



From the desk of Pierre Beaudry



LEONARDO DA VINCI'S {*THE LAST SUPPER*} AND THE CATENARY/TRACTRIX PRINCIPLE



by Pierre Beaudry, 8/16/2009. In memory of Amandus Thoof.

1- THE CATENARY/TRACTRIX PRINCIPLE.

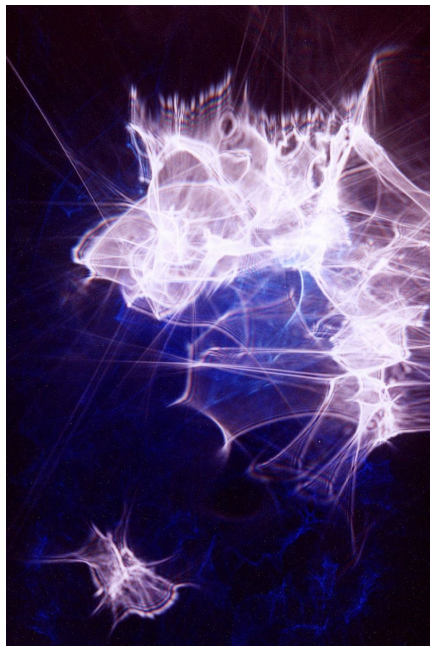
In the original dedication to Duke Ludovic of his book, *Divine Proportion*, written in collaboration with Leonardo da Vinci in 1498, Luca Pacioli described the dynamics of *The Last Supper* as if through the singularity of a catenary/tractrix principle. He wrote:

“It is difficult to imagine a greater form of attention given to the apostles’ animation caused by the sound of the voice of the ineffable truth, when it says: ‘*Unus vestrum me traditurus est.*’ There, the soft hand of our Leonardo has arranged with dignity the moment when, through their actions and gestures, they seemed to be speaking one to another and another to one, in an animated and afflicted state of perplexity.’ (Pacioli, *Divine Proportion*.)

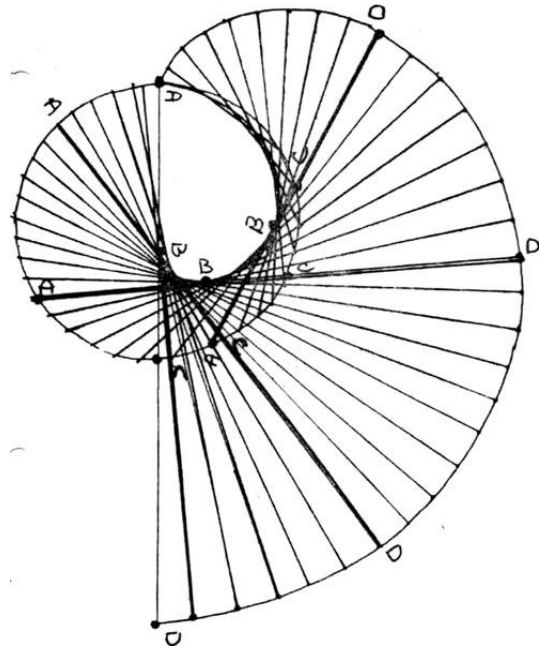
This “*animated and afflicted state of perplexity*” of the apostles represents only the first step of an axiomatic transformation of the human mind, as if generated by the power of an invisible caustic cusp of divine light through which a universal physical principle is being transmitted to future generations: the power of doing something impossible by changing the future. This is the way ideas are created: you project the

impossible, like projecting a skyhook onto a Cusa infinite circle, and then you generate a catenary/tractrix function.

This is the forecasting characteristic of the catenary/tractrix as a function. Do not think of the catenary as simply a static hanging chain, or funicular. That is not what we are looking at here. The physical chain or funicular is merely the end product of gravitation, its resulting effect. What we are looking at is the function pertaining to a universal physical principle. So, accordingly, you should rather think of the catenary and tractrix curvature, together, as the effect of the principle of a constantly changing motion like in a caustic of light in physical space-time. From that vantage point, the physical curvature of the caustic phenomenon represents both an image of the spark of creativity in your soul as the Image of God, and the physical process of a harmonic transformation as a measure of change in least-time. This may sound outrageous, but that's all you need to generate ideas. That is the process of generating ideas that Leonardo caused to happen in *The Last Supper*.



a.



b.

Figure 1 a. shows a multiple caustic. **Figure 1 b.** shows an envelope of developing inversion connecting a geometric curve (circle) and physical curves (caustics). The proportion of the harmonic range of the envelope is 2/1 such that $AD : CD :: AB : BC$.

In *The Last Supper*, each one of the twelve apostles is going through a wrenching moment of developing inversion (Monge's *développante de rebroussement*) resembling the physical measure of change as shown in **Figure 1 a and b**. As in a renaissance movement, the circle is able to grow out of itself into a higher caustic of anti-Euclidean

physical geometry. Similarly, Leonardo is showing how to transform the clinically neurotic emotions of the apostles into a higher state of existence. The emotions that Leonardo reproduced are not pure, but become mixed with the coloration of other emotions that mingle with them to form complex inversion flows of “*reflexive streams*” filled with dissonances. All such dissonances must be resolved into a single and higher unity of effect.

