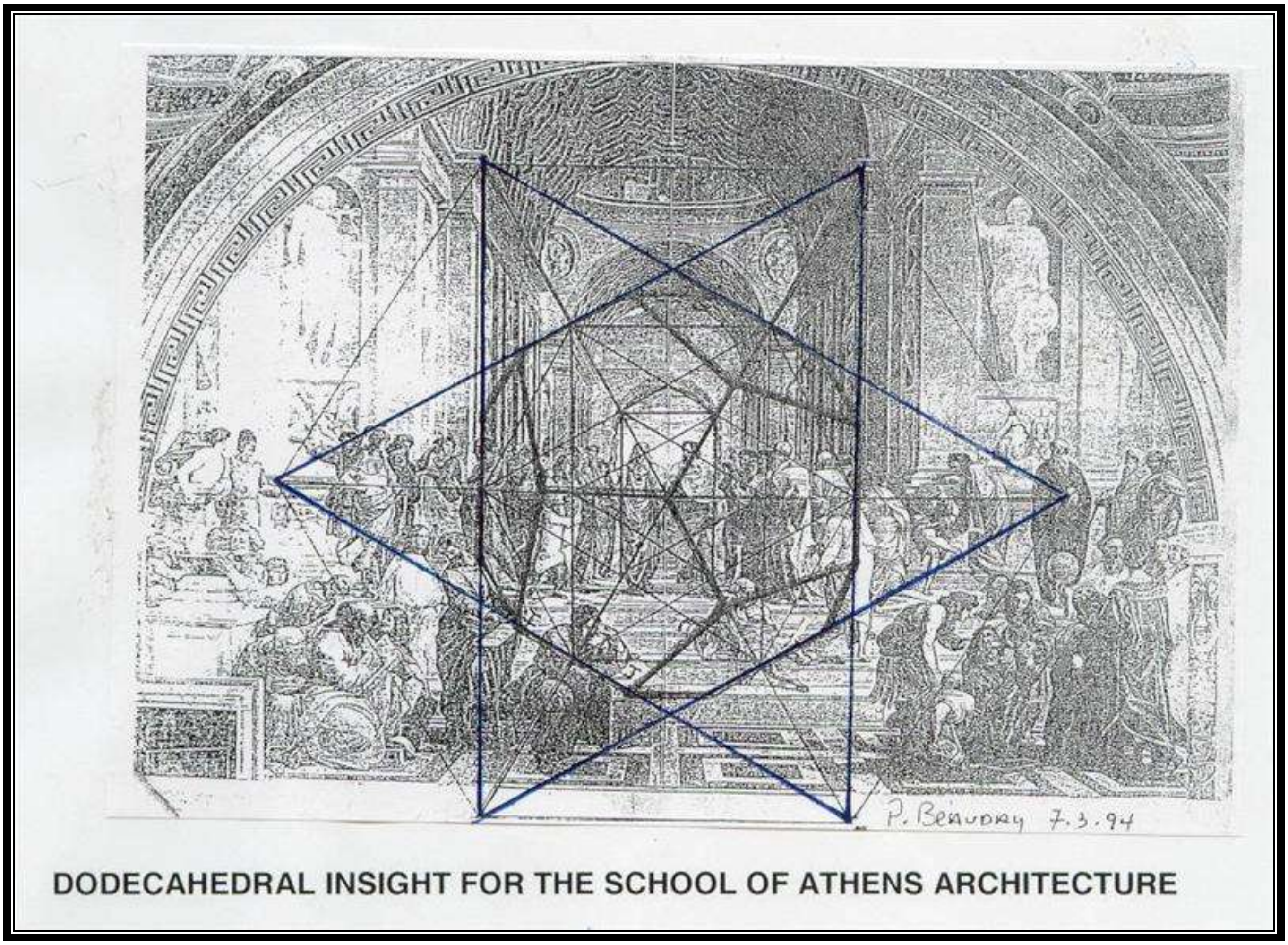


Figure 73

“Some people say this Star of David paradigm is a trick of linear perspective, others call it magic. Don’t believe everything you hear or see,” said Epistemon. “In Lanternland, we call this, “insight”; that is, the difference between the 108 degree dodecahedral angle of the solid edge and the 120 degree hexagonal angle of the Star of David in the plane; otherwise reflecting the difference between Flatland and Lanternland; that is, the difference between Plato and Aristotle. (See the center of perspective in **Figure 73**) As it turns out, that incommensurable difference in magnitude between the two manifolds corresponds precisely to the 12 degree angle of the source of light entering Plato’s Cave,” added Epistemon in a burst of laughter. “And, I should add for your knowledge that in Lanternspeak, “insight” also means wise discernment of what lies below the surface, the discovery of which is only achieved when the eyesight of your sense perception is replaced by the eyesight of your mind.”

Epistemon went on to say: "Raphael chose this method of the *simultaneity of temporal eternity*, because this is the span of spacetime by means of which an individual human being, who has a short physical life span, is capable of communicating with the past, the present, and the future, and is able to share his discoveries with all of humanity, throughout all times. Such *eternal moments* are the greatest moments in any human life because these discoveries represent the principle by means of which humanity is able to grow, from century to century. If you wish to have such an experience, then, you too can make such a discovery and share it with your friends."

"So, my question to you," said Epistemon, is: "Can you relive such a moment of discovery by reconstructing the *geometrical insight* of *The School of Athens*? The same principle applies to *The Dispute of the Holy Sacrament*, which Raphael has also painted on the opposite wall of the Room of the Signature in the Vatican." (**Figure 74**)

