ANALYSIS SITUS OF WHOLE NUMBER RECIPROCITY AND HOW TO MAKE AN AXIOMATIC CHANGE

An epistemological exercise applying the recapitulation method of Saint-Irenaeus of Lyon to the Leibniz-Poinsot *analysis situs* geometry of number theory

By Pierre Beaudry 2/22/18

INTRODUCTION

Recently, someone asked me why I keep saying that the most important thing to look for in a discovery of principle is ***what is not there***. I told him that the reason for this kind of inquiry is simple: you always want to be ready for the unexpected; that is, for something that you thought could never exist.

For instance, if you think you know how whole numbers work, think again, because you may be surprised to discover that the most obvious is not obvious at all. Even with something you know well, always look for what is unexpected; look for something that should not be there, especially when it keeps on not being there repeatedly.

In number theory, there are all sorts of things that mathematicians have done which have complicated things, and yet, most of them have not noticed that whole numbers are essentially expressions of simple circular action, and some of the whole numbers even express multiply-connected circular action with an astounding capacity for reciprocity.

The reason why people don't pay attention to ***what is not there*** is because they think that if it is missing, it can't have much importance. As Aristotle would say: "If it were important, people would see it."

On the contrary, if you think like Plato, you might realize that ***what is not there*** is usually a sign that there is something very important which you should know about, and your life may even depend on it. So, next time you want to come back down from the seventeenth floor of a building in an elevator, don't take a chance. Make sure you notice if something is missing when the doors open.

1. THE ROLE OF LOUIS POINSOT IN DISCOVERING THE IMPORTANCE OF WHAT IS NOT THERE[1]

> ***"And yet, when one takes the time to think about it for a moment, it is easy to see that such a transcendental arithmetic is like the principle and the source of algebra proper."***[2]
>
> Louis Poinsot

On July 24, 1809, a student of Lazare Carnot, Louis Poinsot (1777-1859), introduced, in the first lecture he gave at the Paris Science Institute, the Leibnizian method of ***analysis situs*** for the purpose of grounding the geometric properties of numbers on a solid foundation. Such an axiomless playful form of geometry was characterized by Poinsot as a form of constructive geometry that excluded the Euclidean flat earth types of reductionist formal geometry taught in most French schools. In his opening statement to that class, Poinsot established the following Leibnizian principle of construction for the ***analysis situs*** method. He said:

[1] This section is reproduced with added corrections from my 2010 report: **FUSION POWER IS NOT DEMOCRATIC.** This 2010 report and the present one may be useful for understanding fusion processes.
[2] Louis Poinsot in ***Reflexions sur les principes fondamentaux de la théorie des nombres***, Paris, Bachelier, Imprimeur-libraire, 1845, p. 4.

"The object of geometry of situation (***analysis situs***), as I have said, is to determine the order and the location of objects in space, without any consideration for the size and continuity of figures; such that the part of mathematical analysis, which would naturally apply to it, is the science of the properties of numbers or indeterminate analysis, like ordinary analysis is applied naturally to determined problems of geometry, and the differential calculus is applied to the theory of curves, wherever the curvature changes with imperceptible nuances. I have not found the place in the Acta of Leipzig, where Leibniz talked about the geometry of situation; but it seems to me that the idea he had of it conformed with the one I am giving here, and this is what can be seen quite clearly in this section of one of his letters on mathematical games. ***'Following the games that depend only on numbers, we have the games which further involve the situation, such as backgammon, checkers, and above all chess. The game called Solitaire also pleased me enough. However, I am considering it (analysis situs) in a reverse manner, that is to say, instead of undoing a composition of pieces, according to the rule of this game, which calls for jumping into an empty place, and taking away the piece on which we jump, I thought it would be more beautiful if we reestablished what had been undone by filling in a hole on which we jump; and by that means, we could propose to form such and such a given figure, if it were doable, as it surely could be done, since it was possible for it to be undone. But, some will say: 'what is the purpose!' I would respond, to perfect the art of invention; because we should have methods for solving everything that reason can put before us.'*** "[3]

The Leibnizian idea that Poinsot adopted not only led him to discover two new regular solids, called the Great Dodecahedron and the Great Icosahedron, but also gave him the ammunition to fight the mediocrities of the flatland geometry of Euclid, Newton, and Euler, among others. Thus, Poinsot contributed to the effort of

[3] Gottfried Leibniz, ***Letter VIII to M. de Montfort,*** in Leibniz, ***Opera Philosophica***, quoted by Louis Poinsot in ***Reflexions sur les principes fondamentaux de la théorie des nombres***, Paris, Bachelier, Imprimeur-libraire, 1845, p. 45-46. See Also Poinsot's ground-breaking ***Mémoire sur les Polygones et les Polyèdres,*** read before the Institute on July 24, 1809.

overturning the ***a priori*** system of axioms, postulates, and definitions of what later became known in modern mathematics as ***topology***.

Instead, Poinsot developed the constructive Leibnizian method of ***analysis situs*** whereby the principle of a physical situation had to be included as an essential external component in the construction of physical geometric solutions, but which was not determined by the internal conditions of the objects themselves. He knew the situation had to be conducted from an outside causal agency performing inside of the process.

Moreover, from the standpoint of Lyn's epistemological ***axiom busting*** method, Poinsot's constructive method had the effect of a hand grenade thrown into the foxhole of Euler and his sycophants, during the early part of the nineteenth century, and it can be used, similarly, against today's pessimistic quackademic topologists who haunt the corridors of our universities. Poinsot made the point correctly about Euler's pessimism in his paper on number theory, reporting that the discouraged Euler had renounced all hopes of ever finding the answer to the question of discovering the ordering principle of primitive roots.

According to several accounts concerning judgments made by Euler on primitive roots that are to be found in his ***New Commentaries from Saint-Petersburg*** (Tome XVIII), Poinsot said:

> "Euler admitted that no means of determining these roots could ever be found; that the demonstration which proves their existence indicates, in all cases, that no method exists to discover them; that we cannot find any relationship between a prime number and the primitive roots that belong to it, and from which could be deduced at least one of those roots; that such a law, which rules them, seems to be as profoundly hidden as that which orders the prime numbers themselves."[4]

Aside from demonstrating that Euler was wrong, Poinsot was also self-consciously reminding us that the principle for discovering such an ordering was

[4] Louis Poinsot, ***Reflexions sur les principes fondamentaux de la théorie des nombres***, Paris, Bachelier, Imprimeur-libraire, 1845, p.75.

open-ended like a memory function; that is, ***finite but unbounded*** as Einstein reminded us the universe itself to be. The fact that Poinsot had used the same Leibnizian method of ***analysis situs*** to warn future generations against the pessimism of Euler merely served to confirm the universality of the underlying principle he was promoting. Such a method can be employed extensively in different domains of knowledge. Caution, however, is not always thrown to the winds.