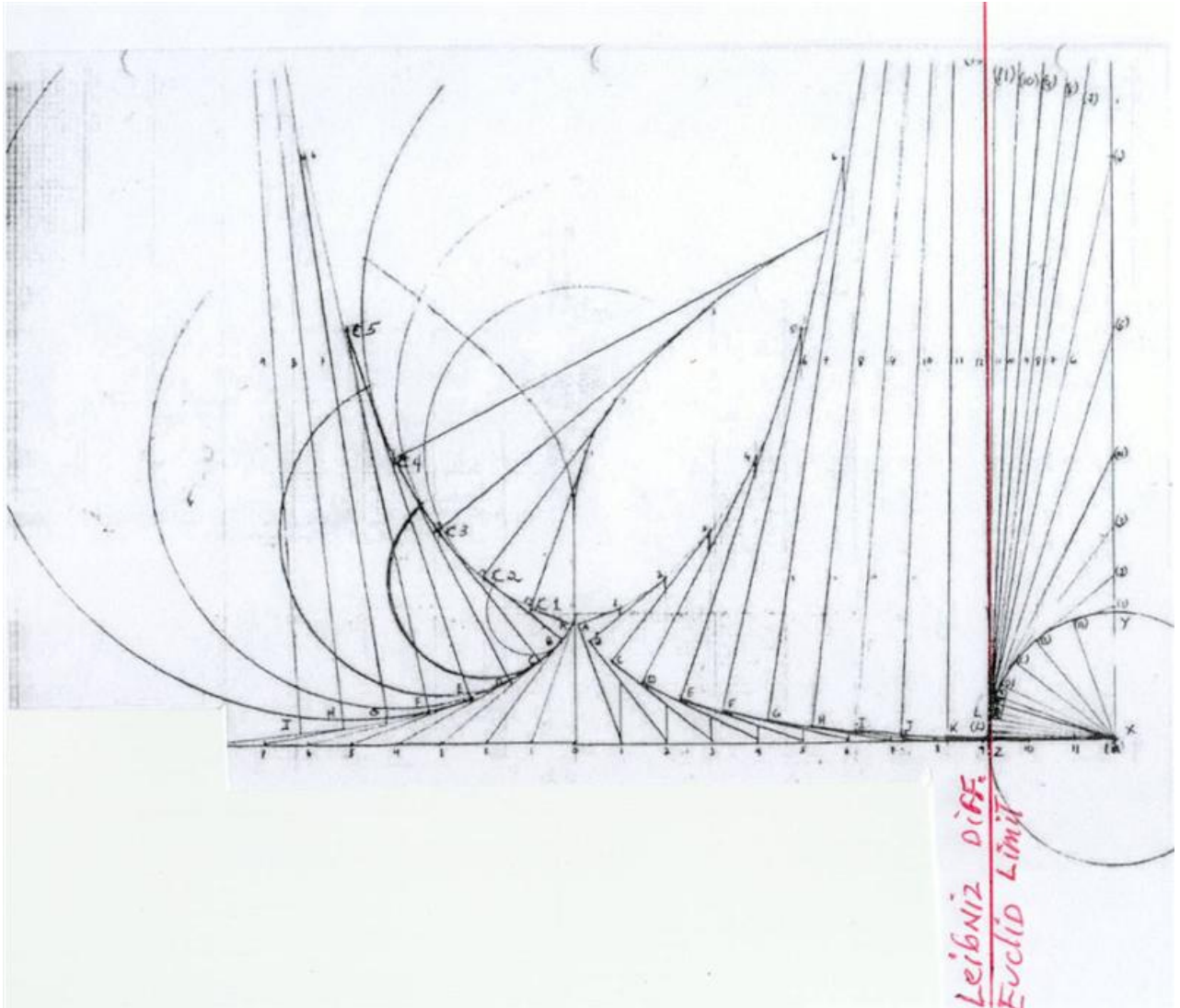


Figure 2. The anti-Euclidean catenary/tractrix function established with the Leibnizian method of inversion of tangents. Note how the tangents and the radii of the circle are parallel to the tangents of the catenary and the tractrix, but they exist in two completely different domains.

At the onset, I wish to make a remark regarding the anti-Euclidean character of both the Leonardo and Leibniz methods of inversion of tangents that I have illustrated here, on the right hand side of **Figure 2**. I have noticed, on several occasions, that some

people had difficulty in grasping the significance of this physical measure of change from the *Euclid limit to the Leibniz differential*. I must say, at this point, that the degree of difficulty in understanding this measure of change is proportional to one's attachment to Euclidean axioms, postulates, and definitions. What Leonardo is provoking us to do with *The Last Supper* is to go beyond the apparent limit of the circle by reaching upward for the optimal condition of development of a new form of existence that strives on increasing outward expansion through optimum inward tension.

The way to resolve this problem of attachment to Euclid is to think of the catenary/tractrix as a future oriented function of change, and that change is the act of building from the future itself, as opposed to from the past; that is to say, from the unknown as opposed to the known, from the imagination of what you wish to shape in the future generations of humanity, in the next 50 to a 100 years, as opposed to what humanity has constructed up until now.

However, you don't break with the past because you want to do something new and original. You break with the past because this past is no longer useful and is becoming destructive for mankind, like the market economy of today. If you think that being creative is being inventive in finding new ways of accounting continuing the present financial swindle of the currently dead monetary system, you are becoming extremely destructive. Monetarism thrives on the fact that raw materials are limited and scarce. However, if you create the conditions whereby raw materials are no longer limited by scarcity, what do you think will happen to monetarism and its insane stock markets?