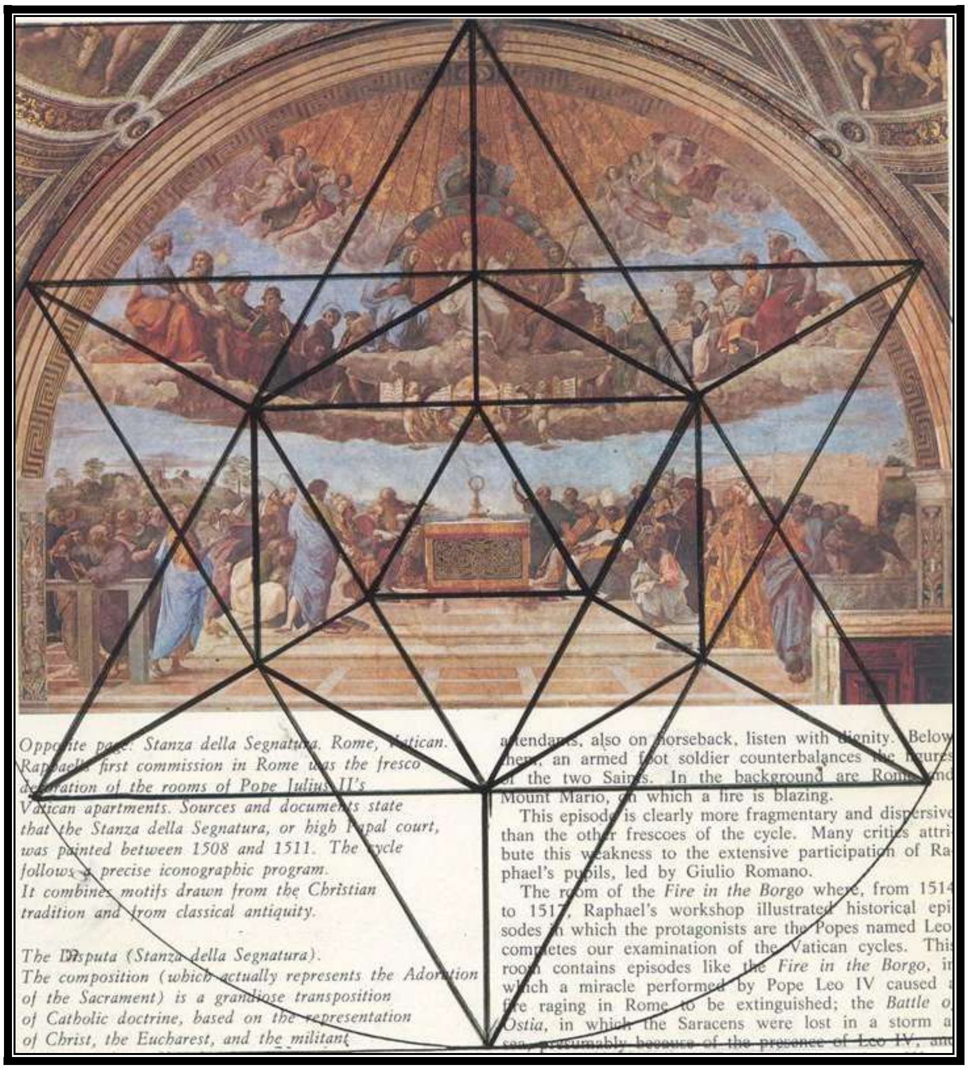
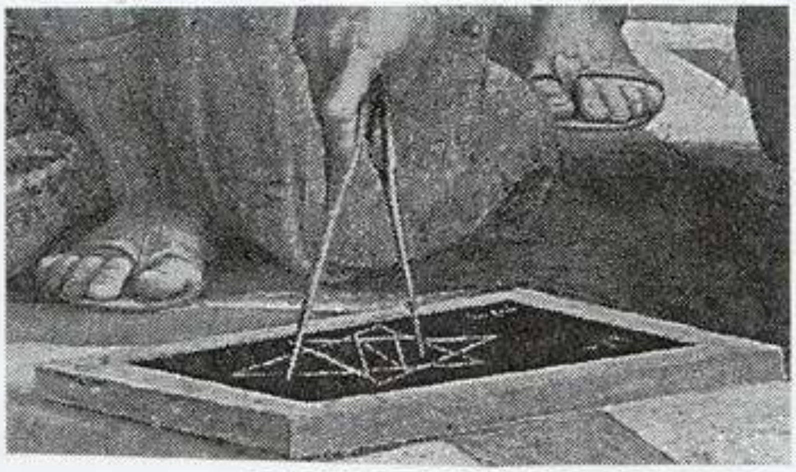
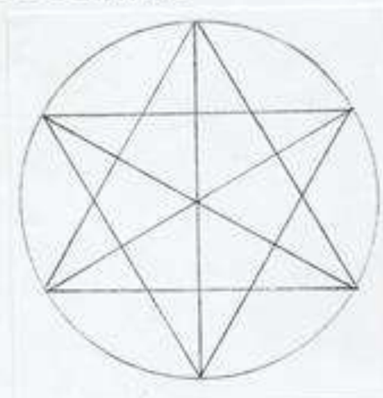


Figure 74 Icosahedral insight of the *Dispute of the Holy Sacrament*. Original picture from Bruno Santi, Raphael, *Scala Instituto Fotografico Editoriale Firenze*, 1977.

THE RAPHAEL STAR OF DAVID GENERATING PRINCIPLE



DETAIL OF THE SCHOOL OF ATHENS



Take a compass and two scalene triangles and construct a Star of David, like this. This is similar to the construction that Archimedes has drawn on his tablet, in RAPHAEL'S painting.

The hexagonal geometry is the geometry of the flat plane which only bees have been able to elevate to a higher dimension, and produce golden honey from it.

Inscribe points of decagons marking a golden section along the hexagonal radii of the Star of David. Project lines from the six points of the hexagon, as if from the inside of a sphere, to the six points of the decagons. You will see emerging from the plane a full size Kepler stellated dodecahedron. You can also generate the Poinsot great dodecahedron.

Figure 75 The Star of David generating *The School of Athens* and nine regular solids.

20- HOW TO TRANSFORM A STAR OF DAVID HEXAGONAL PLANE INTO A HIGHER SPACETIME OF NINE REGULAR SOLIDS

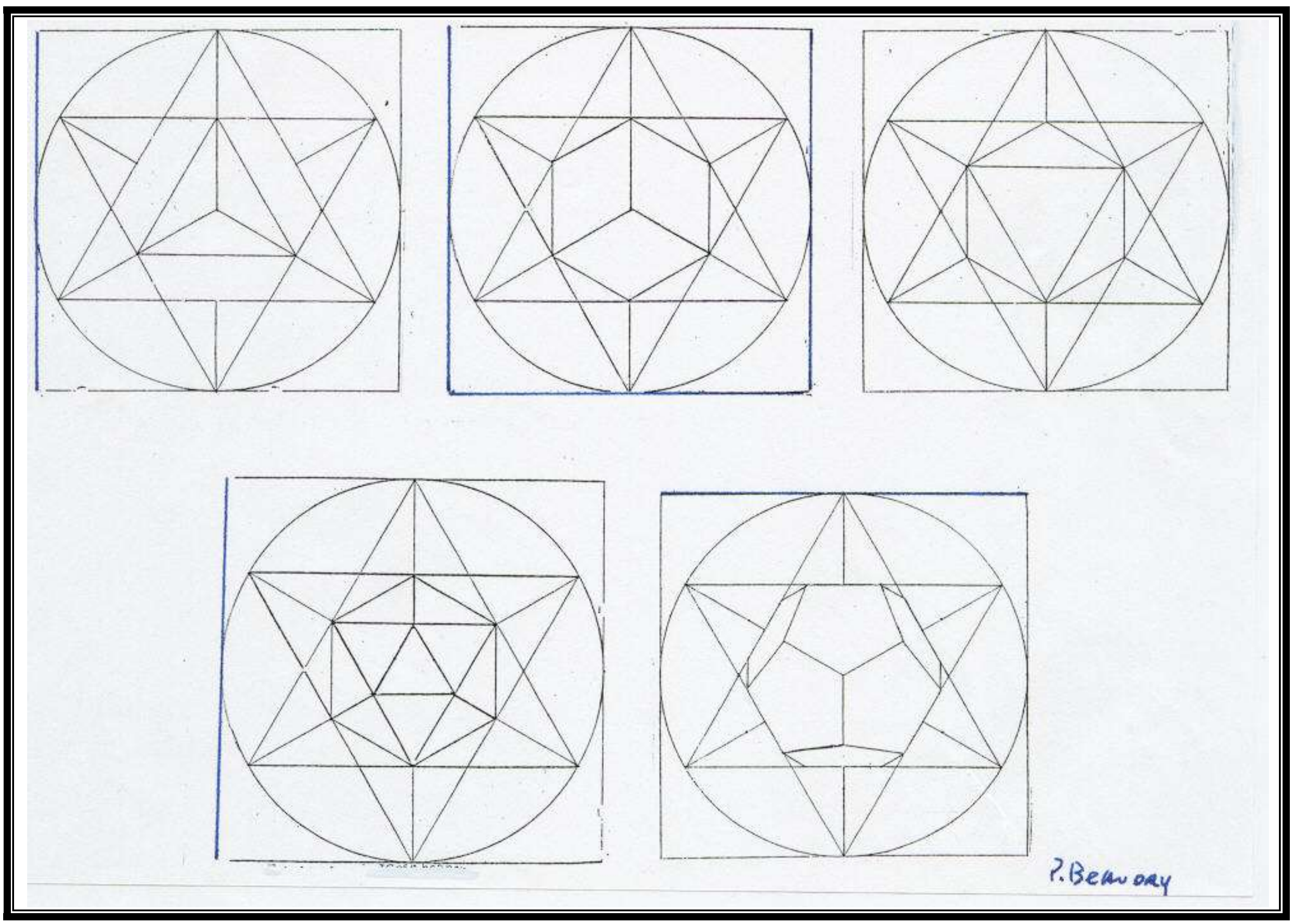


Figure 76 The Five Platonic Solids lifted from the Raphael Star of David.