As Peter Martinson noted about such discoveries: "Gauss recognized the validity of the principle of Quadratic Reciprocity before he completed his first proof in 1796. His proof does not resemble how he discovered that it was valid."[5] What Peter did not say is that Gauss was cautious for personal security reasons. Why? Because if you discover something fundamental that a multitude of other people missed, because they missed their chance to discover ***what is not there***, you will become a real threat to them. As Peter reports later in his paper: "If a person discovers the nature of that continuous domain - that universal physical principle - he can apply that knowledge to generate any singularity he wishes. He has thus gained a power."

Take for example the method that Cardinal Gilles Mazarin used as the primary means of his triply-extended process of negotiation at the Peace of Westphalia.[6] This was the same principle of reciprocal proportionality that Gauss had established as the basis for determining congruence among counting numbers, when he formulated the idea in the first proposition of his ***Disquisitiones Arithmeticae***. The idea is expressed by the fact that a number, **C**, is congruent with two other numbers, **A** and **B,** when **C** is able to eliminate the difference between the other two. Similarly, Mazarin recognized that a lasting European peace could not be achieved unless such a diplomatic power was used, as the Ambassador of the Netherlands did brilliantly before anyone else in achieving the peace a few months earlier than all of the others; that is to say, by succeeding in eliminating the differences between France and Spain.

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[5] Peter Martinson**, QUADRATIC RECIPROCITY.**

[6] See Pierre Beaudry, ANALYSIS SITUS AND THE PRINCIPLE OF RECIPROCITY.

From the vantage point of a similar method, Poinsot discovered that if a number was prime to N, then all of the powers of that number must also be prime to N.[7] From this, he inferred that the number of primitive roots of any prime number P are those which remain after the squares, the cubes, and the fifth powers, etc., have been extracted from the intervals of action of the module P-1. Poinsot gave the example of prime number 61 taken as a module. Since the number immediately preceding 61 is P-1, that is 60, it is clear that the simple factors of 60 are 2, 3, and 5. Poinsot showed that any other multiple could be broken down into these three simple factors, up to the 60th power.[8] That being the case, he excluded all of the squares, the cubes, and the fifth powers. By eliminating the powers of two from 60, he was left with 30. By eliminating the third powers from 30, he was left with 20, and by eliminating the fifth powers from 20, he was left with 16 numbers. Therefore, after this playful exclusion of the superfluous power factors, there remained only 16 primitive roots of P = 61, which were ironically held together, invisibly, by ***what is not there***, just like prime numbers are held together by the missing simple factors, as I have shown in a previous ***analysis situs*** pedagogical.[9] The conclusion of Poinsot was as simple as it was elegant. Poinsot stated:

> "When you wish to find them (prime numbers), one considers all of the simple factors of a given number; and from the natural series 1, 2, 3, 4, 5, etc., one excludes all of the multiples of these simple factors. Here (with primitive roots), instead of excluding those multiples, it is necessary to exclude all of the powers of exponents identified by these factors: the result, as one can see, is an operation of the same type, except from a higher level of ordering."[10]

But, where is that higher anti-entropic principle piercing from? How do you access the domain of the universal physical principle that shines through the cracks

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[77]Louis Poinsot, ***Reflexions sur les principes fondamentaux de la théorie des nombres***, Paris, Bachelier, Imprimeur-libraire, 1845, p.51.

[8] Louis Poinsot, ***Op. Cit***., p. 73.
[9] See Pierre Beaudry, **FERMAT'S GREAT THEOREM** , 2/10/2006.

[10] Louis Poinsot, ***Op. Cit.,*** p. 75.

of the mere quantitative illusions of shadow-numbers? That is the question that remains to be discovered. What is this underlying process showing us with respect to cosmic radiation, for instance? What sort of measuring rod is Poinsot looking for with respect to physical science in general? He is pointing not only to a mere quantitative continuity, but also to a qualitative higher level of continuity. Thus, the Leibniz-Poinsot method of ***analysis situs*** is not merely a playful geometric game; it is also meant for solving anti-entropic axiomatic problems as determined by the function of their location or situation, that is to say, by determining solutions defined from a higher epistemological mode of existence and with an appropriate constructive geometry within the physical or historical context of that higher metaphysical existence.

2. THE ORDERING OF WHOLE NUMBERS BY CIRCULAR ACTION

"Nothing fundamental has been found until one finds reciprocity."

Dehors Debonneheure

Twenty five years ago, I made a little discovery of something among numbers ***that was not there. It is not there*** again today, so I have decided to look into it one more time. It did not appear to be very important then, but in fact, ***what was missing*** was something that was located in the process of counting whole numbers by simple circular motion.

The curious thing was that, as I was counting, ***I kept finding gaps inside of the counting process of rotating most of the counting whole numbers!?*** At first, this simple and playful little exercise didn't bother me because the missing gaps which kept recurring at different places appeared to be arbitrary. But, then, I started to become more and more perplexed as a pattern began to emerge and repeat itself. The gaps appeared repeatedly within the count of similar numbers and not with others. The gaps were like singularities or exceptions to some sort of rule that I had no knowledge of. Then, I began to realize that the circles which didn't have any gaps in them did not have any prime number factors in them either. That was interesting. Why were those factors missing? ***And, why was it so important to look for something that wasn't there?***

As Lyn often reminded us, it is by looking **"*between the notes*"** that one can understand universal history. So then, I decided to look at numbers not as things in themselves, but as **"*intervals of action*."** I began to rotate whole numbers like circular actions and I started looking for the reason why there were gaps or discontinuities among a great number of such circles. To my surprise, what I discovered was that when you count numbers as ***intervals of action*** in a circular manner, you discover that gaps appear to be part of the process of circular action and not as part of the numbers themselves. But, could that have been merely an illusion?

