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From the desk of Pierre Beaudry

THE WELL-TEMPERED DIVINE PROPORTION AND THE BIQUADRATIC UNIVERSE.

by Pierre Beaudry
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INTROCUCTION: AN INVESTIGATION INTO OUR BIQUADRATIC UNIVERSAL

In the domain of Sphaerics, the Pythagoreans developed a *quadrivium* of universal divine proportionality between, music, astronomy, arithmetic, and geometry, simply by using angular shadow measurements. Such a *quadrivium* also expressed this proportionality of the universe by relating four axiomatic discoveries, each of which was based on the hypothesis that the universe as a whole expressed the fundamental form of biquadratic unity of the Pythagorean Tetrad. Those four biquadratic discoveries were 1) the astronomical observatory of the Great Pyramid of Egypt; 2) the doubling of the cube by Archytas; 3) the dodecahedral musical and solar system; and 4) the Five Platonic Solids.

The most exciting part of this form of biquadratic exercise is that it has no required mathematical formula for it: it is entirely constructible by analogue and logarithmic-infinitesimal angular browsing alone. The only numbers required are the shadows of rotations by 360 degrees and the logarithms attached to them. On the other hand, you are required to apply the universal physical principle of proportionality that

Thales, Pythagoras, Plato, Hipparchus, Eratosthenes, Cusa, Leonardo, Kepler, Leibniz, Monge, Gauss, and Riemann have used in understanding the harmonics of non-entropic universal development. But first, let us define what we understand, here, by a biquadratic.

First and foremost, the highest form of biquadratics relates to the domain of the four physical phase-spaces of the universe as a whole which correspond to the four epistemological powers of human creativity as developed by Lyndon LaRouche in his paper on *The Force of Tragedy*, EIR, Nov. 9, 2007. LaRouche posed the four phase-spaces of our biquadratic universe in the following manner: "*Since the relevant summations by V. I. Vernadsky and Albert Einstein, combined, we now know of the partition of the known universe among four rigorously defined phase-spaces: the ordinary (non-biotic), the Biosphere, the Noosphere, and that still higher order of phase-space, which subsumes the Noosphere*."

From that vantage point, the twelve-interval musical system of well-tempering, being universally related to the zodiac as it was understood by the astronavigators of the ancient Peoples of the Sea, and the division of the heavens being assigned to the dodecahedron, as Plato properly made the claim in his *Timaeus*, throughout the history of early civilization from the Egyptians, the Pythagoreans, and the Platonic Academy, it has become a common cognitive heritage since the Italian Renaissance of Nicholas of Cusa, Fra Pacioli and Leonardo da Vinci, as well as Kepler and Leibniz after them, that the divine proportion, including its derivative forms known as the dodecahedron derived golden section, had been the continuous geometrical stamp of recognition of a higher ordering proportionality that properly related the a-biotic, biotic, and noetic domains in accordance with Vernadsky 's understanding of the relationship between the spheres of the non-living, the Biosphere, and the Noosphere. The fourth domain subsuming, as it were, the Noosphere could only be properly identified as the domain of the Divine, whose enveloping higher power, embodied within the universe itself, can only be grasped by human understanding as in a form of negative theology as demonstrated extensively by Cusa in his *De Docta Ignorantia*.

Therefore, unless we introduce an intermediary level of angels, between the Noosphere and the Sphere of the Divine, we shall assume that the universe as a whole, can only be grasped by some form of divinely ordered proportional relationship between the four different and incommensurable domains of abiotic, biotic, noetic, and divine proportionality, but in a manner such that the following can never be an extension of the former. Thus, the general form of our finite but self-bounding and self-developing anti-entropic biquadratic universe will be:

ABIOTIC : BIOTIC :: BIOTIC : NOETIC :: NOETIC : DIVINE

In an artistic form, such a biquadratic harmonic proportion between Vernadsky's Biosphere and Noosphere can best be exemplified by Frederic Edwin Church's rendering of the Cosmos of Alexander von Humboldt in his landscape masterpiece, *Heart of the Andes.*

Figure 1. Frederic Edwin Church, *Heart of the Andes.* (1859). This equatorial scenery expresses beautifully the artistic form of biquadratic integration and distribution of the abiotic, biotic, noetic, and the divine as Humboldt had designed it in his *Cosmos.*

1. HOW THE GREEKS ESTABLISHED BIQUADRATICS AS THE MEANS OF MEASURING CHANGE IN THE UNIVERSE AS A WHOLE.

In his dialogue *The Timaeus*, Plato polemicized against the flat universe of Aristotle and emphasized that if God had created a flat universe, He would have required having only one mean proportional. This is also the "mortifying abasement" that the British Empire would like Americans to live in; but we will soon change that humiliating sort of affair.

The discovery of solutions for doubling and quadrupling the volume area of the cube, as formulated originally by Hippocrates, then by Plato, and later constructed by Archytas with the intersection of a cone, a torus, and a cylinder, is the required measure for determining change in the universe as a whole; but this measure requires to *establish, in the solid domain of Sphaerics, two mean proportionals between two extremes that are in a ratio of two to one*. Since the universe is not flat, but voluminous, it required two mean proportionals to be harmonically ordered. The Greeks addressed this fundamental property of the universe in a very special way that has long been abandoned and forgotten in universal studies and is most important to revive today. Plato pleaded the case for this principle of conspiracy in the following manner:

« *It is not possible that two things alone should be conjoined without a third; for there must needs be some intermediary bond to connect the two. And the fairest of bonds is that which most perfectly unites into one both itself and the things which it binds together; and to affect this, in the fairest manner, is the natural property of proportion.*

« *For whenever the middle term of any three numbers, cubic or square, is such that as the first term is to it, so is it to the last term, - and again, conversely, as the last term is to the middle, so is the middle to the first, - then the middle term becomes in turn the first and the last, while the first and the last become middle terms, and the necessary consequence will be that all terms are interchangeable, and being interchangeable, they all form a unity.* » (*The Timaeus, 32a*.)

This universal characteristic of conspiratorial proportionality applies to surfaces in a manner that their areas are squared in the form of $\mathbf{a^2 : ab :: ab : b^2}$, or its inverse, $\mathbf{b^2 : ab :: ab : a^2}$**.** Thus, a first term is to a second term as the second term is to a third, and conversely, the last term is to a second term, as this second term is to a first. Then, a proportionate surface is established similarly, when a middle term becomes first, like in the case where $\mathbf{ab : a^2 :: b^2 : ab}$, or when the middle term becomes the last as in , $\mathbf{ab : b^2 :: a^2 : ab}$.

Now, in the case of a volume, a higher form of proportionality is required and two mean proportionals are necessary. As Plato put it: « *Now if the body of All had had to come into existence as a plane surface, having no depth, one middle term would have sufficed to bind together both itself and its fellow-terms; but now it is otherwise: for it behooved it to be solid of shape, and what brings solids into unison is never one middle term alone but always two.* » (*Timaeus*, 32b.) This is in direct reference to the Archytas construction for the doubling of the cube requiring a double mean proportion between a series of cubes that are all double of it's immediate predecessor, such that $\mathbf{a^3 : b^3 :: b^3 : c^3 :: c^3 : d^3}$**.**

Now, why does physical space-time require two mean proportionals and not three or four? What is the significance of having two? This is same the question that Pierre Fermat developed with respect light propagation within refraction; that is, "the reason for refraction in our common principle, which is that nature always acts along the shortest and easiest paths." (Fermat letter to de la Chambre, August, 1657.) Thus, finding the reason why nature always takes the most economical pathway is the same as finding why physical space-time requires two mean proportionals. It is also the same as finding that the Cosmos of Humboldt can only be represented in the Quito region of Ecuador in South America.

In first approximation, the geometrical explanation for this appears to come from the growth process of conical spiral action. This may not be obvious to many, but there is something about logarithmic spiral action that defines the growing process of the solid domain in such a manner that there cannot be any more or less than two mean proportionals per octave of a full rotational action. In other words, in order to physically succeed in determining a growing process under the condition of voluminous physical space-time, it is required that the two extremes be in a ratio of two to one. However, it seems that this ratio also requires that there be two mean proportionals in order for a doubling process to be reached. In other words, there appears to be a performative requirement that should be such that the means of achieving the required change is also the change of achieving the required means. The whole process seems to be hinging on this principle of reciprocity. Indeed, if the elements of the problem were to be based on a different ratio, say 1/3, for example, the mean proportionality would be so different that such a universe would be impossible to live in. This is an interesting problem that the Pythagoreans appear to have solved after their return to Greece and Italy from Egypt.