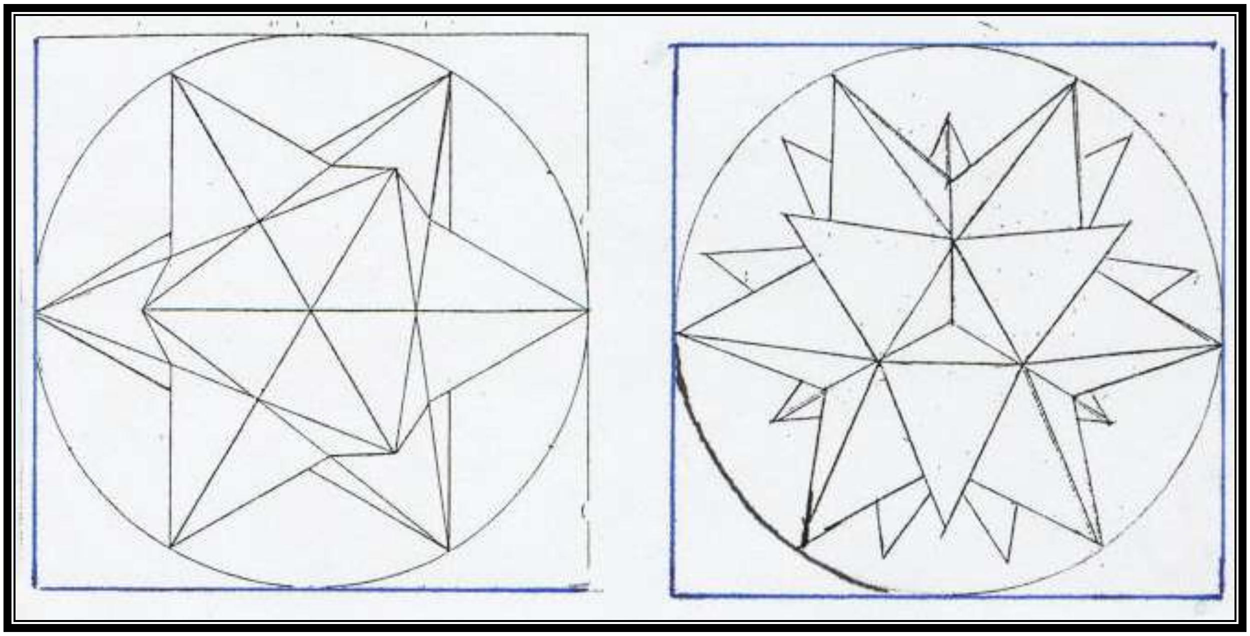


Figure 77 The Kepler small dodecahedron and great stellated dodecahedron.

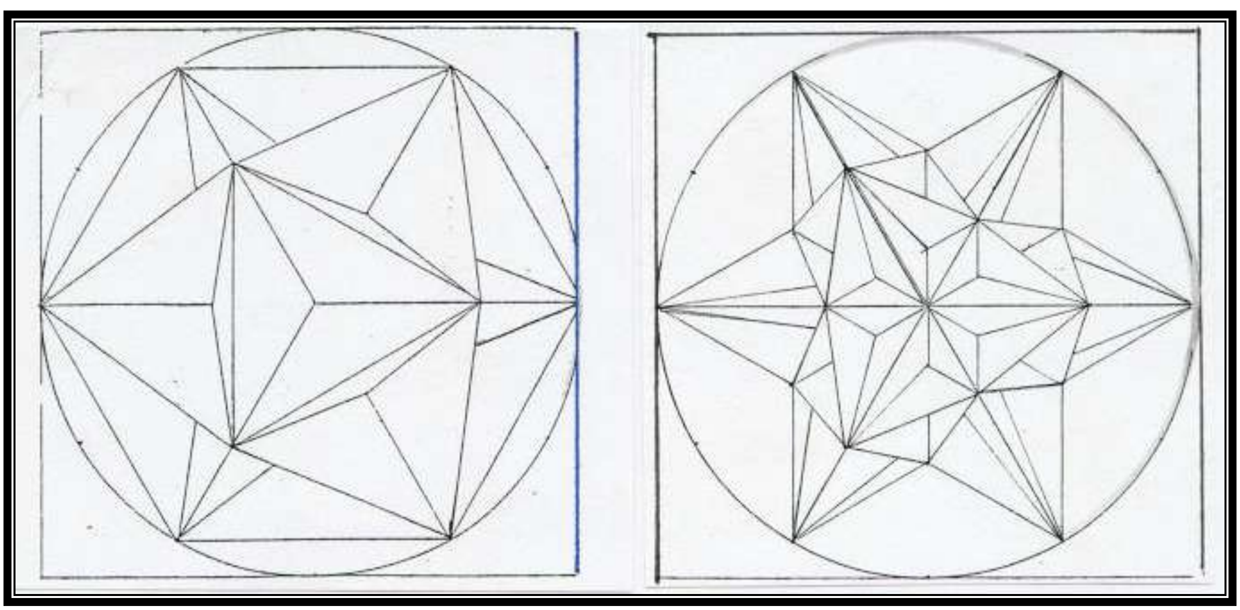


Figure 78 The Poinsot great dodecahedron and great icosahedron.

21- CONSTRUCTION OF THE SPHERICAL 'CHORA'

Take 6 great circles divided into 10 equal parts, and 10 great circles divided into 6 equal parts. Construct them together into a single sphere. The whole mixture of 16 great circles will be everywhere divided into spherical golden sections, in the proportion of 10/6, or 5/3.