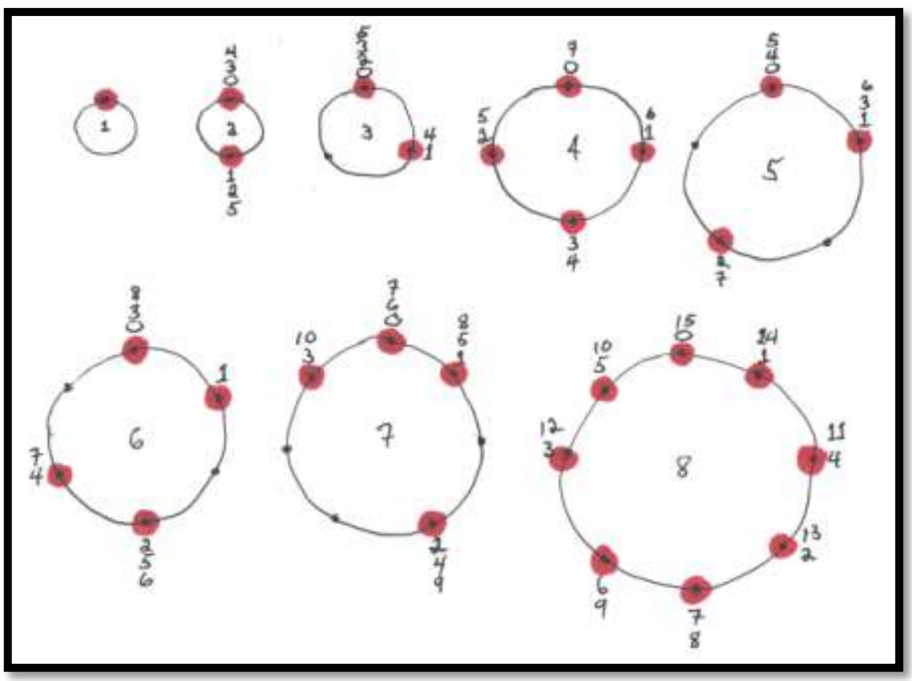

Figure 1. Clockwise counting of whole numbers as intervals of action as opposed to points or things. Imagine the points in each rotation as merely marking the location of "***analysis situs holes***" that you have to fill-up with an indefinite series of numbers which are all diametrically reciprocals.

Consider **Figure 1** and start looking for ***what is not there***. You will discover that what is not there reflects what is missing for connecting the four domains of ***arithmetic, geometry, theology, and artistic composition***. Draw 8 circles on a sheet of paper and divide each circle into as many equally spaced points as you can mark all around each circle. Next, start counting circular actions between each point of each circle and always in the same clockwise direction.

Put the point of your pencil at 12 o'clock; that is at 0, and start counting the spaces between each point as a single unit of circular clockwise action. Identify each number as an interval of circular action equal to any other unit of circular action of any other number of that circle. Count 1, 12, 123, 1234, etc., such that 1 represents a single interval, 2 represents two intervals, 3 is three intervals, etc. Drop the number at the red point where the action stops and start again counting your next step from scratch. Always start counting any new action from scratch, after dropping the last number into the hole next to a red point; that is, every time you go from one interval to the next. Do the same thing for all of the intervals of each of the 8 circles and remember: you are counting internals of action, not numbers or points. You will not understand the significance of this process unless you physically replicate the circular action indicated in **Figure 1**, yourself.

Drop all of the numbers at the end-points of arrival of each action, as if that empty location was a space that had been already prepared in advance for them from the future. If you follow this process precisely, you will notice that you will stop to drop numbers at some points, and you will jump over certain other points where you make no drops. Some of the holes will be filled and others will remain empty. Why are there no numbers in those empty holes? That is what you were looking for: ***why are there no numbers there***. As Leibniz said, the game is to proceed "***by filling in a hole on which we jump!***" So, why are you jumping over those empty places? What do these empty holes represent? Those are the questions we need answers to.

Firstly, note that by counting numbers around the 8 different sets of circular action, only four cases 1, 2, 4, and 8 reflect completeness while the four other cases 3, 5, 6, and 7 all allow gaps or empty holes. I have never been able to account for that curious phenomenon until only recently, and under the strangest of circumstance. Secondly, note also that only circles 2, 4 and 8 reflect complete reciprocity and all of their reciprocals are either prime numbers or composed of prime numbers. What is the significance of this sort of circular action ordering relationship among whole numbers? It seems that the power of two series is privileged by circular action at the expense of every other whole number. Why?

3. THE CIRCULAR RECAPITULATION ORDERING PROCESS OF BIQUADRATIC RESIDUES

> ***"You might not know where you are going to end up at the end of all of this, but at least you will discover how to get there."***
>
> Dehors Debonneheure

While looking back through what I had worked out in this research during 1992, I realized only recently that the ordering of ***biquadratics,*** which I had discovered back then, was not a recondite and secretive science of numbers that only mathematicians could understand, but rather reflected a simple yet profound characteristic of ***Promethean thinking***, as Lyn had identified and which had been developed by Saint Irenaeus of Lyon during the second century AD. The process of this epistemological power can be identified as a ***circular recapitulation form of time reversal reciprocity*** which is common to ***arithmetic, geometry, redemption theology, and artistic composition*.**

I have come to realized only in the last few months that it is crucial to recover such an epistemological Platonic dimensionality of the mind in order to understand the nature of the epistemological change that mankind is going through today without being conscious of it. My purpose is to make this epistemological discovery conscious.[11] As I said in the preface of my report on Irenaeus, this can

[11] See my report: **SAINT IRENAEUS OF LYON'S DOCTRINE OF 'RECAPITULATION'**

no longer be treated simply as a religious matter; this is a matter of epistemology which requires, most emphatically, an investigation into the differences between the ***theology of God*** and the ***theology of man***; that is to say, a matter of ***Epistemology of the Promethean Mind.***

It took me twenty five years to discover that such numbers known as ***biquadratic residues*** behave like the creative process of inversion that Saint Irenaeus of Lyon had developed in his doctrine of **"*recapitulation.*"** In point of fact, ***biquadratic residues*** are recapitulating reciprocal configurations in which each residue has within itself the power to retain the ***memory of the future*** as the driving force of the creative process; that is to say, the power of changing the past by time reversal.

This may sound outrageous, at first glance, but it means that ***biquadratic residues*** have a built-in knowledge of the ***analysis situs*** pathway of where they are going to go next, although they may be blind as to where they are going to end up. In other words, it is as if each residue contains within itself the power of reaching out to the next wave of action, even before the process of that next wave were to begin its actual forward motion of changing the past; it is as if each residue number has the memory value of the next power wave reflected back into the tail end of the preceding wave; and since the next power wave is also inferred to have a residue, which itself has the memory of the subsequent wave; it is as if the very first wave of the entire process also had, within itself, the memory of the system as whole, to start with. As I have demonstrated elsewhere, the Lydian modality of Bach's well-tempered musical system has the same characteristics.[12] Here is how Irenaeus formulated the process in religious terms:

> ***"For the glory of God is a living human being; and the life of the human consists in beholding God. For if the manifestation of God which is made by means of the creation, affords life to all living on the Earth, much more does that revelation of the Father which comes through the Word, give life to those who see God."***[13]

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[12] See, **AN ELECTRODYNAMIC MUSICAL TORUS**

[13] *Saint Irenaeus*, ***Against Heresies***, IV, 20:7.