There are also other surprising aspects to be discovered in this Greek arsenal. For example, the Greeks did not measure extension so much as they measured change, and they used intervals of rotational conical spiral action as means of measuring proportional growth as the form of lawful change in the universe. See the case of building the Acropolis of Athens that I developed in *The Acropolis of Athens: The Classical Idea of Beauty*, New Federalist, June 24, 1988. As I demonstrated then, Greek physical geometry is based on a step-by-step method of constructive geometry that used rotational proportion to measure angular change in the universe as opposed to arbitrary linear extension to measure distances and sizes. This is the method that Hipparchus used to map more than a thousand stars onto a spherical model of the heavens. This is what Gauss had also rediscovered from the Greeks and had expressed in what became known as Gaussian integers.

Our emphasis on angular measurement, here, therefore, must be put on the idea of measuring *intervals of change between magnitudes*, as opposed to measuring quantities pertaining to those magnitudes, themselves. Such intervals may or may not be quantified. As Riemann demonstrated, the measurement of size implied “superposition of two magnitudes in which one is considered a measure for the other.” However, *think of proportionality as intervals of change whose measures are not to express units of metrical determination inside of comparable magnitudes, but rather to express an angular ratio of incommensurable change between those magnitudes*. This is the reason why geometers, in the tradition of Plato, made use of the sign of the colon (**:**) and double colon (**::**) to express the ratio of magnitudes within a relationship of change, rather than the sign of equality (=) within a relationship of quantifiable metric determination. Therefore, think of the colon “is to” and the double colon “as” in terms of intervals of action that measure and compare differences between domains, as opposed to the similarity of their contents. It is in that sense, only, that proportionality can emphasize the axiomatic interval of differences between incommensurable magnitudes, which, otherwise, may never be brought together for the purpose of comparison. Such is the purpose of the analogue as opposed to the digital. This is the reason why Cusa considered that man was to God as the polygon is to the circle.

2. ON THE DIFFERENT MEANINGS OF BIQUADRATICS

Biquadratic is the name usually given by algebraists to an equation of the fourth degree, that is, to an equation of the general form of $x^4 + Ax^3 + Bx^2 + Cx + D = 0$. Such equations are reducible to equations of third and second degrees and are generally useful for determining the intersection curve of two solids, such as two conics or a cone and a cylinder. In other words, a *biquadratic curve* is a curve of intersection of two surfaces of second degree. In the case of the Archytas construction for doubling the cube, for example, the curves of intersection between the cone and the cylinder, and between the cylinder and the torus, are biquadratic curves. Mathematicians have also generated different forms of conical or spherical biquadratic surfaces.

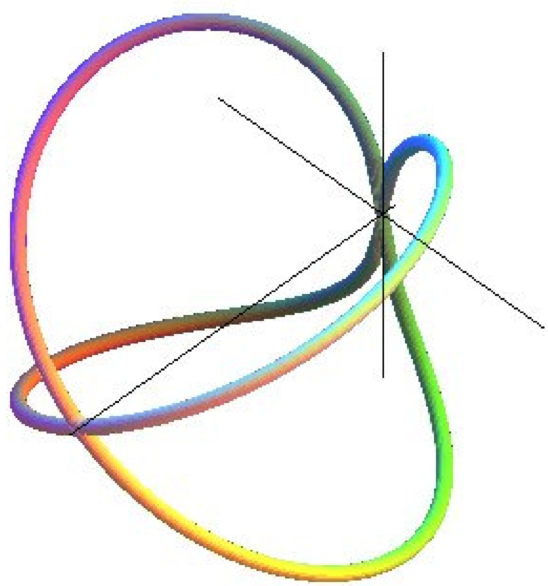

Figure 2. Archytas biquadratic curve of the torus and the cylinder.

From the vantage point of constructive geometry, such biquadratic curves raise interesting questions with respect to the problem of the Archytas doubling of the cube. For example, the three following questions are of the domain of this fourth power:

1- Why is it that, in the Archytas model for doubling the cube, the biquadratic curve intersecting the cylinder and the torus represent, in a continuous fashion, the semi-circumference of the cylinder and of the torus, and yet, its length is greater than the semi-circumference of each of them?

2- Why is the biquadratic curve of the cylinder and the cone of such a length that it corresponds to the edge of a cube that is 64 times the volume of the initial cube that was to be doubled, and yet, its length is greater than that edge? How can you prove it by logarithmic geometrical construction?

3- How can you prove, by logarithmic-geometrical construction alone, that the interval between the two mean proportionals of the Archytas problem corresponds to a Bel Canto interval between register shifts, that is to say, between the vocal registers of a man (bass) and of a woman (alto)?

Furthermore, whatever other usefulness this fourth power may have for mathematicians, it does not, in any way, form, or shape, reflect a so-called fourth-dimension of the universe. The existence of such a four-dimensional universe would depend only on what mathematicians smoke, and would therefore be reduced, at best, to a psychedelic visual imagery of flatland. So, in terms of the real world, there is no such a thing as a four-dimensional sphere or a four-dimensional cube, the likes of which D. Hilbert and S. Cohn-Vossen would have you believe exist. Though Hilbert' s book of *Geometry of the Imagination* may have some merit, his imaginary construct of four-dimensional polyhedra, pages 145-150 of his book, is a complete fallacy of composition based purely on the desire to make a forth degree power fit some sort of geometrical projective reality. This is like Alice in Wonderland; going out window-shopping for a mirror that would satisfy her newly discovered fantasy about a four-dimensional tea pot.

As for biquadratic residues, Gauss discusses them in his Chapter on Congruences of the Second Degree, Section IV, # 114-116 of *Disquisitiones Arithmeticae*. This also led him to discuss the implications for the *fundamental theorem* and the paradox of that theorem that he formulates at #136. I have never been able to prove the theorems of this section, but I can identify for the reader the relevant matter that is pertinent for our work.

Quadratic residues are residues of all prime numbers of the form of 4n+1 and biquadratic residues are residues of all prime numbers of the form 8n+1. For example 17 is a prime of the quadratic form 4n+1 (4x2+1 = 17) and of the biquadratic form 8n+1 (8x2+1 = 17) There are four biquadratic residues for prime number 17. They are 1 and 16, and 4 and 13. As you can see they come in pairs and each one of them has a reciprocal which is also a biquadratic residue. That is, I would imagine, where Gauss got the idea of *biquadratic reciprocity*.