In substance, think of the least-time function that Lyn is working from. Look at how he packs long sweeps of historical developments into very dense and contracted capsules of duration in his mind. Why does he do that? His sweepings include several centuries of human creativity captured in very condensed statements that give you an overview in which you can browse over the essential characteristic features of a whole series of individual discoveries in a very tense form of encapsulation. Why should you want to do that? Because, the more the mind is tense, in that way, the more universally it is sweeping in the simultaneity of eternity, and the easier it becomes to access the truth of universal physical principles.

Consequently, conceive of this process of the catenary/tractrix function as a projected reflection in which each unit of tense historical time is reflected totally in every other period of time in history, past, present, and future, as well as in the total span of human history as a whole. This means that the whole of human development is contained entirely, as a potential, in each of its smallest historical parts. Therefore, each individual human being is precious, because he or she is necessary for shaping the future history of mankind, and by means of which, in turn, the total span of human history must shape each and all of its parts. Think, for example, that by saving the life of the apparently least important living human being on earth today may bring to the science of medicine a breakthrough that will better secure the future of mankind as a whole. Then, similarly, consider that the totality of human history is reflected in each such individual part at the

same time that each part is a unique reflection of all of the others and their totality; thus, the smallest loss in the smallest part of humanity is affecting the whole future of mankind. That is the least-time social function of Leonardo's catenary/tractrix principle applied to *The Last Supper*.

2- PAINTING BETWEEN THE NOTES

In his 1999 paper *On the Subject of Education*, Lyn identified how the principle of irony in Classical artistic composition, as expressed in the form of "*playing between the notes*" by the great conductor, Wilhelm Furtwangler, had also been captured by Leonardo da Vinci, in his timeless masterpiece, *The Last Supper*. Relating this to his own early crucial discovery of 1948-1952, Lyn recognized that such a discovery of principle also pertained both to the social nature of man and to the scientific domain of human knowledge. The real challenge, then, was to make the cognitive connection between a discovery of principle in Classical art form and a validated scientific discovery. It was Leonardo da Vinci who was to provide that cognitive connection by developing the social process of that discovery in *The Last Supper*.

In this challenge, the discovery of principle of Leonardo is pedagogically of momentous importance, because it represents, as Lyn identified, a unique historical example of a discovery of a validatable universal physical principle that unified both of the domains of art and science. That was the challenge that Lyn had put at the center of his education policy. Therefore, I propose to take up Lyn's challenge and attempt to relive the discovery of principle of Leonardo with you, following Lyn's method of connecting the dots, as he proposed.

*"The central issue of this presentation, the focal point, is, 'How should we connect the dots?' With that focus adopted, the congruence of Classical Art and science is made, quite properly, as immediate as possible. The Classical Greek development in sculpture, as compared with the same principle better expressed in Leonardo da Vinci's The Last Supper, typifies the intrinsic non-linearity of the connections which Classical art, like science, makes among the 'dots'" (Lyndon H. LaRouche Jr., *On the Subject of Education*, EIR, December 17, 1999.)*

As Lyn pointed out, you have to imagine yourself moving about inside of the large refectory room of the convent of Santa Maria delle Grazie in Milan, where the Leonardo fresco is located, and relate to it as an expression of the *living in-betweenness* of the *bel canto* method of voice teaching that Leonardo was promoting during the Renaissance, and whereby the principle of the register shifts coloration of a composition by the six adult human voices, here connected three by three, became the basis for all future musical or plastic forms of classical artistic composition.



Figure 3. Refectory of the Convent of Santa Maria delle Grazie, Milan.

Note the paradox in *The Last Supper* which is expressed by dissonances among the four groups of three apostles. Note all of the invisible receding lines of linear perspective all converging on the caustic head of Christ with the purpose of establishing a principle of continuity/discontinuity between the painting and the dining room of the monastery. The illusion of perspective on the wall creates an ironic connection between the two dining rooms. In fact, this is a polemical provocation on the part of Leonardo. He

created a paradox by illustrating the most turbulent dinner located in a perfectly silent monastic dining room. Remember that the monastery rule of the refectory of Grazie was to have the monks eat their meals in complete silence. Furthermore, since the refectory tables were two similar long tables facing each other with monks sitting with their backs against the walls, everyone could see the receding lines under the windows of the opposite wall perfectly aligned with those on each receding wall in the painting. This is the way Leonardo chose to invite the spectator to get inside of his fresco.

Think of the discovery of principle of Leonardo as being an early form of a well-tempered musical modality applied to the domain of plastic arts. Leonardo applied in this fresco the same principle of emotional dissonances of the human voices that appear as anomalies and paradoxes, as expressed in the *St. John Passion*, and the *St. Matthew Passion* later composed by J. S. Bach. A careful study of both those musical compositions and *The Last Supper* reveals not only the presence and the treatment of the same emotions, but the harmonically conjugated presence of the same scientific principle at work as well.