What precedes this statement is:

> ***"2. This, therefore, was the [object of the] long-suffering of God, that man, passing through all things, and acquiring the knowledge of moral discipline, then attaining to the resurrection from the dead, and learning by experience what is the source of his deliverance, may always live in a state of gratitude to the Lord, having obtained from Him the gift of incorruptibility, that he might love Him the more; 'for he to whom more is forgiven, loves more:' (*****Luke 7:43)** ***and that he may know himself, how mortal and weak he is; while he also understands respecting God, that He is immortal and powerful to such a degree as to confer immortality upon what is mortal, and eternity upon what is temporal; and may understand also the other attributes of God displayed towards himself, by means of which being instructed he may think of God in accordance with the divine greatness. For the glory of man [is] God, but [his] works [are the glory] of God; and the receptacle of all His wisdom and power [is] man."***[14]

The **"*Glory of God*"** as stated by Irenaeus is to be found in God's love for fallen man and in His ability to restore the dignity of man and to deify him; that is to say, by projecting into the minds of human beings the divine power of the creative process of the ***Word***. This is what Irenaeus called the doctrine of **"*recapitulation.*"**

When such a recapitulating change takes place in your mind, by circular time reversal, so does the resolution of the unity of opposites within a triply-connected dynamic; that is, when a third discovers the power to eliminate the difference between two others, or as Irenaeus might have said, when the Holy Spirit eliminates the difference between the Father and the Son. Therefore, such a triply-connected process is consummated whenever the mortality of man is united with the immortality of God, a realization which can only take place, as Saint John the Divine put it, through the union of the Word made Flesh or when Christ dies on the Cross for the redemption of mankind. Thus, **"*recapitulation*"** takes up the epistemological value of redemption.

[14] Saint Irenaeus, ***Against Heresies***, III, 20: 2

Figure 2 Exercise demonstrating the self-generating process of the Lydian modality as being similar to the reciprocity of recapitulation.

Similarly, this process of reciprocity is also expressed by the modality of the ordering principle of Lydian forecasting in the well-tempered musical system of Bach and of the Verdi tuning at C-256. Why? Because such a voice-register-shift-process is the only process of change which doesn't leave any holes among all of the interactions of its harmonic composition of musical cycles, and marks, ahead of time, where the next step is going to be. This is because of the triply-biquadratic nature of the Lydian ordering itself, a three over four process of dissymmetry which accounts for all changing transformations in musical composition. Nicholas of Cusa had expressed a similar arithmetic-geometrical conception for his notion of the Holy Trinity.[15]

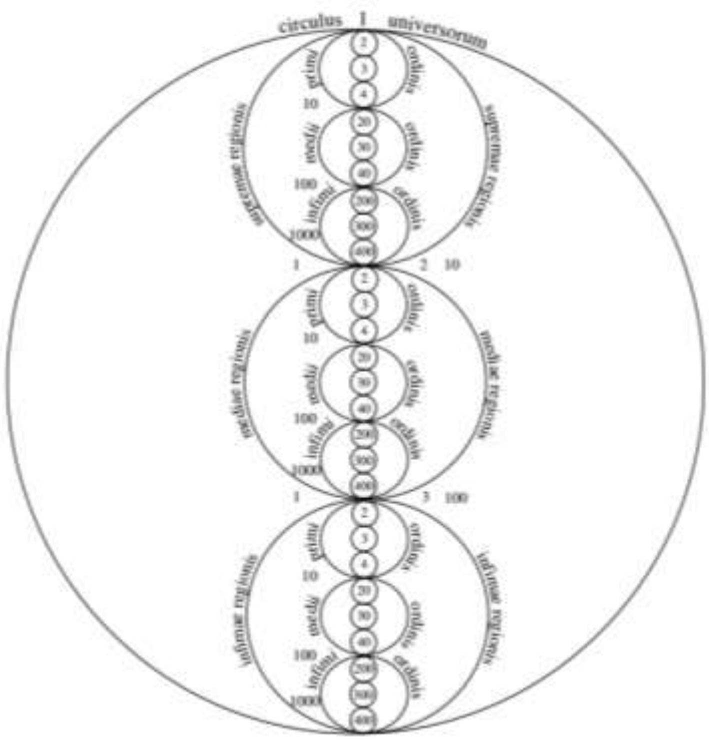

Figure 2 Cusa's idea of a contracted multiply-connected universal least action as a reflection of the Trinity to the third power. As Cusa stated: "If, as the subject-matter requires, you look at the diagram with your mind's eye, then mysteries that are surely important and that are hidden to many will be made known to you."[16]

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[15] See my two reports: **NICHOLAS OF CUSA AND THE PRINCIPLE OF CREATIVITY** and **THE SOLFEGE TORUS**.
[16] Nicholas of Cusa, ***DE CONIECTURIS.***

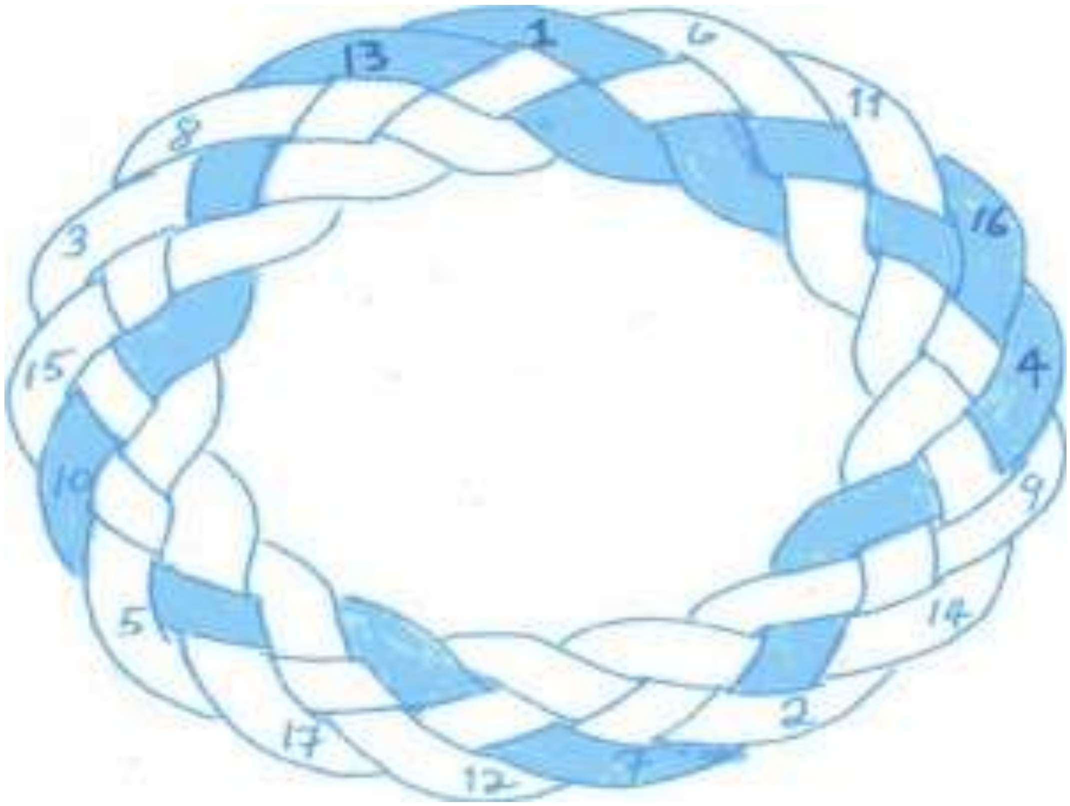

Figure 3 When the Poloidal/Toroidal ratio of module 17 is 4/17, you can find the four biquadratic residues 4, 16, 13, and 1, by ***analysis situs*** alone; that is, and without any calculation.[17]

