# PART II:

## THE GEOMETRIC DETERMINATION OF PRIME NUMBERS BY MEANS OF THEIR INTERVALS.

[First drafted on Wednesday, May 12, 2004]

The question of discovering the underlying geometry of prime numbers is as old as number theory itself, and one of the difficulties in discovering their generative principle is found in the epistemological flaw of treating numbers as things in and of themselves. As LaRouche often emphasized, music is between the notes. So, if we change the rules of the game and stop considering numbers pragmatically as if they represented loose change in our pockets, and begin to look at them as shadows of intervals of some form of productive physical action in the universe, we might be able to discover something fundamental about them.

Since there exists no possible geometry for things taken in themselves, as self-evident entities, the only way to solve that shortcoming is to eradicate the fallacy of considering numbers as things and start considering them as representing something else than what we have been brainwashed into thinking they are. A number is like money, it has no intrinsic value, and it is as stupid as money, in and of itself. Similarly, human beings are not a collection of stupid things that keep bumping into each other in the night or indulge in stupid competition with each other. Thus, from the vantage point of physical constructive geometry, the ordering principle of prime numbers should not be sought for in them, as such, but in the harmonic proportionality that lies between them. It is essentially the harmonic relationships between human beings that define economic science, just like it is the harmonic relationships of numbers that define the theory of numbers. It's that simple.

As LaRouche has taught us, economics is the rate of increase of the productive powers of labor, which leads individual human beings into creating scientific and cultural objects that are necessary to improve the relative population density of our planet. Even though the world economy is more complex than prime numbers, their generative principle of reason and power is the same. So, let us apply the same universal physical principle of proportionality to the growth of prime numbers. By doing this, you will discover that it is the rate of increase of intervals between prime numbers

that establishes their geometric ordering and the density of their distribution and growth, nothing else.

The following will demonstrate that the underlying, or rather the interlaying geometry of position of the intervals between prime numbers, forms patterns of well-ordered multiply-connected circular action, in which all of the multiples of the primes overlap each other as if in some braided form. This means that the patterns formed by the multiples of primes are but shadows of a process of physical action, which produces those braided patterns and which determines the distribution of the primes. We shall demonstrate that the principle of that rate of increase pertains to C-256.

## ORDERING PRIME INTERVALS, AS SUCH.

First of all, order the prime numbers and their multiples according to a repeated simple series of intervals of 2 and 4. For instance, there is an interval of 2 between 5 and 7, and an interval of 4 between 7 and 11; then, again, there is an interval of 2 between 11 and 13, and an interval of 4 between 13 and 17, etc. This way, the multiples of all of the prime numbers will form well-ordered patterns within the lattice. Such patterns could be mapped on a plane, on a cylinder, on a cone, on a torus, or on a sphere, as if they were projected as a reflexion of a higher domain that produces them. However, they don't belong to the same domain that generates cubic roots, or higher roots. They appear to form circular patterns whose straight line shadows can be made visible on the plane of Figure 1, as if you were to "connect the dots" between all of them, and make them emerge from the dimly lit wall of Plato's cave.

Each series of prime multiples, say, for example, the multiples of 5, like 85, 125, 205, 245, 325, 365, 445, 485, etc., forms a unique pattern. Each prime has a unique pattern of multiples, and there exist as many patterns as there are prime numbers. All of them overlap each other and are interconnected by each other's intervals to form a coherent latticework. I let the reader illustrate this, for himself, by experimenting with Figure 1.