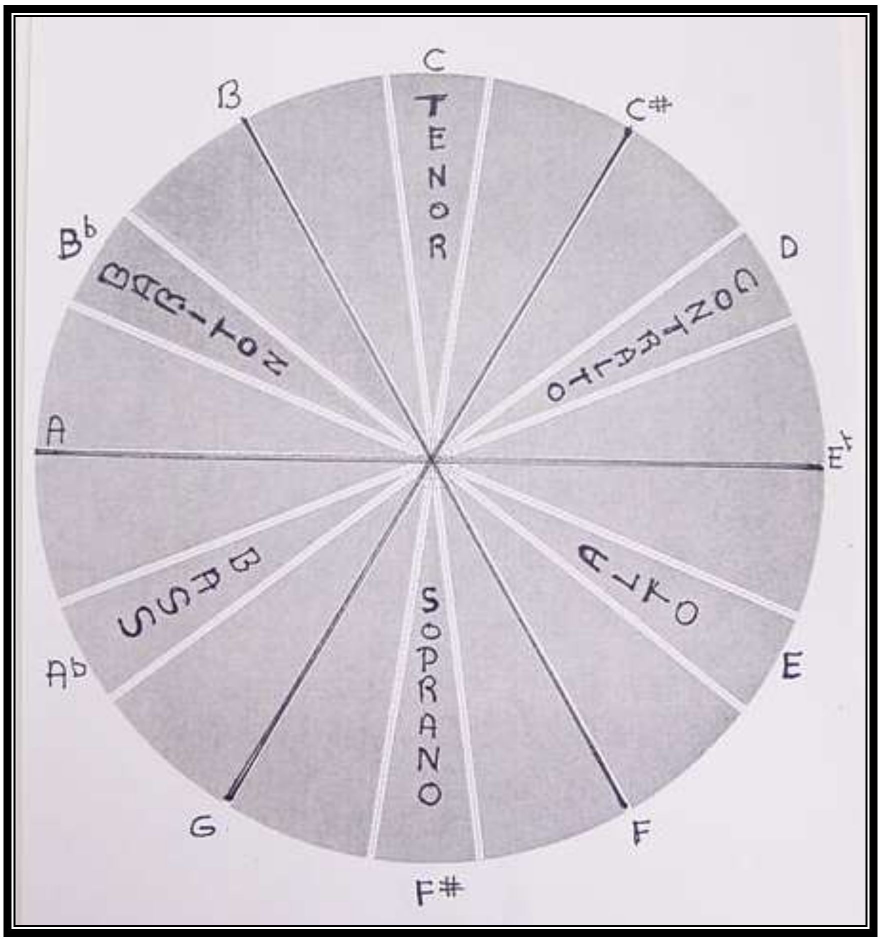


Figure 79 The six human voices are located at their respective register shift. The 120 degree angle accounts for all of the radii of all of circles in the plane.

Insert mid-section cuts of 15 degrees into the center of each of the six-sided divisions. Then, construct half of the sphere with 5 six-sided circles and 3 ten-sided circles. The mixing of the hexagonal and decagonal angles will form the initial starred pentagon.

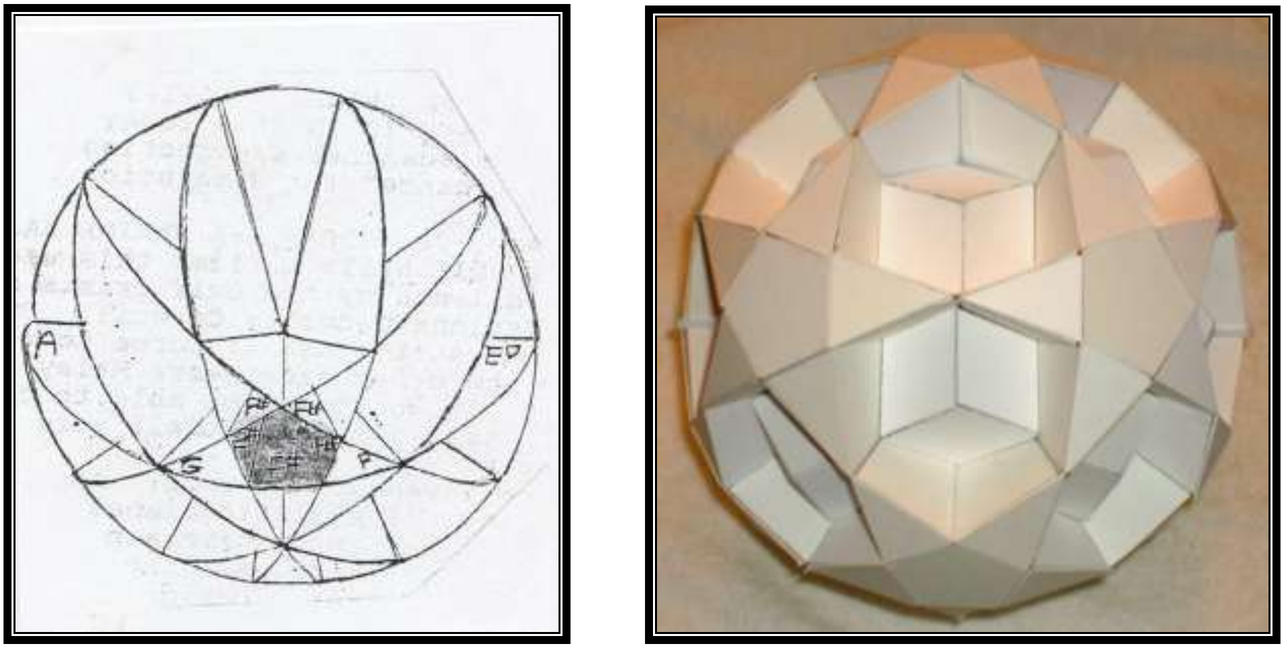


Figure 80

The complete surface of the sphere is made up of 12 regular spherical pentagons, 20 regular spherical triangles, 6 regular spherical squares, and 8 regular spherical triangles. The whole contains and generates all of the Five Platonic Solids.

SPHERICAL ACTION AS THE TRIPLY-CONNECTED PROCESS OF THE *FILIOQUE*

The spherical action as a self-generating action finds its best expression in a Kepler-Gauss approach to the *principle of congruence*, or what Leibniz called the *principle of preestablished harmony* between reason and power.

First, start with Kepler's sphere which represents a triply-connected process of constructive geometry whereby the central region is the core, the surface is its extended product, and the diameter is the unifying connection proceeding from the doubly-connected action of the other two.

The sphere comes into being when the action on the diameter proceeds from both the least-action of the central region of negative curvature and the isoperimetric-action of the surface region of positive curvature. Thus, the sphere is generated by a triply-connected action when the diameter rotates in all directions as the motivator (*Motivführung*) of a self-generating system.

A creative form of least-action is caused by the dissonant rotation of the diameter proceeding by time reversal from the future surface back to the center, isochronically. This form of inverted creative least-action generates lines, points, and surfaces by triply-folding circular action on itself, thus generating the Five Platonic Solids. The same least-action principle applies to classical artistic composition. The net result of the triply-connected process is an increase in energy-flux-density in the human mind.

The secret of this least-action process of *phasespacetime* change of Plato's "Chora" lies in discovering how the diameter is able to rotate through the whole process of spherical transformation in all directions and generate all of these solids without the convenience of sense perception.

THE SPHERICAL GENERATION OF THE FIVE PLATONIC SOLIDS

Figure 81 "Chora", Plato's Nurse of creation in *Timaeus*.

22- HOW VERNADSKY DISCOVERED THAT OUR PLANET EARTH WAS NOT ONE, BUT THREE SPHERES, ALL ROLLED UP INTO ONE; AND HOW FRIAR JOHN DISCOVERED THAT THE GOLDEN SECTION OF HUMAN VISION WAS DERIVED FROM THAT HIGHER HYPOTHESIS

"But, there is another problem that we have not yet discussed," said Friar John, "and I am surprised that you did not bring this up before."

"What is that," replied Bacbuc?"

"You keep referring to the noosphere," added Friar John, "but, you never explain what this noosphere is about. Is that another one of your famous lanterns?"