This is the quadrivium where ***artistic composition, geometry, theology, and number theory*** find their most beautiful and powerful connections. The beauty, here, is that Fermat primes are not ordered in accordance with what the Greeks used in the ordering of polygons such as the equilateral triangle or the pentagon. Here, Poinsot had an extraordinary insight on this matter of principle when he wrote:

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[17] For further investigation in this method of ***analysis situs***, I recommend reading section **4- THE ANALYSIS SITUS OF PRIMITIVE ROOTS: POINSOT** from my 2012 report on: **ANALYSIS SITUS AND THE PRINCIPLE OF RECIPROCITY** in which I have stated the essentials of Leibniz's method of ***analysis situs*** construction underlying the ordering of whole numbers. The method also applies to the ordering of all primes, and they are especially divinely ordered in such Fermat primes as 17, 257, 65537, etc.

> « Thus, the ancients have found that the side of the equilateral triangle, and even the side of the regular pentagon, inscribed in a given circle could be constructed with a ruler and compass; and although they found in these two cases exact constructions, they saw nothing beyond them, and they even believed that they could not go any further. They were able to solve the problem for these two original prime numbers 3 and 5, because the difficulty that comes from numbers here is almost zero, and is not even perceived. But it is not the same thing for higher prime numbers, and they were suddenly stopped in their inquiries, because the true principles of the solution, which can only be found in the theory of numbers, had completely escaped them. And, indeed, if they had understood these principles, they would have seen that the possibility of geometrically dividing the circle into 3 or 5 equal parts was essentially due to a property that is common to these two prime numbers, and that is that each of them, being diminished by unity, makes an exact power of 2; and from there they would have concluded that the solution is equally possible for other prime numbers, such as 17, 257, etc., which also enjoy the same property. However, this is what the solution they had found in the case of 3 and 5, did not even make them suspect it was possible, because they had only found, so to speak, a solution of fact which had not come from this property of numbers, which alone could have led them to succeed."[18]

What Poinsot had discovered is that the ordering principle of numbers comes from the higher ordering of multiply-connected circular least action. Such is the ***analysis situs*** theory of order that Louis Poinsot had established as the founding principle of his theory of numbers.

Let me illustrate the matter with the following modular least action wave process. Take the case of Module 17. (**Figure 3)** If you add in the numbers missing from the empty gaps**,** you will understand why the numbers on the outside rim are in such an apparent geometrical disorder.

[18] Louis Poinsot, ***Reflexions sur les Principes Fondamentaux de la Theorie des Nombres,*** Bachelier, Imprimeur-Libraire, Paris, 1845, p. 6-7.

Their order of position can only be established from the vantage point of the Leibniz and Poinsot ***analysis situs.*** Prove by geometrical construction alone the following theorem: ***Given that the recapitulating motion of the Torus is clockwise, find the four biquadratic residues of module 17 by toroidal action alone.***

Follow the pathway of the wave clockwise by going from 1to 4. Residue 4 marks the end of the first Poloidal wave of four units of action. Next, continue the same wave pattern, starting again from 4 and rotate 4 more poloidal waves around the torus. Your wave motion will end at residue 16 because 4 x 4 =16. If you continue rotating from there by making 16 more Poloidal waves, that is, 4 x 4 x 4 = 64 – [3 x 17], the process will take you to residue 13. From 13, let your finger do the rotating action of 13 more poloidal waves and you will have completed the ***analysis situs*** configuration of the four biquadratic residues of module 17 with a total of 34 poloidal waves.

This process comes from a little discovery I made in 1992 with Louis Poinsot's theorem on the behavior of prime numbers. Poinsot wrote: ***"If you have N points arranged in a circle, and you join them from h to h, h being prime to N, you will necessarily pass through all of the N points before returning to your starting point; and you will have necessarily covered h times the entire circumference."***[19]

Similarly, Gauss was able to find a geometrical construction for the heptadecagon by means of a conical projection.

[19] Louis Poinsot, ***Réflexions sur les principes fondamentaux de la théorie des nombres***, ***Bachelier Imprimeur Libraire, Paris 1845, p. 46.***