# Figure 1. TABLE OF PRIME NUMBERS AND THEIR MULTIPLES.

<u>5</u>    7    11   13   17   19   23   <u>25</u>   29   31   <u>35</u>   37   41
43   47   49   53   <u>55</u>   59   61   <u>65</u>   67   71   73   77   7 9
83   <u>85</u>   89   91   <u>95</u>   97   101  103  107  109  113  <u>115</u>  119
121  <u>125</u>  127  131  133  137  139  143  <u>145</u>  149  151  <u>155</u>  157
161  163  167  169  173  <u>175</u>  179  181  <u>185</u>  187  191  193  197
199  203  <u>205</u>  209  211  <u>215</u>  217  221  223  227  229  233  <u>235</u>
239  241  <u>245</u>  247  251  253  257  259  263  <u>265</u>  269  271  <u>275</u>
277  281  283  287  289  293  <u>295</u>  299  301  <u>305</u>  307  311  313
317  319  323  <u>325</u>  329  331  <u>335</u>  337  341  343  347  349  353
<u>355</u>  359  361  <u>365</u>  367  371  373  377  379  383  <u>385</u>  389  391
<u>395</u>  397  401  403  407  409  413  <u>415</u>  419  421  <u>425</u>  427  431
433  437  439  443  <u>445</u>  449  451  <u>455</u>  457  461  463  467  469
473  <u>475</u>  479  481  <u>485</u>  487  491  493  497  499  503  <u>505</u>  509
511  <u>515</u>  517  521  523  527  529  533  <u>535</u>  539  541  <u>545</u>  547
551  553  557  559  563  <u>565</u>  569  571  <u>575</u>  577  581  583  587
589  593  <u>595</u>  599  601  <u>605</u>  607  611  613  617  619  623  <u>625</u>
629  631  <u>635</u>  637  641  643  647  649  653  <u>655</u>  659  661  <u>665</u>
667  671  673  677  679  683  <u>685</u>  689  691  <u>695</u>  697  701  703
707  709  713  <u>715</u>  719  721  <u>725</u>  727  731  733  737  739  743
<u>745</u>  749  751  <u>755</u>  757  761  763  767  769  773  <u>775</u>  779  781
<u>785</u>  787  791  793  797  799  803  <u>805</u>  809  811  <u>815</u>  817  821
823  827  829  833  <u>835</u>  839  841  <u>845</u>  847  851  853  857  859
863  <u>865</u>  869  871  <u>875</u>  877  881  883  887  889  893  <u>895</u>  899
901  <u>905</u>  907  911  913  917  919  923  <u>925</u>  929  931  <u>935</u>  937
941  943  947  949  953  <u>955</u>  959  961  <u>965</u>  967  971  973  977
979  983  <u>985</u>  989  991  <u>995</u>  997  1001

Find the multiples of any prime number and "connect the dots" of their latticework. All of the multiples of each prime number can be generated in a similar fashion. For example, the multiples of 7 form simple diagonals across the lattice as they alternate by intervals of 8 and 4, such that between 7 and 35, for instance, there are 8 intervals, which are 11, 13, 17, 19, 23, 25, 29, 31, and between 35 and 49, there are 4 intervals, which are 37, 41, 43, and 47, etc. Again, the point to retain, here, is that it is not the values of those numbers in themselves that count, but the geometry of position of their intervals. So, what is the principle behind the construction of those patterns on intervals within the lattice?

# THE DENSITY OF SINGULARITIES BETWEEN PRIMES

The density of intervals between the multiples of primes increases, as the prime numbers get larger. Note that the intervals between the multiples of 5 are 2 and 6, the intervals between the multiples of 7 are 4 and 8, and the intervals between the multiples of 11 are 6 and 14, and so forth. The intervals of those intervals grow arithmetically by even numbers as all of them increase consistently by 4.  Thus, we can establish a table of the different intervals of the multiples of primes, including the quadratic intervals between those intervals.  [Figure. 2]

Figure 2

| PRIMES & MULTIPLES | | | INTERVALS | | INTERVALS OF INTERVALS | |
|---|---|---|---|---|---|---|
| 5 | | | [2 and 6  ] | | 4 | |
| | 2 | 2 | | 2 | | |
| 7 | | | [4 and 8  ] | | 4 | |
| | 4 | 2 | | 6 | | 4 |
| 11 | | | [6 and 14] | | 8 | |
| | 2 | 2 | | 2 | | |
| 13 | | | [8 and 16] | | 8 | |
| | 4 | 2 | | 6 | | |
| 17 | | | [10 and 22] | | 12 | |
| | 2 | 2 | | 2 | | |
| 19 | | | [12 and 24] | | 12 | |
| | 4 | 2 | | 6 | | 4 |
| 23 | | | [14 and 30] | | 16 | |
| | 2 | 2 | | 2 | | |
| 25 | | | [16 and 32] | | 16 | |
| | 4 | 2 | | 6 | | |
| 29 | | | [18 and 38] | | 20 | |
| | 2 | 2 | | 2 | | |
| 31 | | | [20 and 40] | | 20 | |
| | 4 | 2 | | 6 | | 4 |
| 35 | | | [22 and 46] | | 24 | |
| | 2 | 2 | | 2 | | |
| 37 | | | [24 and 48] | | 24 | |