The hypothesis for this pedagogical exercise, therefore, will deal with the lawful ordering of a single universal physical principle of least-time as applied to both the domains of science and of classical artistic composition, simultaneously. I remind you of the specific insight that Lyn provided with reference to Wilhelm Furtwangler and Leonardo da Vinci in this respect. Ten years ago, Lyn wrote:

*« The greatest orchestra conductor of the Twentieth Century, Wilhelm Furtwangler, described his method of conducting, as « performing between the notes. » Leonardo da Vinci, centuries earlier, identified the principle of composition, in painting and plastic art generally, to the same effect. The issue is the same I raised above, in summarizing the significance of the principle of « least time ». What we define as distinct sense-impressions, may each really exist as sense-impressions, but one must not make the mistake of « connecting those dots » in a simply deductive way. This warning, against deductive modes for purporting to « connect the dots » applies as forcefully to art as it does to a mathematical form of physical science. (Lyndon LaRouche, *On the Subject of Education*, EIR, December 17, 1999. p. 29.)*

The point that Lyn was making is that the dots to be connected should not be connected between what looks like and sounds like different sense perceptions. The points to be connected are between non-linear domains and pertain to how universal physical principles connect things together, universally, in your mind. So, you cannot find these types of connections by simple imitation of nature. Imitation, or curve fitting, is a fallacy of composition, as Plato argued the case against the poets who used imitation in Athens, *mimesis*, in their form of artistic composition. And, at any rate, nature, as well as artistic compositions, always have more to give than geometry has to offer, not to mention the stupidities of analytical geometry, mechanical modeling, or benchmarking. Therefore, classical artistic composition is essentially in dynamical opposition to mechanical imitation or copying of natural phenomena. Human beings are not monkeys.

Classical artistic composition, as Leonardo understood it, is intended for the elevation of the soul, not the imitation of nature. This is how Leonardo broke away from the central point-perspective of the early Renaissance. Leonardo wrote the following recommendation for artists to render the intention that is in the mind, as opposed to the imitation of the form that is in nature:

"Represent your figures in such action as may be fitted to express what purpose is in their minds...A picture, or rather the figures therein, should be represented in such a way that the spectator may easily recognize the purpose in the minds by their attitudes...The hands and arms in all their actions must display the intention of the mind that moves them..." (The Notebooks of Leonardo Da Vinci, Oxford University Press, 1952. p.185 and 222)

The point is to pay attention to the intention and to the purpose that is in the mind of the artist. From that vantage point, for any classical artistic composition, it is the intention that counts. It is the intention of the painting that makes visible the thought process that is not seen; just like the musical intention makes audible what is not heard. From that vantage point, think of the principle of composition of Leonardo as having the purpose of replicating the intention of a well-tempered musical modality of the human mind caught in the drama of human life.

The great dissonant moment to be captured in Leonardo's *The Last Supper* is a great anomaly, that is, the irony where one has to turn one's mortality into immortality. Look at the state of minds in the body language of each and all of the apostles and you cannot fail to realize that you are summoned by Leonardo to inquire about what great event has caused them to react in this way. This painting is an explicit physical expression of the process of creativity. The brush of Leonardo is not depicting self-evident visual forms in themselves, but the reflexive shadows of the apostles' thinking processes caused by the central presence of Christ. Leonardo is making visible the invisible process of causality, both the cause and its effect. He is showing you what could never be seen with physical eyes: the catenary/tractrix process of a shockwave tension.

The underlying method of both the musical composition of Bach and the plastic art composition of Leonardo reveals the presence and the treatment of the same principle of *least-time* of Huygens, Fermat, Leibniz, and Bernoulli which is generated by means of the catenary/tractrix whose unity of effect results in the interplay and resolution of the well-tempered dissonances in the composition. This is the way Lyn related the domain of music with this fresco:

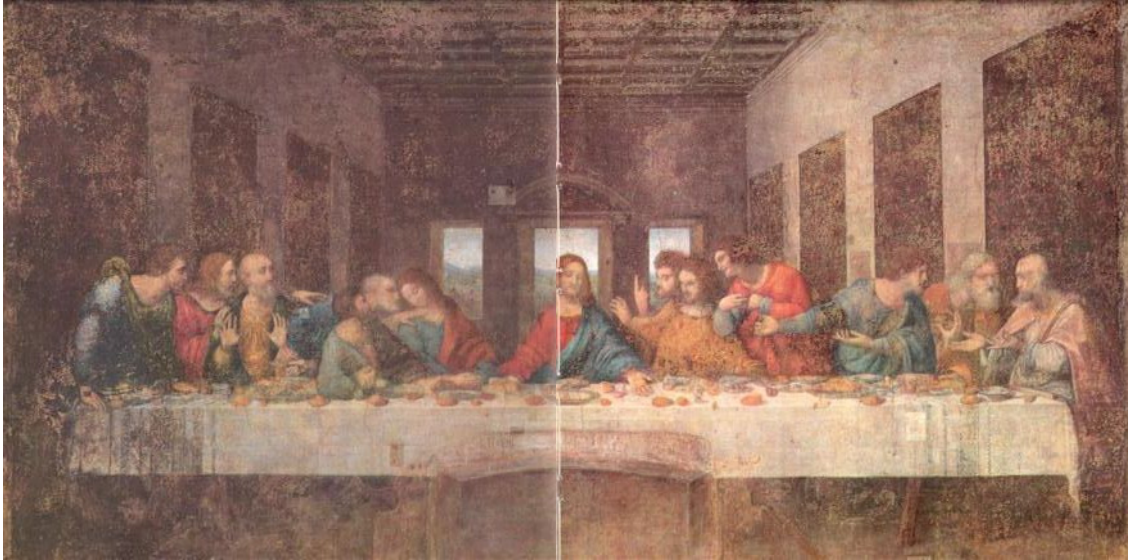
« The developmental principle characteristic of a Classical musical composition, is a nested set of ironies, which converge upon a single, pervasive metaphor. Each of these contrapuntal ironies has the quality of a necessary dissonance to be resolved. One must see the dissonance in this case not as some arbitrary dissonance, but as reflecting the same principle of irony underlying the Classical method of sculpture associated with Scopas and Praxiteles, and Leonardo's {The Last Supper}. It is not dissonance in the sense of falseness, but