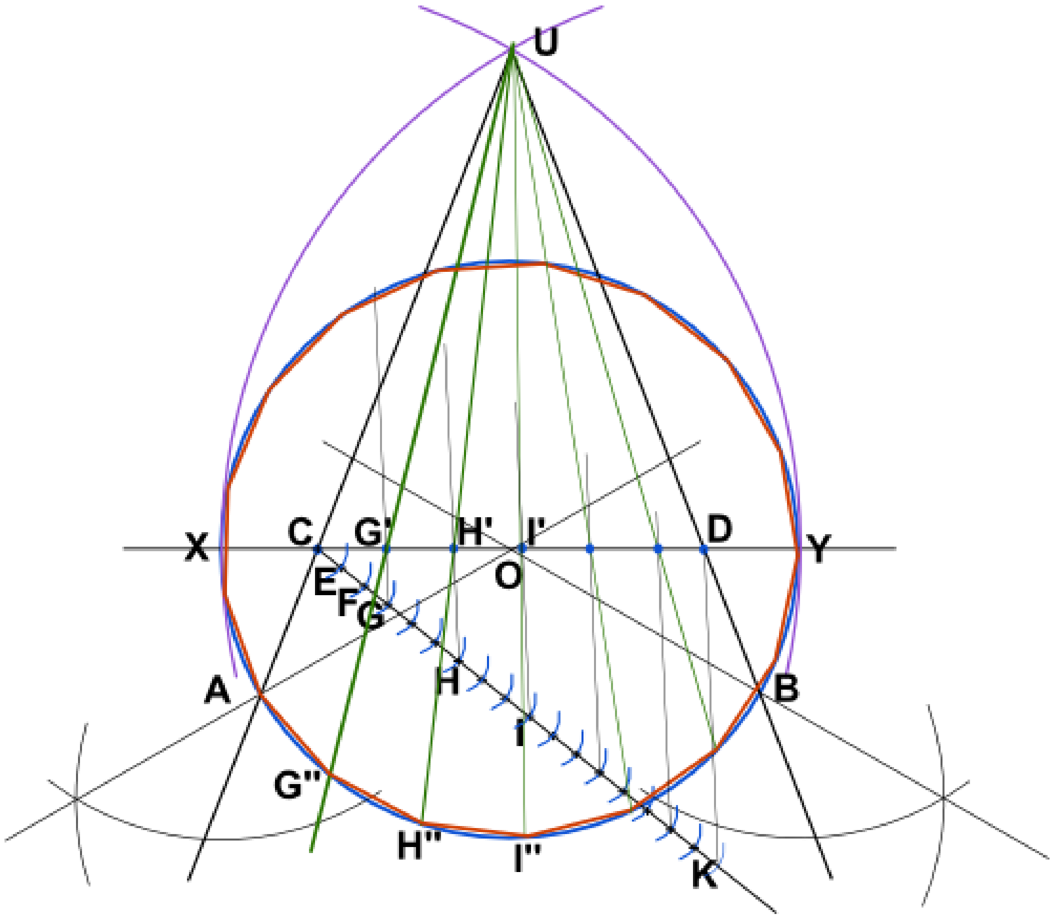


Figure 4 Karl Friedrich Gauss's construction of the heptadecagon.

4. ESTABLISHING THE RECIPROCITY OF THE EIGHT PRIMITIVE ROOTS OF MODULE 17

Here, the reader is required to make a jump to a higher dimensionality; that is, to make a leap from simple circular action to a doubly-connected circular action. Take the following anomaly represented by the discontinuity between the inscribed and circumscribed polygons of a circle and transform it into a donut torus.

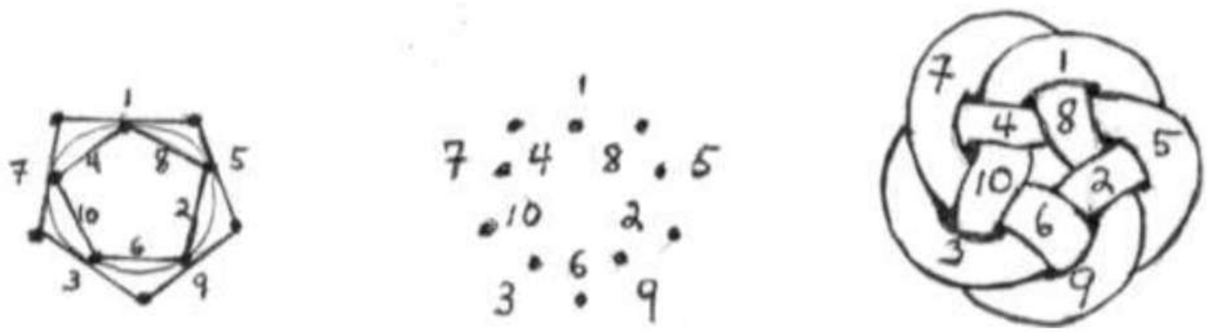


Figure 5 How to axiomatically change from simple circular action to the doubly-connected poloidal/toroidal circular action of the Torus? The two series of polygonal numbers 1, 3, 5, 7, 9 and 2, 4, 6, 8, 10 which are separated by the axiomatic boundary limit of the circle, are no longer separated within the Torus.

How do you find the unity of opposites? How can you resolve the anomaly of **Figure 5** and bring the two series of odd and even numbers together in the same ordering process? How do you go beyond the limitation of the simple circular domain? How do you bridge the discontinuity between the inscribed polygon and the inscribed polygon?

When I applied this sort of number theorem to primitive roots, all I had to do was to realize that the entire module had to be covered *h*[N-1]/2 times by circular construction alone, and not by calculation. That is the ***analysis situs*** solution to the difficulty that Euler was not able to resolve with respect to primitive roots.

Considered as an epistemological memory function, rather than an arithmetical formula, *h*[N-1]/2 encapsulates the same idea of congruence that Karl Gauss identified at the opening of his ***Disquisitiones Arithmeticae.*** As he said: "If a number ***a*** divides the difference of the numbers ***b*** and ***c***, ***b*** and ***c*** are said to be congruent relative to ***a***." Apply this idea of congruence to the function of the "kneeling figure" at the bottom of Raphael's ***Transfiguration*** and you will discover how the creative process is also a crucial epistemological transforming function for artistic composition. The idea is to discover how to change.

Figure 6 Raphael de Sanzio, *The Transfiguration*, 1520. Vatican Museum en.wikipedia.org/wiki/Transfiguration_(Raphael)

Art historian Giorgio Vasari identified this Raphael masterpiece as the "most beautiful and most divine work" ever painted, because it teaches you how to change. Vasari's most significant observation was to identify the kneeling figure in the foreground as the most important figure of the entire fresco, because it represents precisely ***what is not there***; that is to say, ***the reciprocity between the finite and the transfinite with respect to the spectator.***