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          4         2              6
  41                    [26 and 54]                    28
          2         2              2
  43                    [28 and 56]                    28
          4         2              6                              4
  47                    [30 and 62]                    32
          2         2              2
  49                    [32 and 64]            Etc.
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As the value of a prime number increases, so do, proportionately, the intervals of their multiples, up to a certain limit. The first series of intervals of multiples increases by alternating factors of 2 and 6, the second series of intervals of intervals grows by a constant factor of 4. This second growth rate, that is, the intervals of intervals, represents a second derivative form of action, following an underlying ordering of quadratics, that is, an ordering of the second degree, from within which the musical octaves, based on C-256, is derived. This second degree of intervals changes, because the ratio of intervals increases by a conical spiral function, which distributes the primes in accordance with the power of 2.

This does not mean that things stop growing at that point, but that the system has reached a limit, a boundary condition dominated by second degree quadratics, whose heads pierce through the cracks of the universe to show that they belong to some infinite and universal object which eludes the grasp of our sense perception. It is this constant ratio of the power of 2 within quadratics, this invariant interval of interval factor of doubling, which determines the density of distribution of prime numbers.

**PART III**

**A REFLEXION ON THE APRIL 27, 2002 PEDAGOGICAL OF JONATHAN TENNENBAUM CONCERNING ARTICLES 107- 108 of {*DISQUISITIONES ARITHMETICAE*} BY CARL FRIEDRICH GAUSS.**
**[First drafted in Leesburg, May 8, 2002.]**

As Jonathan demonstrated in his pedagogical, Gauss had challenged the scientific community to solve an elementary, but difficult problem of discovering the ordering principle for the distribution of prime numbers, given only a single shadow projected on the dimly lit wall of Plato's cave. The discovery of this ordering principle had led Gauss to develop the fundamental theorem of Algebra, which gave a devastating refutation of D'Alembert, Leonard Euler and Jean-Louis Lagrange's method of algebraic empiricism. This is the way Gauss formulated his crucial concept, again, using the method of Diophante:

>107. "It is very easy, given a modulus, to characterize all the numbers that are residues or nonresidues…But the inverse question, {*given a number, to assign all numbers of which it is a residue or a nonresidue,*} is much more difficult." To wit:

>108. "-1 is a quadratic residue of all numbers of the form 4n+1 and a nonresidue of all the numbers of the form 4n+3."  (Carl Gauss, {*Disquisitiones Arithmeticae*}, p. 71-72)

Think of this question, as a sort of "boot-strap" principle of self-generation, in which the process contains the paradox of Diophante; that is, if the effectiveness of this inversion were not true, there would be an absolute limit beyond which one could not go. However, one is able to go beyond and contradict the limit to show there exist higher-boundedness of action in the universe. So, in that unique way, the universe is "limited and yet increasingly self-bounded." In other words, Gauss is showing how to solve the Leibniz anti-empiricist proposition: {*given the property of a tangent, find the curve.*}

To discover this, Jonathan proposed to proceed in the following manner: Since subtracting -1 from a number has the same significance as adding 1 to it, take the series of squares: 4, 9, 16, 25, 36, 49, 64, 81, 100,

121, etc., which represents the entire series of rational whole numbers, and add 1 to each number such that they are now transformed as:

5, 10, 17, 26, 37, 50, 65, 82, 101, 122, etc.