"No," replied Bacbuc, the noosphere is not a lantern, but I guess this is as good a time as any to discuss this subject matter.

"You see," said Bacbuc, "in the universe, there are three domains of existence which are interconnected by what LaRouche called Riemannian manifolds. They are: ***the non-living, the living, and the cognitive manifolds.*** Those three domains happen to co-exist, and interact intimately, as three spheres composing our planet Earth. The great Ukrainian scientist Vladimir Vernadsky called them: ***the non-living geosphere, the living biosphere, and the thinking noosphere***. Each of those three spheres has its own specific characteristic, yet none of them has a separate existence from the other two, because each requires the existence of the other two for its own survival. You cannot have the noosphere without the geosphere and without the biosphere; and, you don't have the geosphere, or the biosphere, without the noosphere."

"How did Vernadsky ever come up with such an idea," asked Friar John? "Why can't these three domains exist independently of one another?"

"Because they are derived from the harmonics of the solar system, as Kepler has shown," said Bacbuc. "Each planet has its proper orbit, but each orbit affects the orbits of the other planets, and is affected by them as well. Take the case of an ordinary human being. This is a living being who, in order to be in good health, requires iron in his diet. On the one hand, iron comes from the mines of the abiotic geosphere, and belongs to the non-living domain. Now, that doesn't mean that in order to be healthy, you have to go to the mine and chew on a piece of iron ore or a piece of steel for your lunch. Your teeth would not last very long, and besides, your digestive system would become rusty real quick. However, an iron supplement is very much part of getting your body in shape. That is an interaction between the abiotic geosphere and the biosphere.

Now, on the other hand, if you use iron for building a railroad system to run across the Sahel region of Africa, you are developing the noosphere. Iron becomes a function of the noosphere, because the creation of such a railroad becomes part of an economic intention that is aimed at improving the general welfare, and the culture of the entire African population. Thus, a railroad infrastructure brings electricity and technological progress to the heart of Africa, and improves many times over the cognitive power and the growth of the entire African population."

"I see," said Friar John. "Iron is not an isolated resource then. It is not something that has a mere existence in and of itself. Its real usefulness is to interact with the biosphere and the noosphere."

"That is right," said Bacbuc. "Properly used, iron has the power to increase the relative potential-population-density of the whole planet. However, the most important feature of the interactions between those three domains resides in the ability of the higher orders to change, and increase the power of the lower order. An example is how the biosphere changes the geosphere by creating the oceans and the atmosphere, and how the noosphere, that is the cognitive powers of man, changes both the biosphere, and the geosphere by developing new technologies. It is, in fact, the responsibility of the cognitive function of the noosphere to assure the successful transformations of the biosphere and of the geosphere for future generations. This defines fundamentally, and scientifically, the required approach to the question of the environment as a whole, and establishes those three interacting domains as the proper object of physical science as a whole."

"Now, there is a more profound implication in the interaction of those three domains." added Bacbuc. "Consider the universe in its totality. It has been understood from the beginning of time that the universe as a whole is not only alive, but is also universally determined by a cognitive principle. This means that a knowable intention that the Greeks, such as Plato, defined as *Hylozoic Monism* guides the changing geometry of the universe; that is, the universe as a whole is a living being that is determined by a unique law of multiply-connected circular action of change.

"The difficulty that certain people have had with this conception is that they fail to understand it at the level of a constructive geometric principle, because they tend to project onto the universe, an a priori physical, or sense perception image, of what they consider to be a thinking being, or a living being."

"You mean something like those giant animals, bears and dragons, that the ancients imagined were living in the stars," said Friar John.

"No," replied Bacbuc, "those animal forms were merely invented in order to better remember the shape of the constellations. What I am talking about is rather like the case of the Greek stoics, who wore tight corsets all of the time, and wrongly believed that the matter of the universe was made up of tiny little physical atom-like particles, which were filled with life whose mission was to transmit it throughout the universe. Such a silly conception is very similar to some of those nuclear physicists of today, who believe in the existence of quarks, as the fundamental building blocks of the universe. That sort of thing."

"The best example of *Hylozoic Monism* is the *Monadology* by Leibniz," said Bacbuc, "in which every being in the universe is a microcosm interacting with, and being interacted by the macrocosm; that is, a universe in which each of the smallest existing things in the world is acting on the *principle of composition* of the universe in its totality. A good illustration of this would be to conceive, without the need of your sense perception, of a grandiose musical symphony created by God, in which the smallest creature acts like a musical interval expressing the principle of the entire composition; that is, how each and every part carries the living and cognitive process of the whole composition by way of multiply-connected spiral action."

"I think I see what you mean," said Friar John. "I guess this would also mean that physical science is similarly based on the constructive geometrical principle of musical composition. Is that right?"

"Absolutely," answered Bacbuc, "and that is why most of the best scientists in history were also musicians."

"Is it true, also what I have heard about how animals appreciate classical music? I have heard that if a farmer plays Mozart symphonies while he is milking his cows, he will definitely improve the quality, and the quantity of the milk. And in turn, the quality of the manure will improve the soil on his farm. Is that true?"

"Yeah, … and that is probably why the grass will be greener on that side of the road too," replied Panurge, in a burst of laughter.

"Possibly," answered Bacbuc, "but, I am not absolutely sure. In any case, Friar John's idea is not crazy, because the anti-entropic effect of classical music does improve the creative powers of the human mind. That has definitely been proven to be true. So, I don't see why cows, or any other animal for that matter, should not also benefit from the anti-entropic quality of classical music. You see, since anti-entropy is the fundamental characteristic of the universe as a whole, and is present in the succession of growth from non-living to living, to cognitive, this means that such a process of axiomatic change, occurring by leaps and bounds, is like a classical musical composition, and increases the power of the individual over the universe with each new dimensionality. For instance, even the simple axiomatic change between the plane and the sphere represents such a divine anti-entropic change.

"Which reminds me," interjected Friar John. "What about the divine proportion? How does that relate to the geosphere, the biosphere, and the noosphere?

"That is an excellent question," replied Bacbuc, "and, I am glad that you have asked it because it is directly relevant with our subject matter. I will attempt to answer your question the best way that I know how. You see, this question requires a lot more thinking still, so, I am afraid I will not be able to answer it as completely as I would wish. However, I can show you my working higher hypothesis."

"Well, give it your best shot anyway," added Friar John, "because, regardless of the fact that I am a man of the cloth, and that I should know everything about what is divine, I don't even know how to begin answering my own question."

"All right," said Bacbuc let us start with the following divine proportion as a higher hypothesis:

God is to cognition as living is to non-living. Express this, geometrically, in the form of a divine proportion, like this.

GOD:COGNITION :: LIVING:NON-LIVING

"If we assume this proportion to reflect the fact that, first and foremost, man is created in the living image of God, then, we are authorized to consider our higher hypothesis as a direct reflection of that fundamental principle of creativity. Now, let us say that this is the highest form of divine proportion which defines the boundary condition of the three domains of the geosphere [non-living], the biosphere [living], and the noosphere [cognitive], then, how can we demonstrate that the golden section of divine proportion is a derivative of such a higher hypothesis?

"First of all, let us examine the highest ratio of ***God*** to ***cognition***. There is no sense-experience for such a divine relationship. However, such a relationship is the basis for all cognitive human relations, with respect to the reliving of an array of creative discoveries from the past, such as the historical figures represented by Raphael in *The School of Athens*. By rediscovering and transmitting discoveries of principle, agapically, man is demonstrating himself to be a direct reflection of God the creator.