dissonance in the sense of a true ontological paradox. Just as a validatable discovery of a universal physical principle resolves the valid dissonance we see as an ontological paradox, so a great Classical musical composition defines a subsuming musical –ontological paradox, whose solution is the identity of that composition taken as an individual whole. » (Lyndon H. LaRouche Jr., Op. Cit., p. 33)

Now, take a look under the table. Note how Bartholomew, on the extreme left, is rising on his toes, while Simon, on the extreme right, is digging in his heels. That is a very telling anomaly at the two extremes of the fresco. That tells you that Leonardo intends to show the spectator the dynamics of the push and pull motions of flexion and extension characteristic of the catenary/tratrix function. The challenge is to discover the dynamics between those two extremes in order to find the underlying unity of the composition.

Think of the whole scene as a minimum-maximum experiment as Nicholas of Cusa would consider. Look at this ensemble as the framework for a Classical composition of polyphonic counterpoint in which the unity of effect of the whole composition is caused by Jesus Christ, at the center of the field-perspective as opposed to central-point-perspective, and whose action is reflected by each individual within a group of three individual voices, as does a *complex of reflex streams* propagating at different speeds and degrees from the same thought-object. The same *Geistesmassen* responding to the caustic statement made by Christ is already anticipating Gethsemane in the simultaneity of eternity. At the same time, Christ, as the subsuming central figure, internalizes the drama that everyone will be living in the next days by reflecting all of the voices back to a higher unity of transformation, as does a *caustic of developing inversion*.

The question of the unity of composition of *The Last Supper* is really the very first thing to identify, if one is to understand the organizing principle of the painting: how do you explain the explosion of emotions that Leonardo has painted in this dramatic scene? *The issue, here, is really: how do you relive the principle that masters and unifies all of the different clinical emotions that Leonardo has portrayed in this apparent explosive shock-wave reaction? In a paradoxical way, the answer to this question can only come from understanding why Christ is apparently so calm, while the apostles are so apparently agitated in different space-time capsules, why is Bartholomew so jumpy while Simon is in such a state of denial?* That is the central catenary/tratrix anomaly of the whole scene, which begs the question: What must have happened that produced such an apparent opposite paradoxical effect between Christ and his disciples? Somebody must have said or done something to cause such a commotion. What was it? In his notes, Leonardo referenced what Matthew had written in his Gospel; that is, the drama that unfolded immediately after Christ uttered these very emotionally charged words to his apostles:



**Bartholomew,
James the Minor,
and Andrew;**

**Judas,
Peter,
and John;**

Christ

**Thomas,
James the Major,
and Philip;**

**Matthew,
Thaddeus,
and Simon.**

Figure 4. Leonardo da Vinci, *The Last Supper*. 1495-97. (Before restoration)

«‘Verily I say on to you, that one from among you shall betray me.’ And struck by a profound consternation, each one of them asked him, ‘Lord, is it I?’ He replied: ‘ He who has shared this meal with me, shall betray me. The Son of man is leaving; as it has been written of him. But, woe betides he who has betrayed the Son of man! That man would wish he had never been born. ’ »
(Matthew 26; 20-22)

One must fully internalize these terrible and truthful words in order to grasp the shock that must have been produced within the assembly of the twelve. It is that very unsettling moment of transformation between mortality and immortality of the passion that *The Last Supper* is capturing before it actually unravels. This is a forecasting moment in which Christ is announcing a profound change for which He must make his apostles ready. The moment can only be characterized by a tense duration that encapsulates the significance of the whole history of mankind. The question it poses is: what will be your contribution to the immortality of mankind? What will your having lived leave behind for mankind? It is also for that reason that I have chosen to reproduce, here, a copy of *The Last Supper* as it was before the recent restoration, because the so-called recent “restoration” actually destroyed the intention of Leonardo by presenting a sense-perception version that supports the insane fiction of the *Da Vinci Code*.

What Leonardo is conveying to his audience in this sublime event is the experience that each of the twelve apostles lived at that moment, as the truth of those words hit them and forced them to recoil from it, as if they had been projected between two contradictory states: “*Lord, is it I?*” and “*It cannot be me!*” This irony is the

dramatic life and death question of the political organizer of a Renaissance, the difference between the tragic and the sublime, the dramatic paradox that a Renaissance man like Leonardo da Vinci must recreate consciously as the active principle of his daily activities in a period of crisis. From that vantage point, *The Last Supper* is a mirror reflection of any true revolutionary movement!