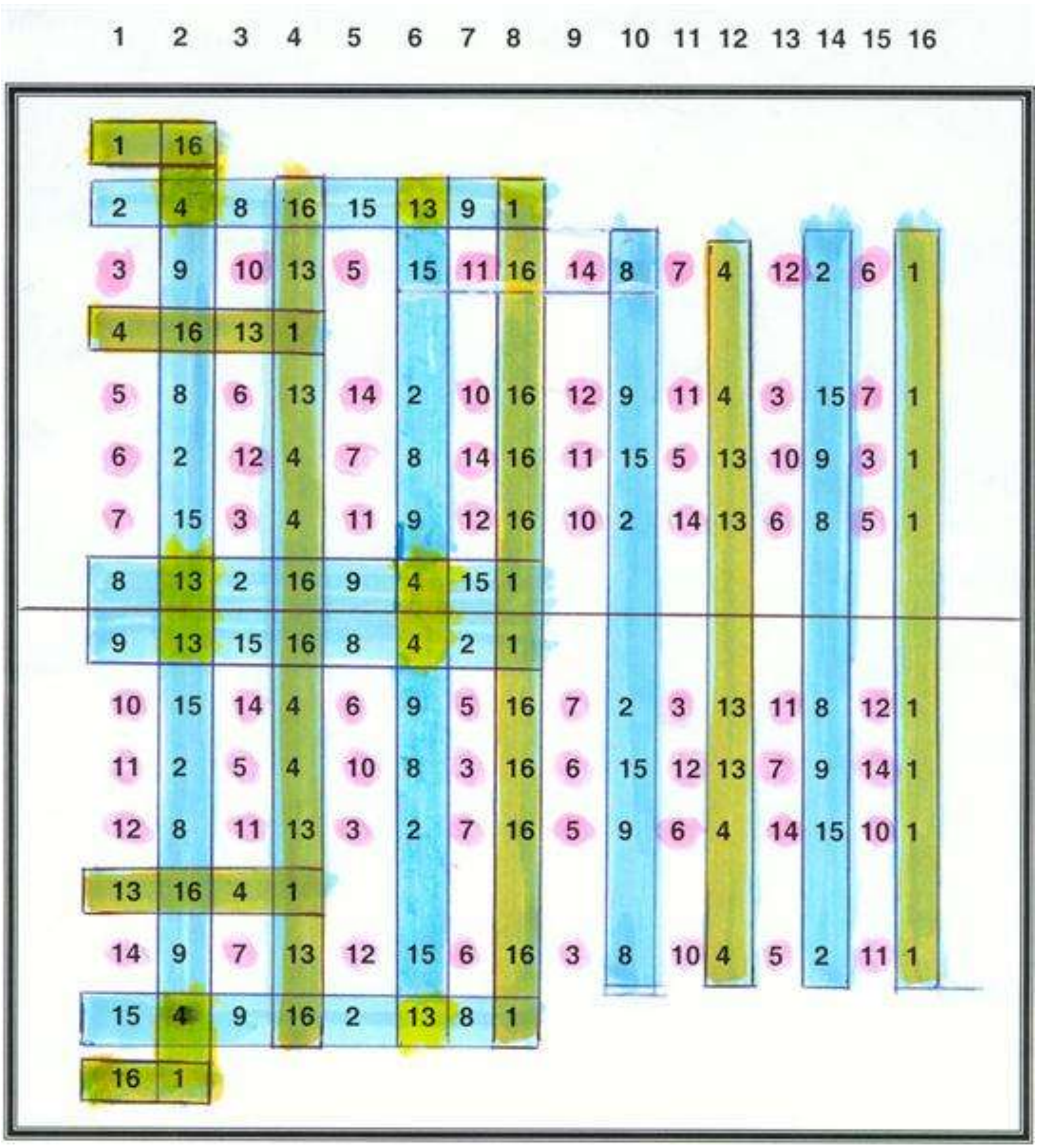
Raphael included that "foreign" outside figure as representing the spectator inside of his painting, because he is the only one who can recapitulate the three different states of mind of the subjects inside of that painting. The spectator represents the uniting principle among the tragic state of the possessed boy's family, the incapacitated state of the Apostles, and the sublime transfinite state of Christ. That is the biquadratic function of the spectator. Therefore, the kneeling figure points to ***what is not there,*** which is what the Apostles are missing in their ability to save the possessed boy. The spectator must therefore discover the blockages which prevent the Apostles from fulfilling their ministry. That's what ***The Transfiguration*** is all about.

Raphael located ***what is not there*** by emphasizing the source of light outside of the painting, which highlights the singularity of this kneeling figure. That source of light is located in the upper left region above the spectators who are standing in front of the painting, as if to impress upon them the necessity to discover that they are directly involved in solving this conundrum of the creative process which is: how do you make an axiomatic change?[20]

Next, look at ***what is not there*** within the distribution of all of the possible combinations of quadratic residues, biquadratic residues, and primitive roots of module 17. See **Figure 7.** Note especially the reciprocity of the entire module. It is your discovering of the ***biquadratic reciprocity*** which holds the process of its elements into a unified whole, as if they were all knitted together from all directions and from opposite ends. All of the residues of the 8 primitive roots are vertically and horizontally reciprocal; and all of the quadratics, biquadratics, and

[20] See Pierre Beaudry **SAINT IRENAEUS OF LYON'S DOCTRINE OF 'RECAPITULATION'**.

primitive roots turn out to be mirror images of each other as if to express the chirality of a living process.



1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
1	16														
2	4	8	16	15	13	9	1								
3	9	10	13	5	15	11	16	14	8	7	4	12	2	6	1
4	16	13	1												
5	8	6	13	14	2	10	16	12	9	11	4	3	15	7	1
6	2	12	4	7	8	14	16	11	15	5	13	10	9	3	1
7	15	3	4	11	9	12	16	10	2	14	13	6	8	5	1
8	13	2	16	9	4	15	1								
9	13	15	16	8	4	2	1								
10	15	14	4	6	9	5	16	7	2	3	13	11	8	12	1
11	2	5	4	10	8	3	16	6	15	12	13	7	9	14	1
12	8	11	13	3	2	7	16	5	9	6	4	14	15	10	1
13	16	4	1												
14	9	7	13	12	15	6	16	3	8	10	4	5	2	11	1
15	4	9	16	2	13	8	1								
16	1														

Figure 7 Eight Primitive roots (pink), four biquadratics (green) and four quadratics (blue) of modulus 17. Note the intervals of eight between all of the reciprocals!