"Secondly, take the other ratio of ***living*** to ***non-living.*** In this case you can associate the golden section of divine proportion by relating the living to the non-living as an expression of sense-experience. You can demonstrate that by showing how the golden section in the sphere is to the golden section in the plane as a metaphor of the singularity between the non-living and the living domains.

"Take the golden section of human vision, as an example of a living process of spherical dimension, and show how visual projection is based on the golden section! First, recall that your visual space is spherical. Now, you may have never paid attention to this, but your visual projection actually has a built in golden section range finder.

"Indeed, since everything that exists in the biosphere is explicitly bounded by the golden section, you can generate the following construction to prove that your physical eyes are also bounded by the golden section: inscribe a dodecahedron inside of a decagon. You can easily do this by inscribing pentagonal points around a circle, and drawing in the plane five equal arcs (**Figure 82-A**) whose radii are each the length of $1/10^{th}$ of the circumference of the circle. Then, reproduce exactly the same circle, a second time, and join all of the points together by straight lines (**Figure 82-B**). In this last case, you have lifted half of the dodecahedron from the plane as if it was coming from a sphere! Now, do you see the anomaly?" Asked Bacbuc.

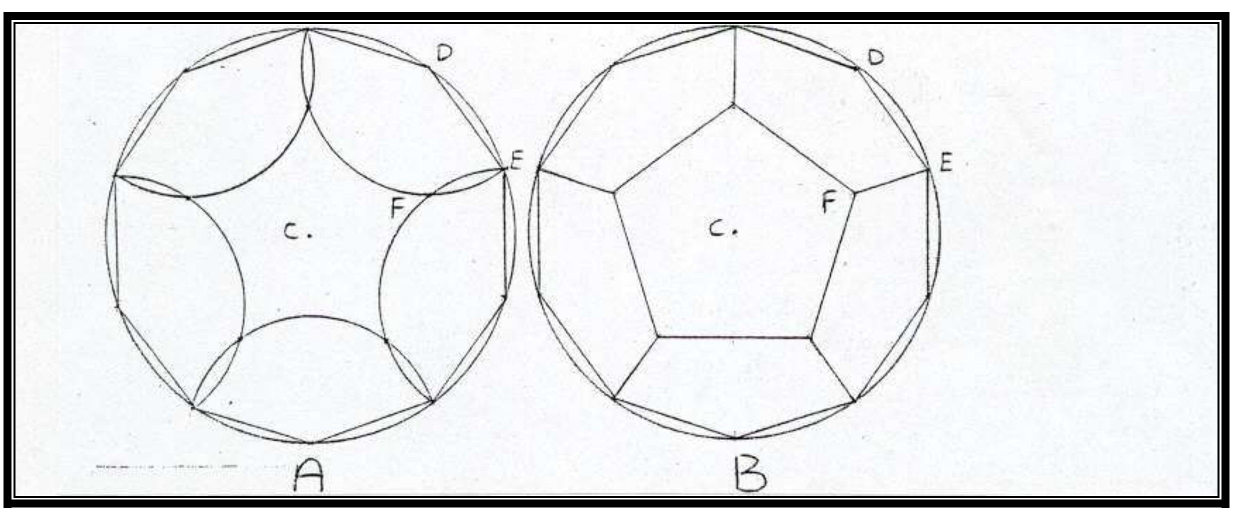


Figure 82 A and B. Projection of a dodecahedron into a decagon.

"What anomaly," replied Friar John?

“Don’t you see the difference between **A**, and **B**,” asked Bacbuc?

“Of course, I see the difference.” said Friar John. “**A** is made up of curved arcs, and **B** is made up of straight lines.”

“What else,” asked Bacbuc?

“Nothing else,” replied Friar John. “What else is there to see?”

“Well, look again,” said Bacbuc. “When you look at figure **A,** you see it as inside of a plane circle. However, when you look at figure **B**, you are seeing it in three dimensions, as inside of a sphere! Why?

“What three dimensions? I don’t see any three dimensions,” replied Friar John, obviously getting upset.

“Concentrate on **B**,” replied Bacbuc, “and you will see that the golden section of figure **B** will pop up in your eyes, all of a sudden, I guarantee you.”

“Yes, I can see it,” said suddenly Friar John. “Wow! It really stands out at you. By the holy plans of Theleme, it is as if God had put a compass in my eye!”

“He did,” said Bacbuc, “and He also put that compass in your mind’s eye. Now, take one of your compasses, and measure the distances between **C, D, E,** and **F**. The relationship between those points will show you the anomaly emerging within the golden section of human vision.

CD:DE :: CE:FE

“Consider this golden section proportion closely, and you will discover that it is false in the plane of figure **A**, but it is true in the sphere of figure **B**!” Said Bacbuc.

“How come?” asked Friar John, getting all itchy and perplexed again. “How can something be true and false, at the same time?”

“This happens with a lot of people,” said Bacbuc, “because **DE** and **FE** are not of equal lengths in figure **A**, yet, they are of equal lengths in figure **B**.”

“No way,” said Friar John, “this can’t be true, because I can see that both lines **DE** are obviously longer than the lines **FE**, in both cases.

“Quite,” replied Bacbuc, “but, since **FE** in sphere **B** corresponds to the side of an inscribed regular pentagon, as seen in perspective, you must discount that difference, because it only appears to be longer than **DE**. Your physical eye doesn’t know that, however, your mind’s eye does. Thus, your mind’s eye knows that **DE** does equal **FE**.”

“Oh! Oh! I see,” said Friar John in a state of wonder. “I feel like the guy who has one eye looking at you, and the other eye looking to the side.”

“Ha! Ha! Now you know how to cook the fish and watch out for the cat at the same time,” said Epistemon laughing.

“Is that the same golden section as in the sphere?” asked Friar John. “Can you measure it with numbers?”

“No! You cannot.” added Bacbuc. “This proof of the golden section of human vision does not require numbers. You can only prove its truthfulness by reliving the discovery that you have just experienced. So, you see, it is the ability of constructing such a pedagogical device, and relating that simple discovery to another person, as the analogue of a metaphor for the creative process, that you can consider having solved the problem and proven it by construction. That is the proverbial ‘proof in the pudding’ that you have proceeded from the divine proportion derived from our higher hypothesis.”

“This is quite beautiful,” concluded Friar John. “But, this is not what I was contemplating at all, when I first asked you my question. But then again, now that you mention it, I guess that must have been what I was thinking all along, but without my knowing it. The divine proportion surely works on you in a funny way, even when you don’t pay attention, doesn’t it,” said Friar John, exploding into a thundering laugh. “God really has mysterious ways.”