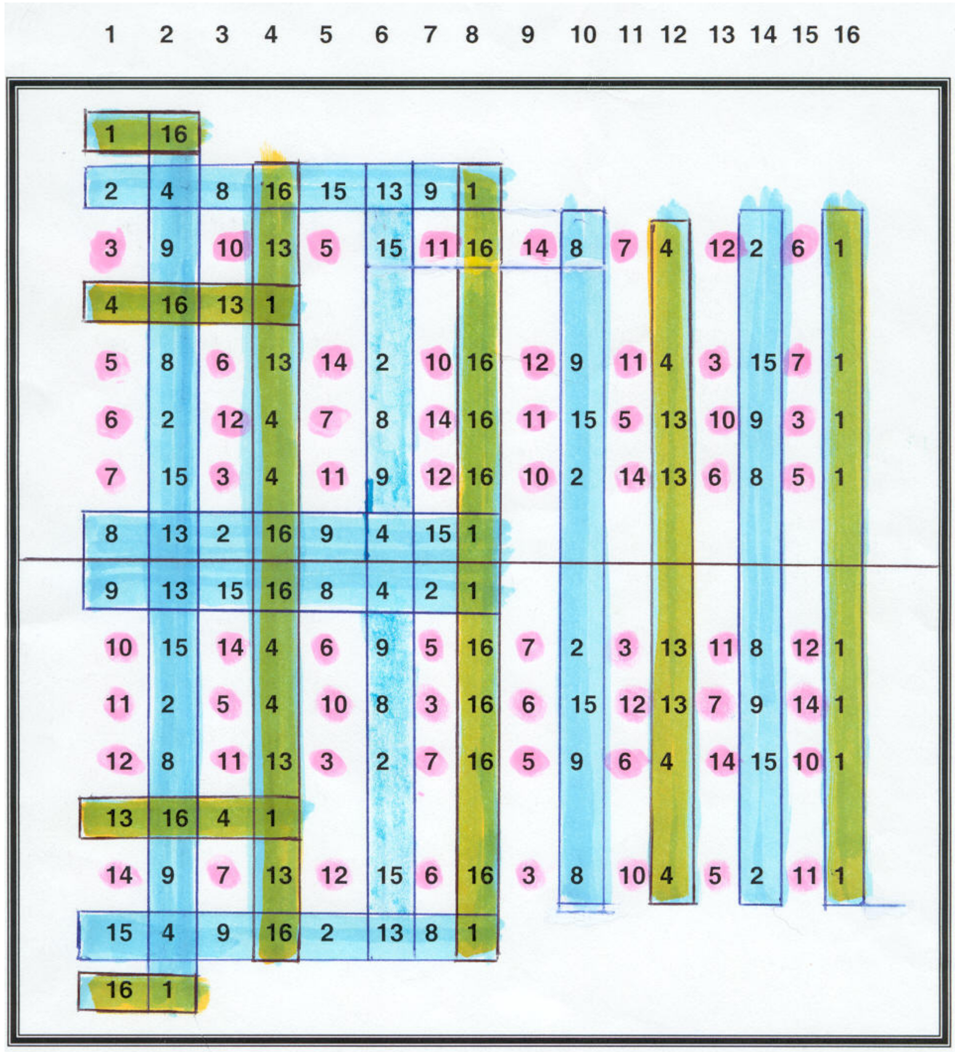

Figure 3. This is the grid of all of the residues and non-residues for prime 17. Note how everything is symmetrically well ordered as in a mirror image. The left column 1 to 16 represents all of the residues and non-residues of 17. The green numbers 1,4,13,16 are the four biquadratic residues, the blue numbers 2,8,9,15 are the four quadratic residues, and the pink numbers 3,5,6,7,10,11,12,14 are the eight non-residues or primitive roots of 17.

What these integers show you are shadows of a least action process that ended up distributing numbers in this mirror image where all of the reciprocals are symmetrical. This suggests that there exists an underlying harmonic process of least action that causes these integers to fall into place in this reciprocal manner and in no other way. I will show this process in another pedagogical and at a later time. It would be too much of a distraction to go through the full demonstration of this process, right now, and I would like to keep the focus riveted on fundamentals. Let's just say that a beautiful Poinsot type of Leibnizian *analysis situs* [See **Figure 4**.] can generate the process that is required, and that, as Fermat put it with respect to his least time principle, it is the power of the demonstration that carries the conviction, because least time action is the punch that forces the truth of reason instead of attempting to convince. As a result, if someone does not feel that force, he simply does not understand: then, it becomes useless to try and convince him.

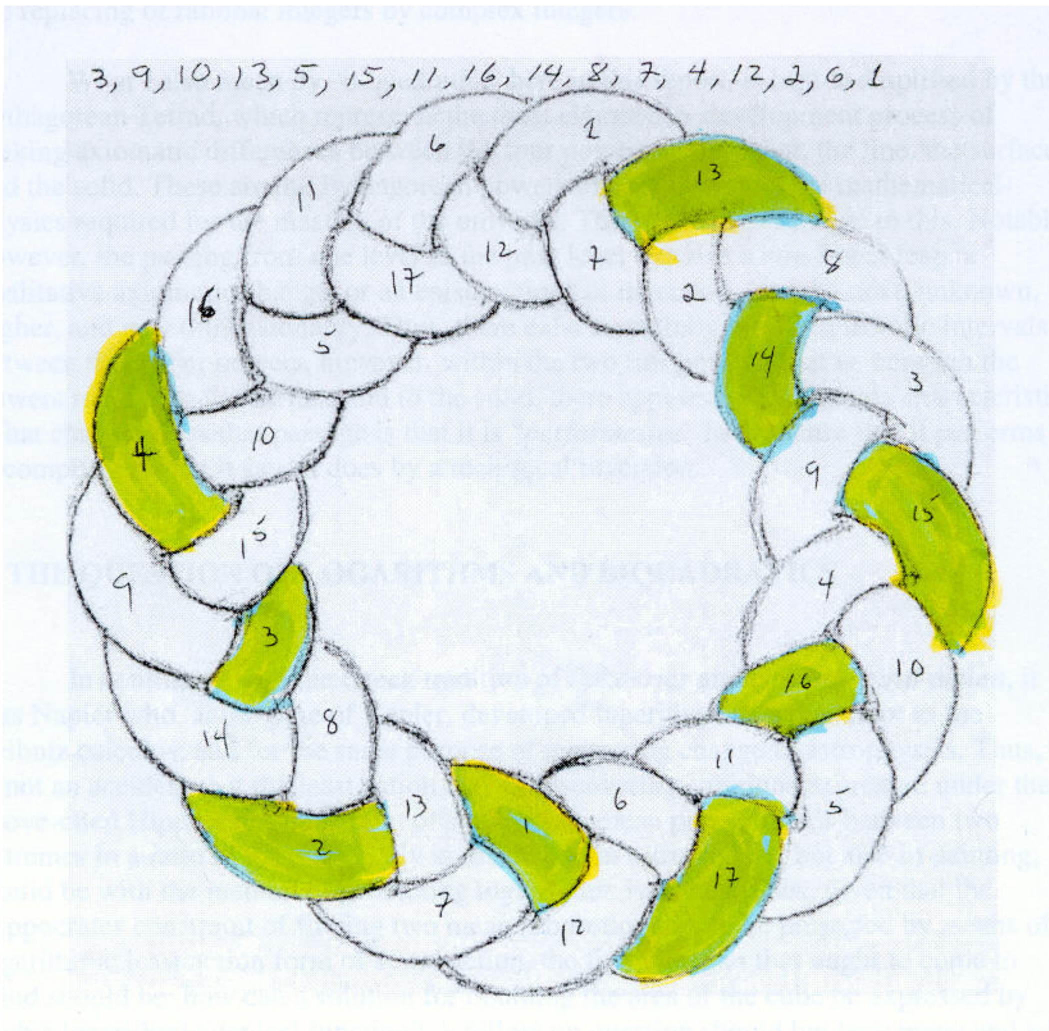

Figure 4. Torus representation of 3 as a primitive root of 17. Note that to find the four biquadratic residues 4,13, and 1,16 for modulus 17, you must generate the least action process corresponding to the number values as intervals of action. This means that going from 4 to 13 requires 4 counterclockwise waves of 3 intervals of action and 13 counter clockwise waves of 3 intervals of action to return to 4. The same process applies in the case of 16 and 1. In fact, all integers follow the same rule of *biquadratic reciprocity*.

Otherwise, what historian of mathematics, Eric Temple Bell, had termed the law *of biquadratic reciprocity* in his *Men of Mathematics*, was essentially the basis for Gauss's *Disquisitiones Arithmeticae*, as well as the basis for his devastating critique of

Euler, d'Alembert, and LaGrange in his 1799 *Theorem of Algebra.* This also leads to the core of Riemann's dissertation *On the Hypotheses which Lie at the Foundations of Geometry.* However, the irony of *biquadratic reciprocity* is that it establishes the fact that the conditions for passing to the next higher level do not exist in the previous level. So, as a result, you have to go against the pricks. Gauss made an explicit note of this in his DM, Section IV, #136. This may be the reason why his discovery required, as in a form of bootstrap of self-bounding expression, the replacing of rational integers by complex integers.

What I also mean by "biquadratic," here in this report, is best exemplified by the Pythagorean Tetrad, which represents the most elementary development process of making axiomatic differences between the four powers of the point, the line, the surface, and the solid. These are the Pythagorean powers of understanding the mathematical-physics required for the mastery of the universe. There is no other magic to this, really, however, it is notable that the passing from one level to the next level requires a non-linear leap, a qualitative axiomatic change, or an epistemological inversion, into the next, unknown, higher, and new dimensionality. Thus, there exists essentially three biquadratic leaping intervals between these four powers, and, within the two last powers, that is, between the powers relative to the surface and to the solid, there appears to be a unique characteristic. What characterizes that passage is that it is "*performative*" in the sense that it performs or accomplishes what it says it does by a reciprocal inversion. This is as far as I can push this question at this point.