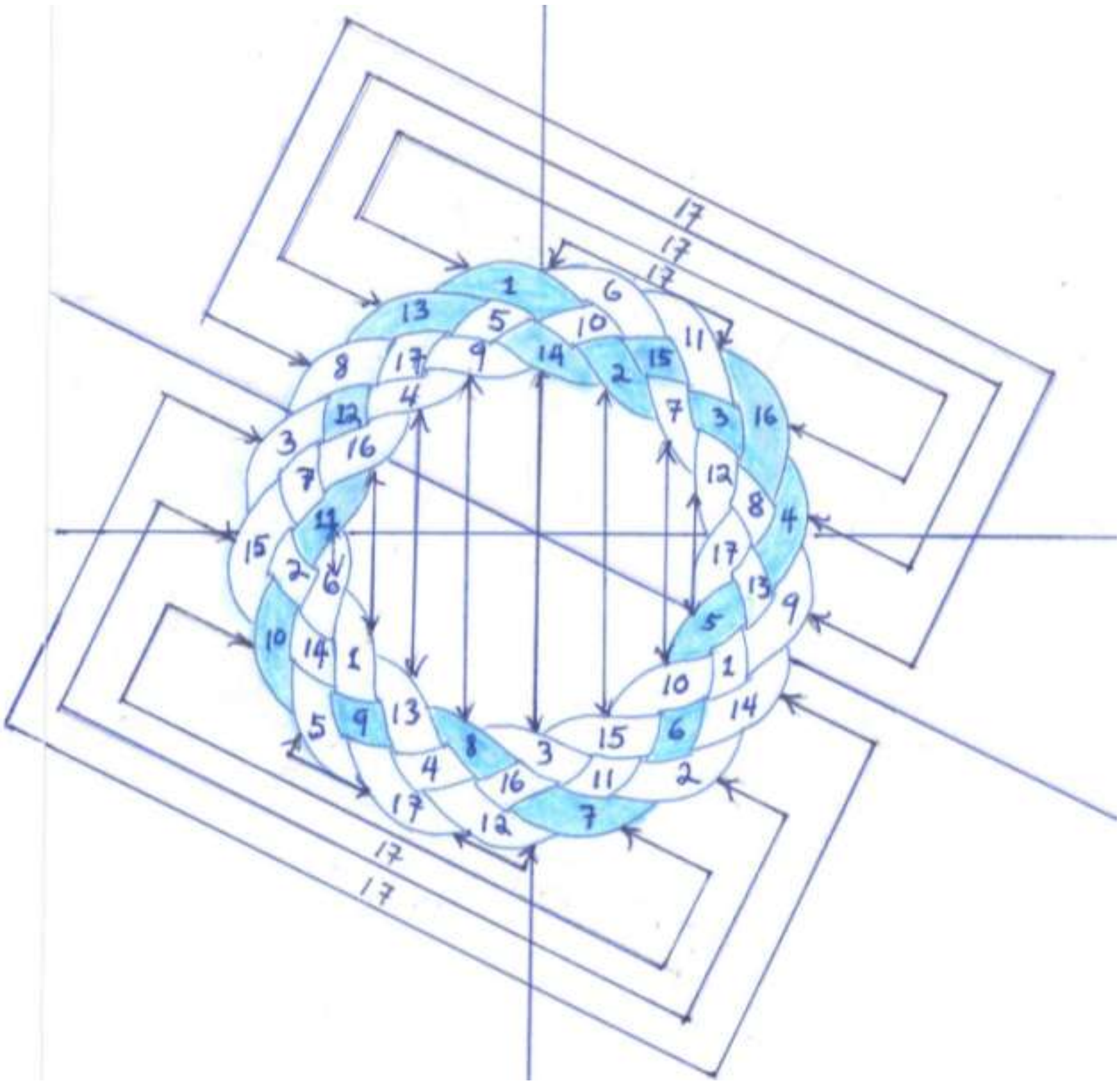


Figure 8 Reciprocals of P/T = 4/17 with intervals of 8 between all reciprocals; clockwise: 4, 16, 13, 1; counterclockwise: 15, 4, 9, 16, 2, 13, 8, 1.The distribution of the four biquadratic residues of P/T = 4/17 shows the amazing reciprocity between 1-16 and 4-13; and reveals the knot they form (in blue) by holding the reciprocity of the entire module.

Lastly, I have included **Figure 9** below as an exercise for the reader to follow the ordering underlying process of ***analysis situs*** of this geometry of whole numbers. Do the following theorem: ***Given a Torus whose P/T ratio is 3/17, (otherwise understood as 3 mod. 17) find by construction alone the 16 continuous modular waves of 3 as the primitive root of 17.***

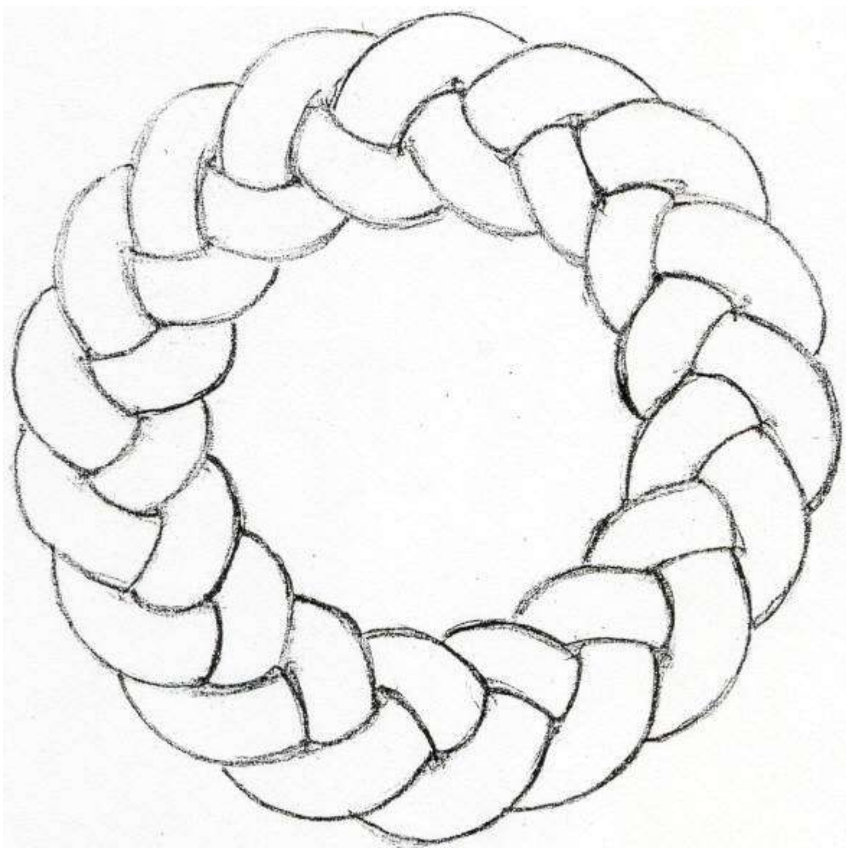


Figure 9 The P/T ratio of a 3/17 torus.

First locate 1 at 12 noon inside of **Figure 9** and fill all of the empty spaces clockwise with two continuous sets of integers going from 1 to 17. Secondly, discover ***what is not there***; that is to say, find the following series of ordered modular waves which generate the following ordering of primitive root residues 3,9,10,13,5,15,11,16,14,8,7,4,12,2,6,1, without relying on any form of calculation whatsoever. For centuries, mathematicians have missed ***what is not there*** because it was not made visible to them as something worth discovering. True, if you don't do the construction yourself, you will never know what you have missed.

Thirdly, let your fingers do the counting around the Toroidal circumference of the donut and let your mind demonstrate to you, by means of ***analysis situs*** construction alone, that the residues of primitive roots can be found without ever calculating them. Let the Poloidal waves express the increasing powers of the process of eliminating the differences between two numbers in such a way that a third is always able to discover where the position of the next residue is located; and that, only from the position of the previous one, as summarized in the Irenaeus method of recapitulation mentioned above. Here, the total number of Toroidal coverings will be P[T-1]/2; that is, 3[17-1]/2 = 24 and the total number of Poloidal waves will be 136.

The wave counting process is the long-handed process of arriving at the next residue. This is done by counting the number of Poloidal waves to be made from the number identified by the previous residue. Thus the "***situs***" of the following residue is already accounted for by the previous one as if it were its generator. However, the short-handed way of arriving at the same result is the shortest route because all you have to do is to count the number of rim-intervals of the 17-sided donut as far as you can go until the unknown residue you are looking for pops up as the one you have arrived at; simply because you have called its number without knowing what it was. That is how ***what is not there*** comes to be.[21]

FIN

[21] Should you have any problems understanding the underlying least action process of these constructions, just write to me at the following email address: pierrebeaudry@amatterofmind.org