A PARTING GIFT

BLOW YOUR WHISTEL BLOW

(A LANTERNLAND SONG)

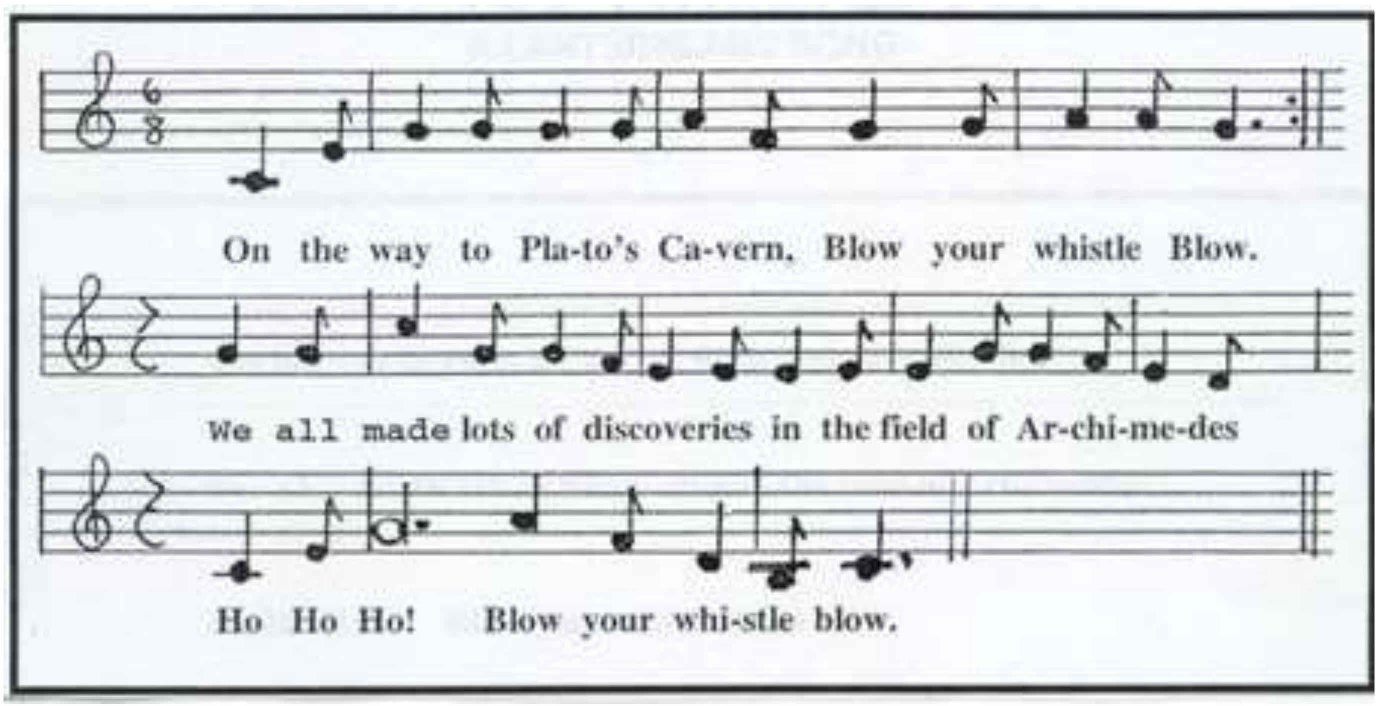


Figure 83

-1-

On the way to Plato's Cavern,
Blow your whistle blow.
On the way to Plato's Cavern,
Blow your whistle blow.
We all made lots of discoveries
In the field of Archimedes
Ho Ho Ho! Blow your whistle blow.

- 2-

We've discovered bees and honey,
Blow your whistle blow.
We've discovered bees and honey,
Blow your whistle blow.
They have neighbors oh so many
Cause they build hexagonally
Ho Ho Ho! Blow your whistle blow.

-3-

We've discovered our frog Henry,
Blow your whistle blow.
We've discovered our frog Henry,
Blow your whistle blow.
He had fallen in the pipe way
But Dario raised him safely
Ho Ho Ho! Blow your whistle blow.

-4-

We've discovered Epistemon,
Blow your whistle blow.
We've discovered Epistemon,
Blow your whistle blow.
He's the greatest midnight oiler
With a wonderful demeanour
Ho Ho Ho! Blow your whistle blow.

-5-

We've discovered Pacioli,
Blow your whistle blow.
We've discovered Pacioli,
Blow your whistle blow.
He had found the shortest pathway
With the angle one and twenty
Ho Ho Ho! Blow your whistle blow.

-6-

We've discovered Pantagruel,
Blow your whistle blow.
We've discovered Pantagruel,
Blow your whistle blow.
He's the greatest knowledge drinker
Axiom buster like no other
Ho Ho Ho! Blow your whistle blow.

-7-

We've discovered Midnight Oilers,
Blow your whistle blow.
We've discovered Midnight Oilers,
Blow your whistle blow.
Each would guide us with a lantern
To the site of Plato's Cavern.
Ho Ho Ho! Blow your whistle blow.

-8-

We have built Platonic Solids,
Blow your whistle blow.
We have built Platonic Solids,
Blow your whistle blow.
They reflect the golden section
From the clear dodecahedron.
Ho Ho Ho! Blow your whistle blow.

-9-

We've discovered our friend Panurge,
Blow your whistle blow.
We've discovered our friend Panurge,
Blow your whistle blow.
He had mastered Kepler's knowledge
Without having gone to college.
Ho Ho Ho! Blow your whistle blow.

-10-

We have met Nic'las of Cusa
Blow your whistle blow.
We have met Nic'las of Cusa
Blow your whistle blow.
He had prov'd that for the circle
There had to be zero angle.
Ho Ho Ho! Blow your whistle blow.

-11-

We have met Raphael Sanzio
Blow you whistle blow.
We have met Raphael Sanzio
Blow your whistle blow.
He conceived *The School of Athens*
As a ga-the-ring of good friends.
Ho Ho Ho! Blow your whistle blow.

-12-

We've discovered Max and Mirca,
Blow your whistle blow.
We've discovered Max and Mirca,
Blow your whistle blow.
They have helped us oh so nicely,
Oscar did, and also Mattie,
Ho Ho Ho! Blow your whistle blow.

ADAGIO IN CANON FORM

-13-

I have seen you all so happy,
Blow your whistle blow.
Richard, Matthew, Jacob, Roman,
Blow your whistle blow.
Gabby, Adrie, Chris, Umberto,
'Long with Michael, Sam'n Cory.
Ho Ho Ho! Blow your whistle blow.

APPENDIX 1

DARIO'S DIMEMMA

Figure 84 A true story

A long time ago, during the warm days of summer camp, the air was filled with the brassy songs of crickets and cicadas, and all of the children were having a good time, jumping and swimming in the cool waters of the lake.

Dario, however, was very sad because he thought he had done something terrible, and did not know how to solve his problem. He was in a real dilemma. He had mistakenly put his friend Henry, the frog, on top of one of the pipes that held the pier together, thinking he would be safe, and he would not lose him in the water. Well, something terrible happened. Henry fell inside of the pipe, out of reach.

Dario was very upset. "What should I do," he cried out desperately? "I don't know," said the swimming instructor, he might be hurt down there. Henry cannot climb or jump out of that hole by himself." Dario was terrified suddenly, as he looked inside the dark hole and could not see Henry. Dario was beginning to despair when he heard Henry calling him: "Burpuit! Burpuit!"

“He is alive,” cried Dario, wiping his eyes with a dash of hope. He ran to the nearest bush to get a long twig. “What are you going to do with that,” asked the instructor. “I am going to help him out, “replied Dario, happy to have found the solution to the problem. “All he has to do is grab hold of the twig, and I will pull him out,” he said.