3. THE QUESTION OF LOGARITHMS AND BIQUADRATICS

In continuity with the Greek tradition of *Sphaerics* and *conical spiral action*, it was Napier who, at the time of Kepler, developed logarithms as a precursor to the Leibniz calculus, and for the same purpose of measuring change in astrophysics. (See **Figure 5**.) Thus, it is not an accident that the least action way of discovering solutions expressed under the above-cited Hippocrates constraint of finding two mean proportionals between two extremes in a ratio of 2/1, especially in music and in astrophysics, but also in painting, would be with the method of generating logarithmic intervals. Thus, given that the Hippocrates constraint of finding two mean proportionals can be projected by means of a logarithmic least action form of spiral action, the first question that ought to come to mind should be: how can a solution for doubling the area of the cube be expressed by such a logarithmic conical function? A follow up question should be: how many and what form of intervals of logarithmic spiral action must necessarily be generated between those two mean proportionals and those two extremes, in order to satisfy the original condition of Hippocrates? Those are the questions that have led me to investigate the domain of biquadratics as a form that was best suited for defining the fundamental proportionality of anti-entropic change in the universe as a whole. This subject of *geometry of change* requires some explanation because it also involves the reciprocity of a *change of geometry*.

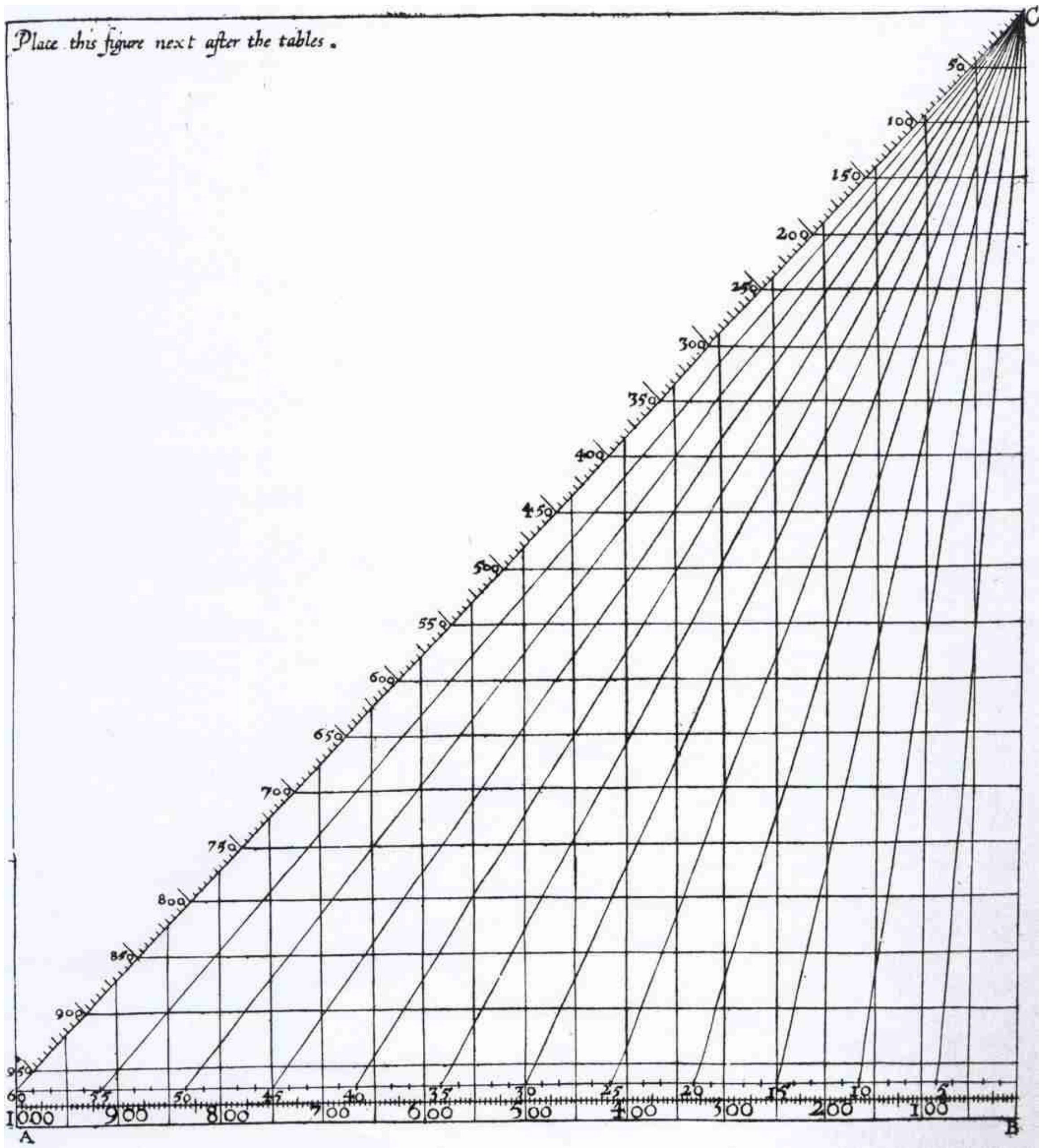

Figure 5. Napier's conical triangular logarithmic grid.

If measuring in the universe were no longer to be expressed by the linear measure of extension, but by measuring angular change, then such a measure would actually change the universe by its very introduction as a self-fulfilling form of physical action. It would be a self-bounding process such that the change of the means would also become the means of the change. However, such a condition always becomes intelligible after a willful jump into the future has been made, and, such a jump always anticipates and proceeds from the will to change the tragic state of misery of mankind. Now, based on this self-generating principle, let's have a look at the traditional Greek form of selfsimilar conical spiral action in **Figure 6,** and see how it applies to both music and astrophysics.

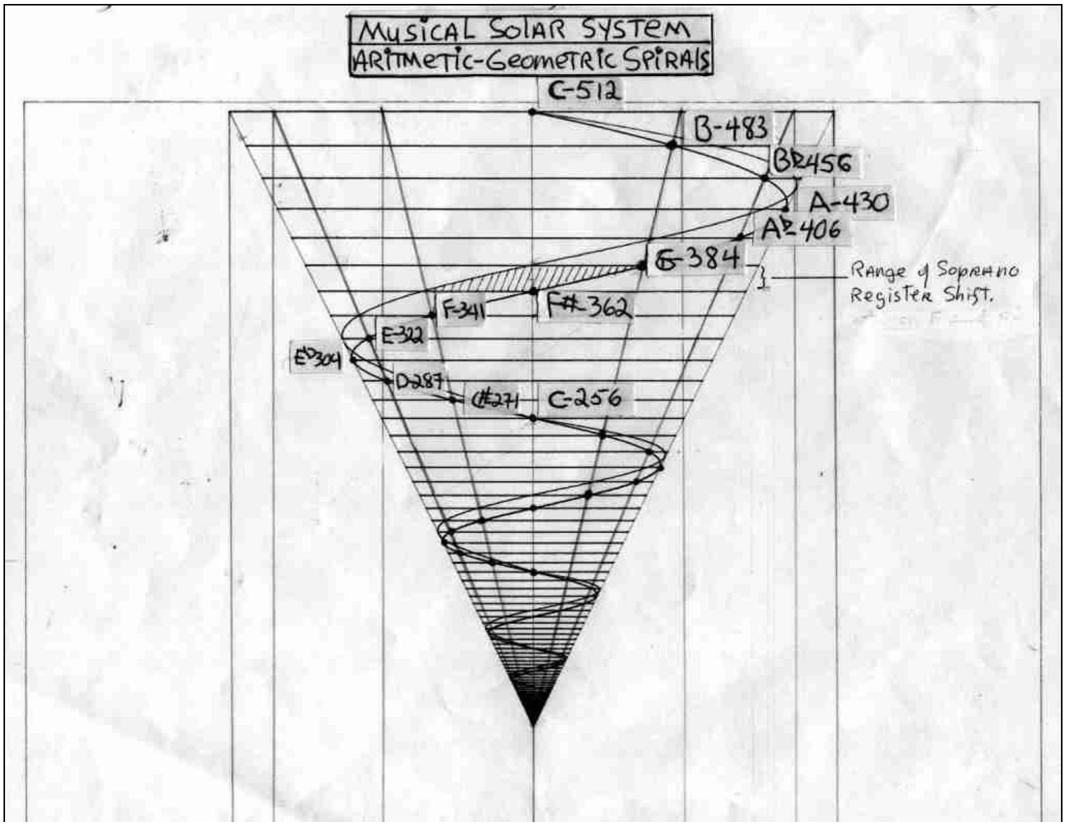

Figure 6. A conical logarithmic spiral, coupled with an arithmetic spiral, projecting musical intervals and dividing the cone into a series of equal-tempered octaves with their twelve logarithmic intervals. The octave that is identified with cycle numbers represents the range of the musical octave between C-256 and C-512. The accompanying arithmetic spiral is aimed at showing, heuristically, the difference between the arithmetic mean and the geometric mean in a spiral form, in order to locate the voice register shift of the soprano voice as passing through the geometric mean F# and the arithmetic mean G. The intersection points between the twelve apex projective rays and the logarithmic spiral locate all of the logarithmic circular conical divisions of the cone. Any set of four intervals of any octaves, anywhere inside of the logarithmic spiral, will establish the Hippocrates requirement for doubling the cube. Here, we have chosen C-256, E-322.54, Ab-406.37, and C"512.

