DIOPHANTUS AND THE ART OF INVENTION

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From the vantage point of Fermat, I have found that the most important thing to discover about Diophantus is that his method was entirely based on creative constructive geometry, that is, the constructive art of discovering the unknown by means of cognitive inventiveness. So, provided that the solutions of the following constructed problem were rational, that is to say, expressed by whole rational numbers, or their fractions, the general rule of thumb of Diophante could be stated simply by saying: {***Given something that is known, find the unknown.***}

However, I must warn you about the fact that this art of invention is always froth with paradoxes. For instance, Diophantus always challenges you to discover the unknown by giving you something that is known. But, as you rapidly discover, the unknown does not proceed directly from the known. You may object that you can only access the unknown from what you already know, but that is false. It does not work like that. What must be done is to create the indirect means that will lead you to what needs to be discovered. Let's take an example.

« Find two numbers such that their sum and the sum of their squares correspond to two given numbers. {*Example*} The sum of the two given numbers is 20, and the sum of their squares is 208. One of the numbers shall be 10 + N, and the other, 10 – N, the sum of their squares ; $2N^2 + 200 = 208$, N = 2. »

Here, the formalist student will simply tend to grab the result N = 2 and apply it immediately to the formula 10 + N and 10 – N, and be satisfied with the discovery of the two numbers, 12 and 8. The inquisitive student, however, will look for something else, because he will have realized that the question was not so much how to get the result, but rather, how to discover the way the problem was constructed in the first place. So, he will seek to find shadows of that process. He will be looking for the intention, not the result. The truth is not in the sunlight, but in the shadow. As an old painting teacher of mine used to say: "Don't paint the trees, paint the shadows of the forest. Then, you will understand the trees." So, the seeker will look for how to construct any arithmetical problem whose purpose is for someone else to discover the unknown? That is the intention of the Diophante method.

Therefore, how did Diophante go about constructing this problem? The answer is very simple. Diophante simply started his construction by determining what the two unknown numbers would be!

CONSTRUCTING FROM THE UNKNOWN.

As I stated above, the solution to this problem cannot proceed from investigating the known numbers, 20 and 208. You can stare at these two numbers for hours; they will never yield the solution. You will find yourself in the same state as the phone team organizer who stares at a contact's phone number and never decides to make the call. On the other hand, the way to discover the two relevant unknown numbers is to create those two unknown numbers.

So, let's say 10 and 14 are those two unknown numbers, whose sum is 24, and whose sum of squares is 296. The key to the construction of the problem is {***the creation of two other numbers, 12 + N and 12 – N,***} which are created out of the blue by dividing 24 by 2. That is the necessary in-between step that is required in order to solve the whole problem. Since the invention of this minimum-maximum device has only a limited number of variables, it is easy to discover that the value of N is 2, and therefore, the sum of their squares corresponds to $2N^2 + 288 = 296$. If you were to decide that $N = 3$, then, $12 + 3 = 15$, and $12 – 3 = 9$ would also give you a sum of 24, but the sum of squares would have to be changed to 306. That's all there is to it.

So, you see, what you were looking for were not the two unknown numbers, as such, but the unknown construction by means of which any two numbers could be found. That is the trick. Like the old Lakotah spiritual leader, Iktomi, you have to become a trickster and seek to discover what lies, as Lyn put it, « between the notes. »

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