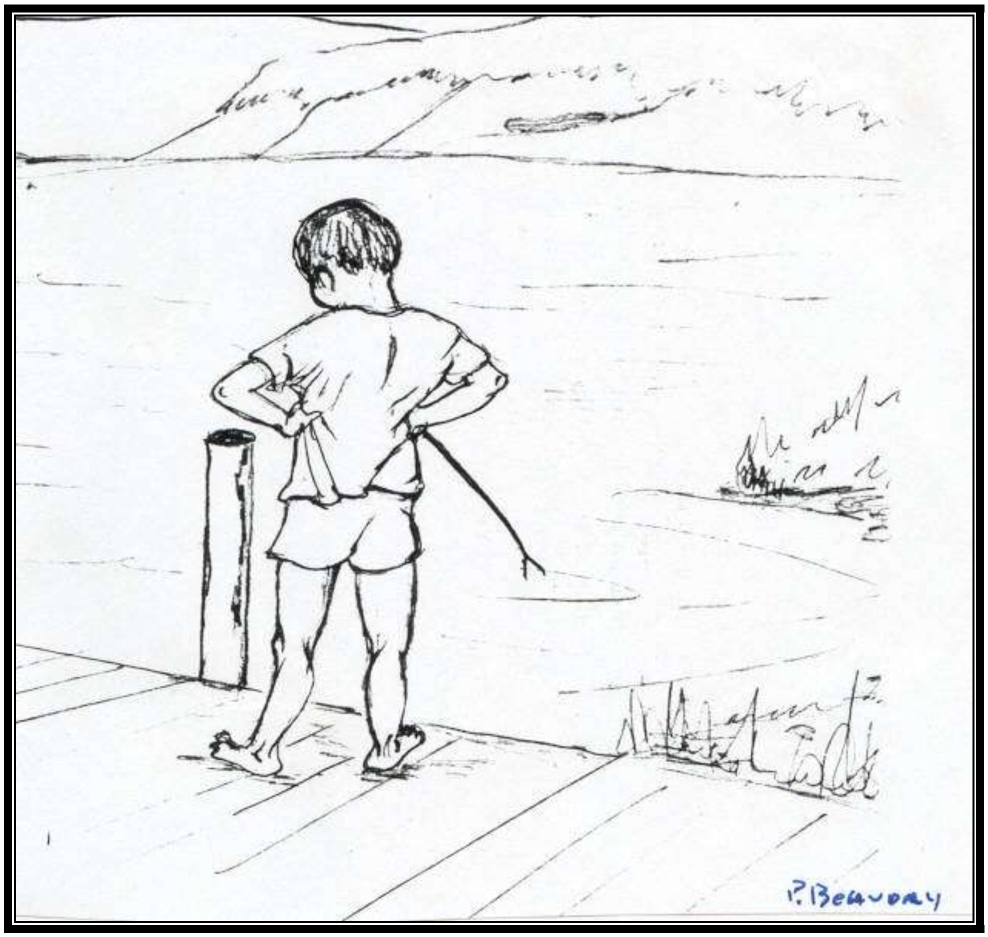


Figure 85

“Silly boy,” replied the instructor in a stern voice, “don’t you realize you could kill him if you poked at him with this stick. And besides, what makes you think that he can grab that stick? Henry doesn’t have fingers. “Dario was so confused that he felt his mind was about to burst. “You see,” said the instructor calmly, you are thinking of your own resources for survival. What about Henry’s resources for survival?”

Dario stared at the instructor for a moment, then wiped his tears off again, and smiled. “I know what I am going to do,” he said, fully confident that he had found the real solution to his problem.

Do you know how Dario solved his dilemma?

DARIO TOOK A PAIL OF WATER AND POURED IT INTO THE PIPE. THEN, HENRY FLOATED RIGHT UP TO THE TOP!!

Figure 86

Sometimes, out of purely selfish interests, we inflict on our friends a fate worse than would otherwise happen if we let them decide what they know best for themselves. Give a friend a chance to discover his own resources. That is the idea. And a good way to start is to do an investigation of your own talents as well as those of your friends, and share this awareness with them. Your increased consciousness of your own capabilities, and that of others, will increase your confidence in yourself, and in those who have put their trust in your friendship. Sharing these talents with others will help them discover their own capabilities, and will also show you where you can improve with respect to them. That is called *agape* of the common good. The ancient Egyptian pyramid builder, Imhotep, called it the Maat Principle, the principle of balance and reciprocity. This is the reason why the suffering of a little tension over the concern for others is always necessary if you wish to make a discovery.

APPENDIX 2

NEOLITHIC PLATONIC SOLIDS

Figure 87[18]

Archeologists have uncovered in Aberdeen shire, Orkney, Skye, and in other sites in Scotland, hundreds of carved stone balls dating back to the Neolithic period; that is, between 3200 to 2500 BC. Most of the carved rocks are generally the same size, each less than three inches in diameter, and can be easily carried by hand.

Note that some of the regular features of the stones reflect early forms of spherical platonic solids.

1. The ball on the extreme left is divided into 8 equal sections forming a rounded cube.
2. The second ball from the left is divided into 4 knobs suggesting the shape of a spherical tetrahedron.
3. The ball in the middle has 12 knobs, defining the oldest known dodecahedron.
4. The second ball from the right has 14 knobs forming an incomplete icosahedron.
5. The ball on the extreme right divides the spherical space into 8 equal parts, close packing 6 knobs forming an octahedron.

[18] Illustration from Dorothy N. Marshall, ***Carved Stone Balls,*** (Proceedings of the Society of Antiquaries of Scotland 180), pp. 40-72, 1976-77.

Although the function of these stones seems to be unknown, their construction indicates a well-ordered sense of spherical composition. Many other stones have more knobs, and some are intricately carved with spirals and cross-hatching on their faces. Some show decorative concentric circles that required great craftsmanship as well as a keen knowledge of the material. The degrees of elaboration in the designs were proportional to the malleability of the stone, but most of all, reflect a real tour de force in the determination of multiply-connected circular action. The material used varies from sandstone, serpentine, to very hard stones such as granite, greenstone, gneiss and quartzite.

The most amazing aspect of these Neolithic polyhedra, however, is that they were produced about 2000 years before they were first known to exist in the Greece of Plato. Here, you have the oldest set of spherical regular solids reflecting early man's attempt to master the idea of dividing the sphere into regular parts. This is the most beautiful proof of the power of cognition of ancient man as distinct from the beast. Those were the days when Neolithic man knew how to organize his Platonic Cave!

BACK COVER

Pierre Beaudry is a well known political organizer of the Lyndon LaRouche movement, who has been teaching constructive geometry for the last 40 years and who has gained international recognition primarily through his online axiom busting blog known as **Pierre Beaudry's Galactic Parking Lot**. He is an epistemologist who has been able to construct his own knowledge, himself, on the model of the Rabelaisian ***alpheste***; that is, in the image of the industrious miller who likes to grind old ideas in order to improve on them.

If you wish to change your own bad habits of believing everything people tell you and you are tired of company manners, then, ***LANTERNLAND*** is the book for you to investigate and to master, because this sort of exercise will make you discover outrageous Platonic and Rabelaisian principles which will make you lose the fat of your false underlying assumptions, will help you escape the boring plane of Aristotelian FLATLAND, and will uplift you to the exciting galactic domain of Platonic SPHERICS.

In short, LANTERNLAND is a self-teaching and self-transforming method of constructing your own knowledge without being forced to go along to get along. It shows you how to change and it changes you at the same time.