First of all, if you develop a logarithmic spiral by way of a purely synthetic and constructive geometric method, which I have elaborated elsewhere, and relate it to the equal-tempered musical system corresponding to a properly tuned piano, you can see and hear that the projections of all of the intervals of the spiral actions are based on the principle of twelve logarithmic divisions, no more no less, as reflected by the construction of the dodecahedron. Such harmonic divisions are based on biquadratic forms of triply extended power intervals dividing the logarithmic spiral action into four different dimensionalities, as expressed originally by Pythagoras in his Tetradic or biquadratic function of the point, the line, the surface, and the volume.

Such a harmonically ordered Pythagorean Tetrad represents the earliest form of the four dimensionalities developed by V. I. Vernadsky and Lyndon LaRouche, notably, the four phase-spaces of our universe as expressed in the Divine Proportion whereby the Abiotic is to the Biotic as the Biotic is to the Noetic in the same proportion that the Noetic is to the Divine. It is, therefore, the changing nature of these universal biquadratic phase-spaces of our growing universe that is measured by double mean proportionality. This is also what Thales of Miletus had identified in first approximation as the *dynamis* of the ancient Greek principle of Hylozoic Monism.

As Rabelais demonstrated explicitly in Chapter 36 of *The Fifth Book*, conical spiral action represents the proper *Sphaerics* function of the Pythagorean Tetrad. The crisis point of axiomatic change, or the Riemannean jump, was quite extraordinarily expressed by Panurge and was located by Rabelais at the appropriate register shift location of the arithmetic-geometric mean of the spiraling Tetradic staircase. This geometrical-mathematical construction by Rabelais is quite amazing, since it appears that the required sundry calculation did not exist before Gauss discovered it. The construction for such an amazing development can be found in my class to the Montreal LYM of November 29, 2007, on *How Panurge dealt with his axiomatic change,* and part of which is added in section 5 at the end of the present report. Just to summarize the three essential steps of the Tetradic Powers, consider the following:

The first power is the shadow power of changing from the domain of things to the domain of shadows. It is ***the point shadow of the power of two,*** which has no existence, in and of itself, and which reflects ***the end of an interval of action,*** that is, the shadow of rotational angular action intersecting two lines of the second power. This shadow power is also the power of sophistry and deception, like the power of money in our foolish societies. It is the use of this power that freemasons have been abusing the world for so may centuries.

The second power represents the ***linear power of two***, which results from the division by the spiral into ***arithmetic intervals of action*** between points reflected along the axis of the cone as a whole, thus, marking the limits of a series of 10 equal-tempered octaves, numbered 1, 2, 4, 8, 16, 32, 64, 128, 256, 512, etc., on the vertical axis number line and whose sum total of n + 1, at any point, always follows the predecession principle that establishes the value of the next higher octave.

The third power represents the ***lydian surface power of the three previous domains,*** that is, of the point, the line, and the surface combined, which divides the appropriate octave along the line of the axis as well as around the surface of the cone into a complex arithmetic and geometric division by half. The double arithmetic-geometric spiral reflects such a power. This is the ***lydian interval of action*** reflecting the dissonant discontinuities of the voice register shifts pertaining to the six human Bel Canto singing voices. This arithmetic-geometric discontinuity is generated by a dual geometric-arithmetic spiral action representing the anti-entropic unstable vibrato of a passing tone of the human voices at C for Tenor, D for Contralto, E for Alto, F# for Soprano, Aflat for Bass, and Bflat for Baritone. This is the first level from which the anti-entropic nature of the biquadratic universe can be discovered.

The fourth power represents the ***biquadratic power of the solid domain of physical space-time*** subsuming the three previous powers within itself and, thus, determining the density of register shift discontinuities in the musical domain. This is the LaRouche-Riemannean domain as such. Within the singular octave range from C-256 to C-512, there are, naturally, two biquadratic double mean proportionals composed of three register shift discontinuities each: one includes C, E, Aflat, and the other includes F#, Bflat, and D. To this power pertains the ***biquadratic interval of action*** underlying the logarithmic spiral and dividing the complex spiraling octave into three intervals of two double mean proportional sets pertaining to non-entropic cubing in solid physical space-time, and especially the method of doubling of the cube by Archytas and the Gaussian method of solving the Fundamental Theorem of Algebra. Each ***biquadratic interval of action*** represents a spiral extension made up of a set of four logarithmic intervals. Such intervals are also directly reflected in the isoperimetric circle of Cusa's isoperimetric theorem and are also an active force within our solar system as a whole.

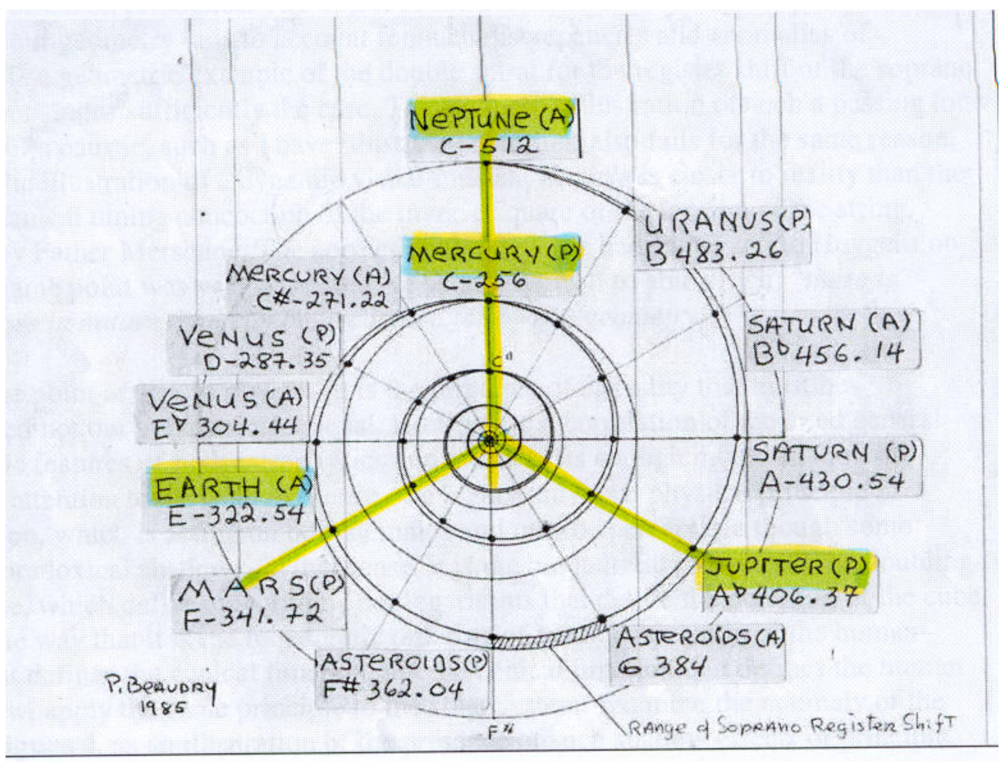

Figure 7. Projection of the musical intervals onto the approximate positions of the planets of our solar system. The true aphelion and perihelion distances are obviously not respected, but their proportionality is, provided that one locates the position of the sun, as

Kepler did, at the apex of the cone. Note the biquadratic relationship between **Mercury (P), Earth (A), Jupiter (P), and Neptune (A).**

Secondly, if you project this first logarithmic musical spiral action onto the corresponding astrophysical logarithmic domain of our solar system, the projection from a continuous manifold onto a discrete manifold of such a series of musical/planetary interval correspondence, reflecting both the least action principle of voice registering of Bel Canto singing and the well-tempered gravitational tuning of the planets of our solar system, points to the existence of a lawful universal ordering principle which informs both domains of musical artistic composition and of astrophysical scientific composition. Kepler proved the existence of this relationship, in a most beautiful way, with his planet shattering, *Mysterium Cosmographicum*. This may be further confirmed by studying the following table of correspondence, which has been provided to me a number of years ago by an old friend of the LaRouche movement, William Bohdan, from Calgary, Canada.