When you look at this last series, you realize that all primes of the 4n+1 species form half of that series of squares +1, that is, 5, 17, 37, 65, and 101, but such is not the case for the other half of those numbers. My surprise was to find that those other numbers were not directly derivable by means of the form [4n+3] either.  I was perplexed, until I found that the other numbers of that series, 10, 26, 50, 82, and 122, were all of the form of a close relative of 4n+1, which is 2[4n+1], a biquadratic. Even though Gauss had pursued the matter by demonstrating that –1 was a residue to a prime modulus of the biquadratic form of 8N + 1, I did not see how helpful this would be until I found the ordering principle behind the numbers of the biquadratic form of the square root of 2[4n+1]-1. That looked to me as a very curious sort of complex number.

As the history of science shows, a discovery of physical principle is, most of the time, caused by a nagging anomaly, a real ambiguity that keeps knocking in the back of your mind. My investigation into the series of numbers in the form of the square root of 2[4n+1]-1 led to the realization that these numbers could integrate the two species of primes, [4n+1] and [4n+3], that Gauss was talking about. So, I asked myself: What if the principle underlying this form of the square root of 2[4n+1]-1, alone, were to provide a least action pathway to integrate all of the primes dressed up in the form of squares, in accordance with Fermat, Diophante, and Pythagoras? I first lined up the series of numbers as follows to look at the ordering principle of the change between them, in my mind's eye:

```
  5    =     [4x1+1]
 10    =  2 [4x1+1]
 17    =     [4x4+1]
 26    =  2 [4x3+1]
 37    =     [4x9+1]
 50    =  2 [4x6+1]
 65    =     [4x16+1]
 82    =  2 [4x10+1]
 101   =     [4x25+1]
 122   =  2 [4x15+1]     etc.    Figure 3.
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I noticed that all of the values of [4n+1] corresponded to squares +1, that is, [4x1+1], [4x4+1], [4x9+1], [4x16+1], [4x25+1], forming directly the series of numbers 5, 17, 37, 65, 101. However, if one pays attention to the other numbers of the form 2[4n+1], that is, a process of simple doubling of primes, as opposed to a squaring, a most fascinating series of intervals emerges, and a series of intervals of intervals takes hold of the whole process, whose harmonic ordering generates all of the primes!

The intervals between each multiple within the 2[4n+1] brackets, that is, between 1, 3, 6, 10, 15, have intervals of 2, 3, 4, 5, an increase by 1, while the intervals of intervals between the corresponding numbers 10, 26, 50, 82, 122, are 16, 24, 32, 40, thus expressing a constant increase by 8, a biquadratic interval. I will show that there exists, here, a {*harmonic ordering principle of these complex biquadratic intervals*} which generates all of the primes from any of the modules of the power of two, that is, the 256 series.

## NUMBER THEORY AND TORUS GEOMETRY

From this vantage point, it became clear to me that the {*intention of numbers*}, established by God himself, so to speak, had never been to empirically count things as self-evident things, as if you were counting your loose change, but to express a different kind of change, a change in direction, a change of orientation, even a change of axioms by means of intervals of physical action; that is, by means of adding a new dimensionality to the cycles of universal physical processes. In this case, the added dimensionality is that of the physical geometry of the torus. Thus, what was implied here, was a physical principle of action such as that of a least action form of the catenoid curvature of a torus which functions in such a way that it is motionless, yet is, paradoxically, everywhere in motion.

For example, take any module of the 256 series, say 16, and put it to work into a complex circular form of a torus. It doesn't move, but it changes all the time.