Here a barrier is being broken through, somewhat similar to breaking through a sound barrier. Leonardo is breaking away from all of the previous treatments of the Last Supper up until his time. He daringly broke with all of the axioms, postulates and definitions of central-point-perspective in order to concentrate exclusively on the caustic mental intentions of real human beings expressing the crisis of their time as a group as opposed to individually. Leonardo composed his caustic-field-perspective as a group function in the manner that Lazare Carnot had described as “*generating ideas by means of the senses, of acting on the soul by the organ of vision.*”

Now, from that non-linear perspective, look at the group as a whole, then at the sub-groups of three, as having the function of responding to that single action of Christ. The dynamics is not one on one, but three on one, in which the social group is the unit of action, not the individual. Here, each apostle is required to go from a lower manifold to a higher manifold through a group dynamic. It is the group that makes the jump representing the resolution of all of its dissonances into the unity of the composition represented by the sublime serenity of Christ.

The task of Leonardo was not merely to reproduce the mental anguish of the apostles. He treated every single apostle as a dissonance by means of which a well-tempered composition could become a standard of truthful artistic composition, as a higher form of resolving such dissonances. In other words, Leonardo was creating a scientific revolution in which the series of expressive human reactions, depicting each and everyone's response as a visible form of the invisible state of mind of that existential crisis, was merely a first step in representing the process of creativity. But that is not the subject of the painting. The sub-groups have to be transformed into a higher domain, where the passing from a lower state to a higher state of existence represented the resolution of their dissonances into a higher unity of effect of the composition represented by the dramatic serenity of Christ, or the domain of the true reality of principles.

That is the true intention of the aerial or field- perspective which, as Carnot said, in his class at the Ecole Polytechnique, can only be grasped by the sentiment, that is, cognitively. This is to be conveyed as if a shock-wave were to have come from the dynamic yet peaceful center where Christ is sitting, with His arms peacefully extended and His eyes lowered in the serene acceptance of what is to become of His mortality, and had traveled simultaneously to the two ends of the long table, hitting every apostle differently, and resonating off of each of them, in a polyphony of dissonances; and from which all are recoiling, back to Christ and to the spectators in the refectory room. Like the drama of a Classical Shakespearean play, Leonardo's *The Last Supper* forces the spectator to relive this drama on the stage of his own imagination, and to come out of the

experiment a better human being than he was before entering that room. So, you have to look at *The Last Supper* as having the impact of such a drama.

But first, look at the Furtwangler-like hands of Leonardo. Imagine Furtwangler's hands, as he conducts a chorus and orchestra at the precise moment when he summons the voices and instruments to replicate a change in voice registration that the composer has written into the musical score. Study how he leads the instruments to sing as different human voices, and relive the human emotions of phase changes, only by the slightest motions of his hands, in the way that Lyn had identified as the crucial characteristic of "*playing between the notes.*" A good conductor knows how to transmit to the musicians the singularities of a classical composition by very specific movements and expressions of his hands, of his face and of his entire body. This is how he conveys to the musicians of the orchestra and the singers of the chorus the moments when they should bring out, softly or dramatically, the emotional idea of change and transformation from inside of the composition. Leonardo did the same with the intricate interplay of all of the twenty-four hands of the twelve apostles and the two hands of Christ, as if they were dots to be connected non-linearly, three by three, as he is painting between the notes.

Now, look at the drama. Take the four groups of apostles, one by one, starting from the left, and begin to consider this complex *Geistesmassen* as a drama of artistic composition. Thanks to some of the notes that Leonardo recorded in his notebooks, we are able to confirm how he was, indeed, showing us that he was painting a tragedy between the notes. Note that the in-betweenness of each character is reflected by the ambiguity of the emotions as expressed in the faces, the hands, and the body motions. Each apostle is both pulling back and pressing forward at the same time.

In the first group on the far left are identified Bartholomew, James the Minor, and Andrew. Bartholomew is so shocked by the announcement that Christ just made that he sprang up from his chair to the tips of his feet. Is he reacting out of disbelief, as he puts his two hands firmly on the table to better see if Jesus will identify the culprit by name, or, is he so shocked that he is springing out of fear into a flight forward? Next to him, James the Minor, with his right hand on Andrew's right shoulder, as if to say: "Don't worry, He doesn't mean us," while he reaches out, at the same time, to touch Peter's back with his left hand, as if to find out what Peter is saying to John. This is the tension of "in-betweenness," of painting between the notes. Here, Leonardo wrote a note saying: "*Another (Andrew) showing the palm of his two hands, shrugs his shoulders up to his ears, making a mouth of astonishment.*" Is Andrew denying the whole affair by showing his clean hands, or is he simply saying: "Don't look at me. This is the first time I hear of this." How do these emotions, flight forward fear, quiet resentment, and astonished denial, get resolved within that group?

In the second group are found Peter, Judas and John. Start with Peter and note how his left hand is barely touching John's right shoulder, while reclining towards him, as if to whisper something in his ear, like: "Of course He doesn't mean you. You are his most beloved from among all of us." At the same time, Peter has his right hand twisted behind his back, firmly gripping a knife, as if to say in a vengeful tone: "I am going to

kill anyone who tries to harm Jesus.” Remember that Peter was the hot headed one who sliced the ear off of one of Christ’s assailants later that same night. Again, Peter is caught in the “in-betweenness” of two different and opposite emotions. Meanwhile, Judas is paralyzed with fear, and is in a state of total stupor, realizing that Christ has just put his case on the table. Here, Leonardo broke with the tradition that always isolated Judas on the other side of the table. He chose to integrate him among the twelve in order to reflect the fact that all of the apostles have the potential for treason. Fearing that he is about to be discovered for his treason, Judas is hanging onto his purse as if dear life depended on it, and his left hand is about to steal a piece of bread before he attempts to make good his escape. But, is he staying or is he fleeing? It’s not clear. Next, look at John. He is an inverted image of Christ. He is dressed in the same manner as Christ, except the left and right colors and shoulder garments are inverted. His face is in a state of intense, yet calm meditation, with his hands resting in a prayer-like fashion on the table, as if to reinforce the idea that he is the only apostle who has absorbed the shock of the announcement, by having internalized, with *agape*, the suffering of Christ himself. All three disciples of this grouping are complete dissonances and inversions with respect to one another. Their bodies are so filled with these emotions to the point that they physically deform them. How do you untangle these contradictory states that are dormant in all of them? How do you resolve the anomalies between Agape, Treason, and Revenge, all at once, and without smoothing them over?