A word of caution, however, is necessary at this point. This projection is not intended to be a one on one curve fitting mapping of the musical system onto the solar system. That would be silly and meaningless. Each domain is a dynamical living system that must account for internal changes that allow for significant discrepancies and anomalies based on principles. The object of the geometrical projection is precisely to show how our geometry fails to account for such discrepancies and anomalies of principle. The geometric example of the double spiral for the register shift of the soprano voice demonstrates sufficiently the case. The geometric illustration of such a passing tone by means of a caustic, such as I have illustrated it earlier, also fails for the same reason, although the illustration of a dynamic visual-musical *wavicle* is closer to reality than the silly mechanical tuning concoction of the inverse square of the tension of the string, invented by Father Mersenne. The correction that Leibniz had suggested to Huygens on that very same point was very wise, and we should do well to abide by it: "*there is always more in nature than can be accounted for by our geometry*."

The point is that, in physics, it is the principle of causality that must be investigated not our geometrical arsenal. However, the correlation of the cited general logarithmic features of both astrophysics and music casts enough mixed shadows to direct our attention toward the existence of a higher universal physical principle of composition, which is acting on both domains and may be perceptible though the fine angle of some definite paradoxical shadows. In that sense, it is the biquadratic ordering of the doubling of the cube, which defines logarithms not logarithms that define the doubling of the cube, in the same way that it is the biquadratic ordering of the register shifts of the human voices that defines the conical function, not the conical function that defines the human voice. Now, apply the same principle to the solar system. Examine the anomaly of the table in **Figure 8**, as an illustration of the presence of such shadow effects of principle.

Planetary Orbits and the Equal Tempered system — Rene 11-28-90

Planets	Astronomical Units	Log 10^x	Added Constant	Multiple Constant	Cycle Equivalent	Musical Cycles	Planets
Mercury	(P) 0.310	−0.5086	+2.496	x 128.8	255.97	C = 256	Mercury
	(A) 0.470	−0.3279	" "	" "	279.25	C# = 271.22	Mercury
Venus	(P) 0.715	−0.1457	" "	" "	302.72	D = 287.35	Venus
	(A) 0.725	−0.1397	" "	" "	303.49	E^b = 304.44	Venus
Earth	(P) 0.983	0.0074	" "	" "	320.52		
	(A) 1.017	0.0073	" "	" "	322.42	E = 322.54	Earth
Mars	(P) 1.379	0.1396	" "	" "	339.46	F = 341.72	Mars
	(A) 1.661	0.2204	" "	" "	349.86		
Asteroids	(P) 2.2	0.3424	" "	" "	363.32	F# = 362.04	Asteroids
	(A) 3.6	0.5563	" "	" "	393.13	G = 383.57	Asteroids
Jupiter	(P) 4.95	0.6946	" "	" "	410.95	A^b = 406.37	Jupiter
	(A) 5.45	0.7364	" "	" "	416.33		
Saturn	(P) 9.006	0.9545	" "	" "	444.43	A = 430.54	Saturn
	(A) 10.074	1.0032	" "	" "	450.69	B^b = 456.14	Saturn
Uranus	(P) 18.288	1.2622	" "	" "	484.05	B = 483.26	Uranus
	(A) 20.092	1.3030	" "	" "	489.31		
Neptune	(P) 29.799	1.4742	" "	" "	511.36		
	(A) 30.341	1.4820	" "	" "	512.37	C = 512	Neptune

Figure 8. The table illustrates the correspondence between the octave span of Mercury and Neptune and the equal-tempered octave of our musical scale. Compare the astronomical unit values of the planets and the equal tempered musical cycles of intervals between the octaves of C-256 and C-512. The anomaly of that correspondence is based on transforming the astronomical unit values for each planet's aphelion and perihelion into cycles per second equivalents by means of arithmetic and geometric adjusted logarithms in congruence with my conical projection.

Third and lastly, the shadows of the anomaly shown in this table confirm the fact that a higher universal physical principle is actually acting on both the musical and the astrophysical domains; and that this higher principle is reflected in the biquadratic power of the mixed shadows that are being cast throughout the table. Therefore, for the purpose of our demonstration, let this projective anomaly show how two musical mean proportionals between two extremes, in a ratio of two to one, solves the problem of doubling the volume area of a given cube. ***Between those two extremes of Mercury projected at perihelion C-256 and Neptune at aphelion projected at C-512*, *there are two mean proportionals which are respectively Earth at aphelion, E-322.54, and Jupiter at perihelion, A-flat-406.37. Those two latter musical cycle values represent the sides of two cubes whose volume areas would be respectively double and quadruple the volume area of a cube whose side corresponds to the cycle value of Mercury at C-256.*** That is to say, Mercury is to Earth as Earth is to Jupiter in the same proportion that Jupiter is to Neptune. Thus, we have the following biquadratic double mean proportionality demonstrating the congruence between the solar system, the musical system, the doubling of the cube, and the new proportional biquadratics.

Mercury : Earth :: Earth : Jupiter :: Jupiter : Neptune
C-256 : E-322.54 :: E-322.54 : Ab- 406.37 :: Ab-406.37 : C-512

Those two mean proportionals, Earth and Jupiter and their extremes Mercury and Neptune correspond to the three register shifts of the Tenor voice at C, the alto voice at E, and the bass voice at A-flat. The two mean proportionals and their two extremes are, thus, separated by three sets of *biquadratic intervals* which, when connected together by their four conical spiral points, form a *biquadratic conical surface of negative curvature*. That biquadratic conical surface is constructible with wire and may be made visible in a soap solution; but there is reason to believe that you might not be able to *see* this properly as a *wavicle* biquadratic surface unless you go to Lanternland and *hear* the secret caustic word of the Oracle of the Holy Bottle that Panurge discovered, there, after breaking through his Pythagorean Tetradic ordeal!

4. ANSWERING A QUESTION ON THE PROPORTIONALITY OF THE ARITHMETIC-GEOMETRIC MEAN ITERATION WITH RESPECT TO THE BIQUADRATIC LIMIT OF AN ELLIPTIC FUNCTION.

In answer to a question from Oscar Cardenas and Pedro Rubio of the Bogotá LYM on the physical geometric proportionality between the Rabelais arithmetic-geometric mean and the human power proportionality that Panurge discovered, in Chapter 36 of *Book Five*, I replied that you can construct the equivalent of a proportional *spiral action range* by using a *biquadratic elliptical range* and look at the harmonic relationships between the semi-minor and the semi-major axis of each of the four ellipses for determining your changing measurements.

It is the iteration of those pairs of semi-minor and semi-major axis, taken two by two into a biquadratic form, which shows the double proportionality of the iteration between the different ellipses and the two different means. Take the following example of a minimum-maximum elliptical range and follow how rapidly the rate of change occurs from a quasi-straight line to a quasi-circle. The values for A , A', A", A'" correspond to the arithmetic means and the values for B, B', B", B'" correspond to the geometric means.

The harmonic proportional relationship of the ellipses in the series is such that the major axis minus the minor axis of one ellipse is equal to the distance between the two foci of the next ellipse in the series. This is how those ellipses are harmonically self-bounded together. This reflects a proportional rate of change between them, a harmony of harmony within the elliptical function.

This is as far as I have gone in constructing these elliptical functions from proverbial scratch with Mark Fairchild, a number of years ago. Anyone who wishes to bring an improvement to this construction is quite welcome. Here is a challenge for you: How do you construct, on your computer, an audio-visual *wavicle* animation where the elliptical function, as shown below, would express the increasing rate of change in relationship with the appropriate musical intervals? What you would hear in this experiment would, of course, be different from hearing just the Lydian interval of C-F#. Think that you are hearing and seeing the density of dissonances that actually produces the complex wolf-sounds of the arithmetic-geometric mean. This must, therefore, sound something eerie, like Panurge identified with Cerberus, which, if I recall correctly, had three heads. Thus, you would require three intervals making up three wolf-sounds!!! Something like C-G, C-F#, and G-F# in progressive succession, and then the three of them culminating into a planet shattering sound.