```
              1[3]    11[23]
        *           *           *
       0                      2[5]
    *              →              *
  5[11]                         8[17]
   *                              *
 12[25]                         6[13]
  *         ↑        16      ↓     *
 9[19]                          3[7]
  *                               *
 7[15]                         10[21]
   *              ←              *
  13[27] *         *         * 15[31]
            4[9]      14[29]
```
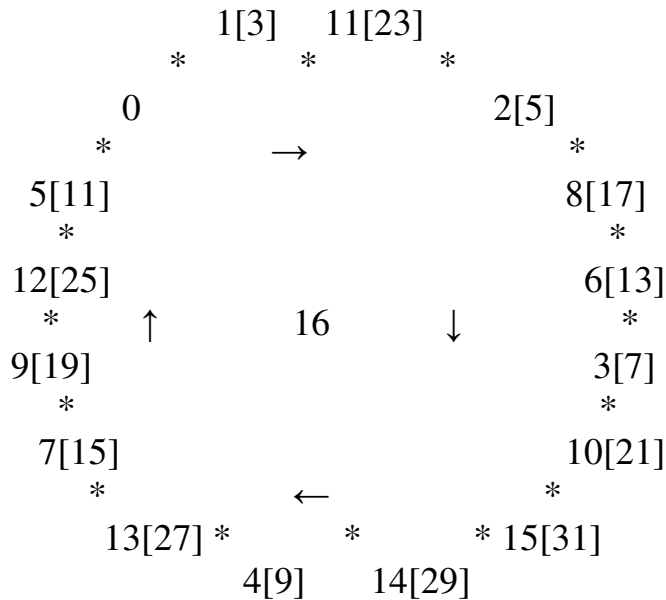
Figure 4

First, mark the simple 16 intervals of action by whole numbers 0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, clockwise, count every unit of interval of action starting from 0 to 1, noting that you are increasing each interval of action by one, after each number, as you move forward, clockwise. Fill the entire module until all of the 16 intervals are covered.

Next, overlap this first series of simple intervals by a second series of the complex numbers of the form square roots of 2[4n+1]-1, which changes the direction, ever so slightly, by adding a new dimensionality. Go from the square root of 2[4x1+1]-1 up to the square root of 2[4x120+1]-1. This generates all of the primes from a second-degree derivative interval of intervals, which overlaps the intervals of the first degree, binds them together proportionately, and modifies their course by an infinitesimal degree of change in each interval of action. This should demonstrate the point made by Fermat, which is that, with this least action process of the torus, you have reached the boundary condition of all rational numbers with roots of the second degree. Thus, as if in a fast spinning sun, the self-

bounding physical geometry of the process of change is reflected in the characteristic of the torus.

For example, the first prime in the series is the square root of 2[4x1+1]-1, which is [3], because the square root of 10-1 is [3]. That is your first prime, whose value overlaps the first interval from 0 to 1 in the circle of Figure 5, thus, modifying its position course ever so slightly.

The second prime in the series is the square root of 2[4x3+1]-1, which is [5], because the square root of 26 - 1 is [5]. That is your second prime, whose value overlaps the two intervals between 1 and 2 in the circular form, and maintains the same changed course.

The third prime in the series is the square root of 2[4x6+1]-1, which is [7], because the square root of 50-1 is [7]. That is your third prime, whose value overlaps the three intervals between 2 and 3 in the circular form, and maintains the same changed course.

The missing biquadratic square roots of 2[4x2+1]-1, of 2[4x4+1]-1, and of 2[4x5+1]-1, are not rational whole numbers, and therefore, have simply been overshadowed, leaving the necessary spaces open for subsequent primes to come later and fill their pre-established harmonic positions within the cycle of the modular function.

The remaining primes (and multiples of primes) found in this manner shall be, [9], [11], [13], [15], [17], [19], [21], [23], [25], [27], [29], [31], all of which shall find their pre-assigned intervals in due course within this cycle. Although there exists a simple formula to achieve such results, it is better to construct these biquadratic second-degree {*intervals of intervals*} long hand, and enjoy the complex beauty of their relationships in the geometry of your mind. This way, you experience the amount of playful least action work that God has put into them.

In a nutshell, this exercise intended to show how the "great theorem" of Fermat implicitly led us to generate and distribute all of the prime numbers in a Diophantine biquadratic form of the square root of 2[4n+1]-1, and thus, enabling us to develop all rational roots of the second degree as a {*torus modular function*}. As LaRouche indicated in {*What Connects the Dots*}, the "great theorem" of Fermat is not merely mathematical, as such, but "ontological" in character, that is, it pertains to the universal principle of

least action underlying all physical processes, including primarily economic processes. This is why Fermat was right in establishing a limitation whereby it is impossible to divide any power into two other powers of the same order beyond the second degree, because the universe is "finite and yet not bounded." [pierrebeaudry@larouchepub.com](mailto:pierrebeaudry@larouchepub.com)

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