The third group of Thomas, James the Major, and Philip is in complete turmoil. Here, the shock of the words “*One of you shall betray me*” has resonated in such a way that Thomas has left his seat to find himself standing behind James the Major. He points the index finger of his right hand upward in a very ambiguous way. Is that the inquisitive finger that will later empirically verify if Christ was really resurrected? Or is it raised to ask the question: “Do you mean me, Oh Lord?” Or is he asking: “Oh Lord! Was this not prophesized?” Then, James the Major, in front of him is pushed back by the shock and extends his hands as if to say: “Oh Lord! How can you say something like that?” On the other hand, he could be thinking the exact opposite: “Maybe I will be the treasonous one, and I don’t yet know it.” The situation remains ambiguous. His emotional state could either be that of outrage or of anticipated guilt. On the other hand, Philip is bending forward with his hands turned inward, in a manner that Leonardo described as “*morbidissimo*,” a state of extreme morbidity that forced him on his feet in a completely guilt-ridden state. All three are unresolved as a group, reflecting different stages of doubt, outraged guilt, and morbidity. But, how do you resolve this group dynamic?

The fourth group of Matthew, Thaddeus, and Simon, has almost entirely folded back onto itself, as Leonardo continues his clinical study of the whole process. At the extreme right end of the table Simon appears to be in a total state of shock. He seems to be saying: “This does not make any sense at all. How can this happen in my own house?” And Leonardo added a note saying: “*Another ... turns with stern brows to his companion.*” In fact both Matthew and Thaddeus seem to turn their own fear into anger against Simon by saying: “Hey! Cut it out. Stop blocking on this. You have heard what the Lord said. This is a tragedy.” However, they may also be saying something quite different, like: “Look, you are the master of this house. Call your servants to this table.

Could it be that one of them is the traitor?" Again, the situation is uncertain. It is like the undecided quality of the Hamlet question: "*To be or not to be?*" Every one is caught in mid-motion as if Leonardo had captured the Praxiteles emotional quality of all of the apostles' minds at the most vulnerable moment when their very existence was being challenged.

Now, think of this whole dynamic process as a passionate organizing process in which each of the clusters of what could be called groups of *thought-emotions* are all expressions of a negating process; that is to say, a process in which Leonardo demonstrated how the paradoxes of well-tempered dissonances negate the system in which they have been introduced and from which they must break out. None of the individual apostle understands what is happening to him because he has no significance outside of the group that is acting as the unit of action of the whole dynamics. The question posed is personal, but the answer to the question is social. Since everybody is interpreting the event individually, from the standpoint of his own tragic experience of the past as opposed to what the future holds as a solution, each apostle has been inserted in a group in accordance with a profile of clinical neurotic behavior inside of which the group of three must resolve the crisis at the limit of their boundary conditions. Each individual of each group of *thought-emotions* has to find a social reality principle from which he must stop negating individually and find a proper resolution to the existential crisis and break out of his tragic fishbowl containment.

Now, let's look at the intervals between the groups as if you had pebbles dropped in a pond of calm water. The different waves do not interfere with one another, but the entire field is changing by the action of the transversal waves. On the other hand, the central figure of Christ is isolated from the rest of the group in order to underscore the solitude of leadership and the silent acceptance of having to bear the responsibility of all of the sins of mankind. "*All of you will fail me!*" Christ said to his apostles the next day. Thus, the four groups of three are distinctly separated and united by similar emotional barriers.

But here, suddenly something very singular is happening. It seems that the different individuals of each group are incapable of understanding what is happening inside of the group as a whole, or inside of other individual groups. It appears that each apostle can only perceive his own dissonance with respect to the group he belongs to, and inside of which he resonates. In other words, only Christ seems to be capable of having a sense of the unity and totality of the event, which also includes the foreknowledge of what each and every one of the twelve apostles will do, or will not do, in the Garden of Gethsemane and during the following two days of the Passion. In that sense, Leonardo had the same sort of foresight that Furtwangler had before beginning interpreting a symphonic composition. He had the entire finished painting in his mind at the very moment before he gave his first brush stroke.