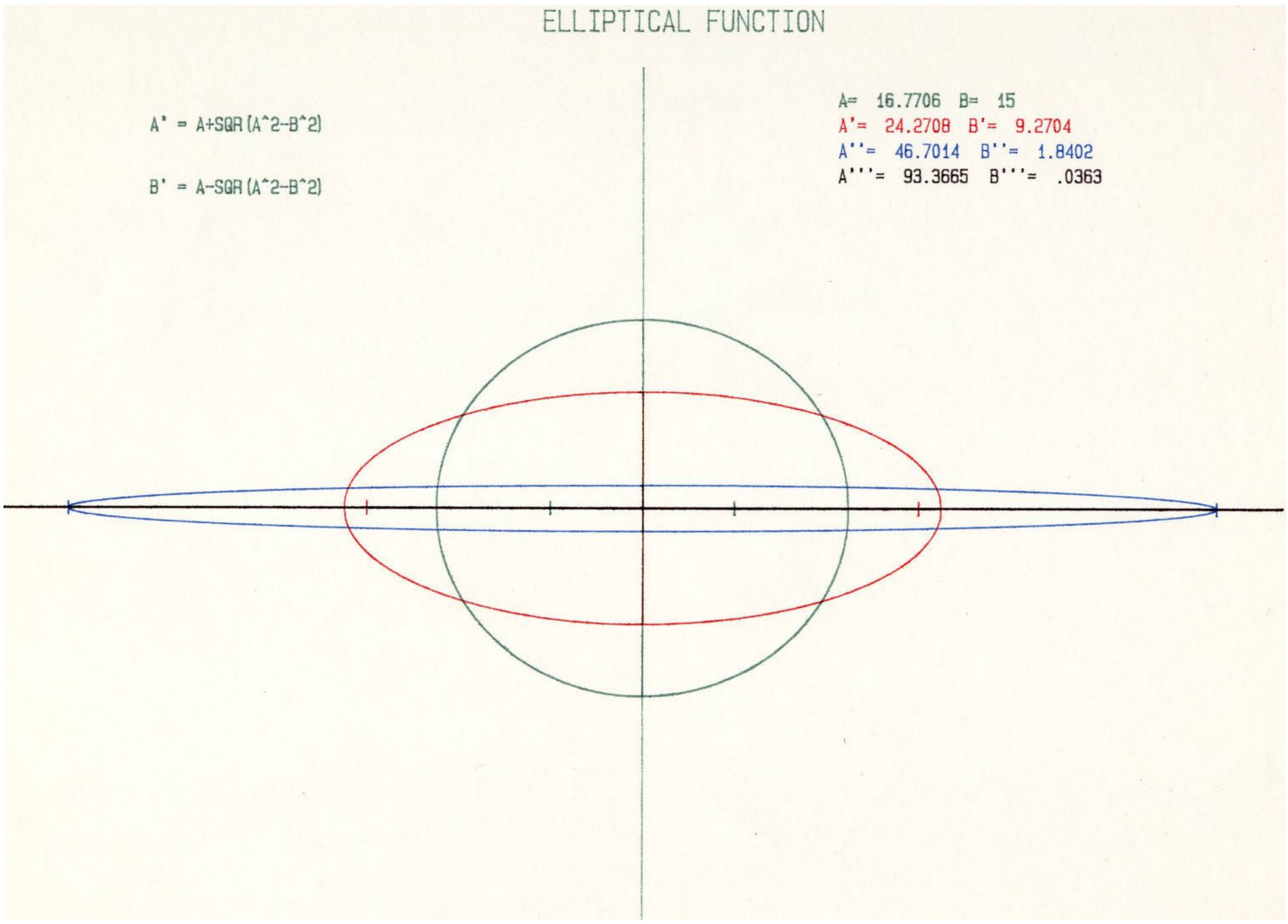

Figure 9. Elliptical Function.

5- RABELAIS AND THE SINGULARITY OF THE PYTHAGOREAN TETRAD.

Just to establish for you the context of this story, let me start with the forecast that Rabelais made to all visitors of Lanternland. He used the statement of the Greek Stoic philosopher, Cleanthe, which was translated into Latin by Seneca, and which said: {***Ducunt volentem fata, nolentem trahunt.***}: ***"Destiny leads the willing, but drags the unwilling."*** (***Les destinées meuvent celui qui consent, tirent celui qui refuse.***) This statement is found in Book Five, the last book that Rabelais wrote about the last adventures of Pantagruel, Panurge, and Friar John who had traveled to the Island of Lanternland, on their last expedition. When they arrived at their destination, the visitors were greeted by Midnight-Oilers "Lanternois" who immediately started having philosophical discussions with them, especially on the subject of final causality, whereby "*all things move to their ends*." The habit of Midnight-Oilers is to stay up all night and feast on ideas generated exclusively from their lanterns which are modern forms of *Pythagorean Sphaerics*.

I refer you, most emphatically, to Chapters 32 to 48 of that last book, because, of all of Rabelais' writings, it is in that last section of the *Fifth Book* that the axiom-busting method of Rabelais is best mastered and displayed. But, for our purpose, here, I will only use ***Chapter 36: Our Descent of the Tetradic Steps; and Panurge's fright,*** which you should all have electronic copies of. (Page 685 of the 1955 Penguin Books edition.) Study this well. It is a pure delight of *Pythagorean Sphaerics*. Here, Rabelais brings the reader to making a fundamental discovery of universal physical principle by using what Leibniz later called his *principle of continuity.* Think that this discovery is of such significance and importance that it may one day save your life. It may also put your life in danger. This is what it did to Panurge.

In *Chapter 36*, Rabelais restored playfully to civilization the Pythagorean Tetradic principle of growth by means of an experiment with a conical spiral action. He wrote the story with such a tongue in cheek measure and such gusto that the reader cannot help but be provoked to investigate the seriousness and truthfulness of his mathematics. That's why I e-mailed you this little puzzle in advance. However, this is a joke that is not a joke. It is a funny story which has a deadly serious twist to it. So, as LaRouche once put it: "*Believe nothing that for which you cannot give yourself a constructive proof.*" Go work it out for yourself, and see what the Rabelaisian arithmetic adds up to. Count the number of steps and draw your own conclusion. You don't need a mathematical degree to do it. Here is the relevant part of Panurge's crucial experiment.

Book Five, Chapter 36: *Our Descent of the Tetradic Steps; and Panurge's fright.*

"Then we descended an underground marble staircase, and came to a landing. Turning to the left, we went down two other flights, and came to a similar landing. Then there were three more to the right, ending in a similar landing, and four to the left again.

‘Is it here? Asked Panurge at this point.
‘How many flights have you counted?” asked our splendid Lantern.
‘One, two, three, and four’ answered Pantagruel.
‘How many is that?’ she asked.
‘Ten’ answered Pantagruel. [1+2+3+4=10]
‘Multiply this result by the same Pythagoreal tetrad,’ said she.
‘That’s ten, twenty, thirty, forty,’ answered Pantagruel.
‘How many does that all make?’ she asked.
‘A hundred, answered Pantagruel.

‘Add the first cube,’ she said, ‘which is eight. At the end of that foreordained number of steps [108] ***we shall find the Temple door. And note most carefully that this is the true psychogony of Plato, which was so highly praised by the Academicians, but so little understood. The half of it is made up of unity, of the first two plane numbers, two squares, and two cubes.’*** [1+2+3+4+9+8+27 = 54 x 2 =108]

In descending these numbered stairs, underground we had good service from, firstly, our legs, for without them we could only have rolled down like barrels into a cellar; secondly, our illustrious Lantern, for we saw no other light as we descended, any more than we should have done in St. Patrick’s hole in Ireland, or in the cavern of Trophonius in Boëtie. When we had gone down seventy-eight stairs [78]***, Panurge cried out to our most luminous Lantern:***

‘Most wonderous lady, I beg of you with a contrite heart, let us turn back. For by God’s truth, I am dying from sheer fright. I agree never to marry. You have taken great pains and trouble for me, and God will reward you for it in his great rewarding-place. I shan’t be ungrateful either, when I get out of this Troglodyte’s cave. Let’s turn back, if you please. I’m very much afraid that this is Taenarus, which is the way down to hell. I think I can hear Cerberus barking. Listen, that’s he, or I have a signing in my ear. I’ve no liking for him at all, for there’s no toothache so bad as when a dog has got you by the leg. And if this is only Trophonius cave, the ghosts and goblins will eat us alive, as they once devoured one of Demetrius’s bodyguards, for lack of scraps. Are you there Friar John? I beg of you, old paunch; keep close to me, I’m dying of fear.’
”(François Rabelais, {*Gargantua and Pantagruel*}, Translated by J. M. Cohen, Penguin Books, London, 1955, p. 685-87.)

Figure 10. Panurge passing over the register shift of the Pythagorean Tetrad by P. Beaudry, *Lanternland* report, 2001.

Now, after having gone through this astonishing psycho-epistemological drama, investigate the processes of the three numbers that Rabelais generated. What are they? The explicit process is the arithmetical generation of the Tetrad. The implicit process is the underlying arithmetic-geometric singularity function of Gauss. Project the shadows of these numbers on the wall of Plato's cave. The three numbers are 108, 78, and 54. Is that what you have calculated? How did Rabelais arrive at these three numbers? And, what is their significance? Have a look at the Gauss arithmetic-geometric mean function and see how it works. What is the significance of those three numbers with respect to the Gauss A-G mean? How do they relate to what Panurge has gone through? What is the significance of the geometric relationship to the psycho-epistemological behavior of Panurge? This experiment is very similar to the one that Benjamin Banneker made when he related his mathematical puzzle of proportionality to the issue of slavery with the master of Monticello. You can find this earlier pedagogical of mine on the LYM website.

If you take the total number of steps in the spiral Tetradic staircase, the conical function as a whole has 108 steps forming a musical octave starting from step 54. Then, there is the complex halfway rotating step between them. It is an arithmetic and geometric step 78, which represents the singularity of a threat that Panurge perceived as deadly when he was about to put his foot on it. What is the threat? What does it have to do with number 78? Is this merely an imagined fear or is it a real fear of death? Note also the caustic inversion of the dog bite and the toothache on his leg!

This is the tragedy of not being able to go beyond an apparent axiomatic limitation of character, such as the flaw of Hamlet, or as the flaws of the current members of the U.S. Congress, or the flaws of the general population. This is the excruciating moment of a high density of singularities that a political leader experiences at a crucial historical moment of decision. This is the Lydian moment of Gethsemane as expressed by Brahms's *Four Serious Songs*! This is also, quite literally, what the arithmetic-geometric mean function represents at the complex halfway mark of a spiral progression of an octave. It is represented as a conical function in the {***So You Wish to Learn all About Economics)*** book of LaRouche, the arithmetic-geometric mean function of the whole spiral action progression, p. 51. If you do the calculation yourself, you will find the A-G mean of that octave as being more precisely, 78.666! Rabelais did not include the 666 parts for reasons that should be obvious. The foolish freemasons are still trying to figure out where the satanic number 666 comes from. Do the following construction yourself and you will see:

Find the A-G mean = 78 of the octave that Rabelais gave us, that is, between 54 and 108. How do you do that?

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1) First, take the arithmetic mean of those two values, which is:

$$\frac{54 + 108}{2} = 81$$

. Then take the geometric mean of the same two values, which is the

square root of 54 x 108 = 76.3675...

---

2) Second, take the arithmetic mean of the last two values, which is:

81 + 76.3675
-------------------- = 78.6837... Then take the geometric mean, which is the square root of
2

81 x 76.3675 = 78.6496...

---

3) Third and lastly, take the arithmetic mean again of the last two results: which are:

78.6837 + 78. 6496
-------------------------- = 78. 666... Then take the geometric mean, which is the square
2

root of 78.6837 x 78.6496 = 78.666... the A-G mean of that octave. Simple isn't it?

Thus, you have arrived at an apparent limit of 78.666... after three iterations, which generate the delta volume of the Leibniz calculus, the singularity of the quantum of action of the A-G mean, which had been associated with the fearful *devil's interval* during the Renaissance. This infinitesimal interval was used to scare the hell out of people during the Renaissance and made them politically impotent for fear of being burnt as a witch at the stake for telling the truth. Now, what is interesting, here with Rabelais, is that he used this as the creative singularity of an axiomatic change. He used the crisis as an opportunity. The interval describes and explains how a creative moment is always fearful, because, at the point where one has to make a decisive step that changes one's entire life, the subject becomes totally perplexed, freaks out, and wants to run back to a *comfort zone*, for fear that one would not be able to break through to the meet the next higher degree of responsibility that history put on his shoulders. This crucial experiment, therefore, has universal implications and carries with it a heavy load of consequences.

For example. This crucial experiment is one of the best pedagogical representations for the creative process itself, but it also locates the astrophysical significance of Kepler's discovery of an exploded planet in the register shift region of the solar system, between Mars and Jupiter. So, you see, this is a nice little problem that Rabelais posed as an **axiom buster** to the reader, about 300 years before the young 20-year-old Gauss developed the same mathematical-physics approach, and made one of the most astonishing discoveries of modern science. This is the highroad that Gauss took to discover the asteroid Ceres, with only a couple of observations. So, anyone investigating the scientific methodology of Gauss's discoveries cannot avoid taking into account the fact that Gauss, in every one of his discoveries, also consciously conducted this crucial Rabelaisian experiment during his entire adult life. This is what Rabelais refers to as the "*little understood psychogony of Plato*," the nature of the infinitesimal development of the human mind, yet incommensurable, gap between two different manifolds. This is, also what Riemann had identified as the nature of the "*causal connection of phenomena*" within the domain of the incommensurably small. It should not surprise anyone, therefore, to discover that this is also the hook where the Riemannian-LaRouche economic method hangs its hat.

Now, if you have done some serious Bel Canto voice exercises, you will recognize what I am talking about, because you will have gone through such an experiment and you will have constructed, for yourself, this joyful creative experiment of the passing tone register shift by discovering how to place your own voice in changing from the chest register to the head register. Similarly, the same axiomatic change is generated after you willfully organize politically, in the field, as a world historical figure. This crucial Rabelaisian experiment, therefore, will be at the center of every organizing day of every political leader shaping the future of human history for the thousands of centuries past, present, and still to come. It is the experiment that decides whether you are *leading willfully* or you are being *dragged unwillingly* behind the *Manifest Destiny* of western civilization. Politically speaking, that's where the monkey sleeps! Any questions?

That is the secret of Panurge passing the test of his moral commitment to changing history. It is based on the will to change; that is, the will to risk discovering, after the fact, what *Manifest Destiny* had called on him for, and why *all things move to their ends*. This is the test for the entire population of the United States, today, in response to the LaRouche challenge. This is the test for you in Canada who are willing to tell the parliamentarians in Ottawa what needs to be done. In a few weeks from now, all of those Canadian politicians will be terrified like Panurge. They don't realize it yet, but they will need to know that we have now entered into *crisis politics*, and that LaRouche organizers are the only ones who know how to turn that crisis into an opportunity. So, they will need to know that even though Panurge told the Lantern lady that he was no longer willing to get married and wanted to go back to his *comfort zone,* he was merely reacting like a Baby Boomer. That was a cover up for the historical crisis that needed to be faced and solved with steel nerves in the Renaissance period. This is what Panurge did when he jumped over his fears of Cerberus.

But, you see, even today you don't need to be terrified by Cerberus because they just went bankrupt in the current ongoing financial collapse. You should now, and you will see this in the briefing of tomorrow morning, that the private-equity firm, Cerberus, is howling because it is going out of business. The mad dog firm just pulled out of a $4 billion deal to buy up a power-tool rental company called United Rentals. They claim to have $10 billion of available liquidity, but that's a lie. GMAC, in which Cerberus has a 51% share just reported a loss of $1.6 billion and they cannot sell their $4 billion Chrysler debt, despite the 3% discount from the 11% they borrowed at. They are finished. That's why they are howling. The satanists are crawling back into their holes.

The proof is in the pudding. Listen to Panurge claiming his sublime victory: and stating that he is *willing* to go on to the next battle: "***Let's go on, then,"*** said Panurge, ***"and charge ahead foremost through all the devils. We can but perish, and that is soon***

done. I have always been preserving my life for some battle. Let's move, let's get moving, and let's press onward. I have enough courage and more. It's true that my heart is pounding. But that is from the chill and staleness of this cave. It's not fear, oh no, it's fever. Let's move on, let's pass on, push on, and piss on. My name is William the Fearless." (Francois Rabelais, *Gargantua and Pantagruel*, Penguin Books, 1955, p. 686.) This reminds us of the famous speech made by Roosevelt during the last depression, and in which he said: "*all you have to fear is fear itself*." But, Panurge made the wise decision to act like Lyn said: "with cold blooded enthusiasm."

FIN