This is the social power of dynamics that Lyn has identified as the power relationship of the individual voice and the *field of change* that the composition as a whole represents in its progression. Each voice produces on the total outcome of the

composition a definite effect in the form of a potential, as from the Dirichlet Principle, but without really seeing how its role would be integrated into the whole, nor how it would effect change in that whole. The effect of the individual voice is masked to itself within the chorus, and the individual singer can only hear the effect of the polyphony on his own local region where he is performing. I refer you to this note on Furtwangler that Lyn wrote a few years ago:

“For example, what conductor Wilhelm Furtwangler sometimes identified as performing between the notes. In a Classical polyphonic work of many performers, unlike the case of the accomplished string quartet, the individual performing voice does not hear the functional interaction of his or her own voice within the array of voices as a whole. What is heard is the impact of the polyphony upon the volume of the region in which the work is performed and heard. This is heard not as a collection of voices, but as a field, as I have identified the notion of a field in reference to the case of Kepler’s principle discoveries and Dirichlet’s Principle. The exceptionally able conductor, such as Furtwangler, hears the whole in a way which the performers do not, thus, seeing and shaping those subtleties which craft the effect of the field of the performed composition, in that acoustical setting, as a sensed indivisible whole.” (Lyndon H. LaRouche Jr., *SCIENCE: THE POWER TO PROSPER*, Morning Briefing for April 16, 2005.)

In other words, what Leonardo has executed in *The Last Supper* is the same type of *field principle of organizing*. The dynamic principle is a powerful means of shattering the fishbowl domain of socially accepted public opinion. The dissonances that an individual voice seems to be expressing as a personal choice in this scene are not free expressions of individual taste, or of their independent thinking. They are the expressions of social axioms being broken. What Leonardo painted was the relationship between the visible behaviors of human beings as they are defined in and by a social environment of their culture, and the invisible principle unifying their reactions in the real world. This means that it is not the particular perceived individual that counts, nor the specific “fishbowl” he belongs to that is relevant, but the significant change of intervals of their relationships that the field-perspective connects among them between the dots. The apostles relate to one another with respect to Christ, their unity of principle. Such is the manner in which social functions behave within classical artistic composition. What Leonardo painted was not the individual reactions, but the relationship between characteristic clinical behaviors of social interactions and the invisible principle underlying them within the real world. That is an artistic expression of Leibnizian dynamics.



Figure 5. Leonardo. Pastel study of the head of Christ for *The Last Supper*. 1495.

So, the question that arises is: Since all of the four groups of apostles can be identified as negative clusters of neurotic cross-voices, how can all of these voices find the only social way out of these predicaments? How can you find a positive inversion against fear, resentment, denial, revenge, etc? Leonardo has developed the expression of *agape* on the face of Jesus for that explicit single and higher purpose. Unfortunately the deterioration of the Christ figure of *The Last Supper* is too advanced to show the extraordinary mastery of Leonardo on that account. However, there is a pastel drawing

study of a Christ head that Leonardo made in preparation for the fresco which reflects the required sublime agapic quality that Leonardo intended. (See **Figure 5.**) Note how this fundamental human emotion is expressed, softly and serenely, especially in the treatment of the shadows in and around the eyes and the mouth of this figure. Leonardo chose to paint Christ beardless, and without eyebrows and eyelashes, in order to better emphasize His divine state of mind.

Here, Leonardo is deriving his artistic conception from a real life Platonic notion of power; that is, the power of love and justice, the power of *agape*. Ultimately, what *The Last Supper* represents is the power of making an axiomatic change from a neurotic state of interpersonal relations to an agapic social state exemplified by Christ. In an amazing way, Leonardo showed how the solution could only be found in the discovery of the universal physical principle of love of mankind, the same principle that Cusa had used in the Council of Florence in 1439, and that Mazarin had established as the principle of the *advantage of the other* in the Peace of Westphalia of 1648.

3- THE FIELD-PERSPECTIVE OF LIGHTS AND SHADOWS.

Leonardo was the first to follow in the footsteps of Nicholas of Cusa and to develop the principle of least-time that was to become firmly established scientifically by Pierre de Fermat two centuries later. During the beginning of the 1480's, Leonardo sought the application of this principle for every vibrating phenomenon such as water, sound, light, shadow, color, odor, heat, magnetism, gravitation, etc. As he stated: "*Each natural phenomenon is generated by the shortest pathway.*" (*Anatomy Notebook*, IV, fol., 16 recto, and *Cod. Arundel*, 1508, fol. 85 verso)

The quest for such a principle of least-time is, itself, very important to establish historically and should be put in its proper perspective, because history is the laboratory where discoveries of principle incubate and need to be discovered "*in time.*" With Leonardo, this process started early with the study of physical phenomena of traversal wave motion and vibration (*vibrazione*) in water, which he successively applied to sound, to light, and to the idea of gravitation.

"What must be investigated are not only the quantity of vibrations (vibrazione) and their measure, but also the quantity of vibrations in sounds, in weights, in the seasons, and in the regions inhabited by planets and every other form of harmony." (*Codex K*, Institut de France, fol. 49 recto.)

«I say: if you throw at the same time two little pebbles sufficiently far from one another into a calm pool of water, you will see appear around the two shock points two series of circles, which by expanding will penetrate one another, while their center struck by pebbles will remain the same. Even though it appears to be moving at that moment, the water is not displaced at all, and it is these sort of little wounds which by opening and closing suddenly, impresses

upon it a certain reaction that owe more to trembling than to motion. In order to better understand this, think of debris that float on water and which do not change their places regardless of the little waves formed by the said circles that cradle them. These circles cannot be broken when they incorporate each other because water is homogeneous in all of its particles, this sort of trembling will be transmitted from one to the other of these particles without any displacement of the water itself; because water remaining in the same place, can easily receive the said trembling from the neighboring particles, as it can communicate them to the following ones, with a rhythm that goes on gradually decreasing to the end.” (Codex Atlanticus, Institut de France, (fol. 61 recto)

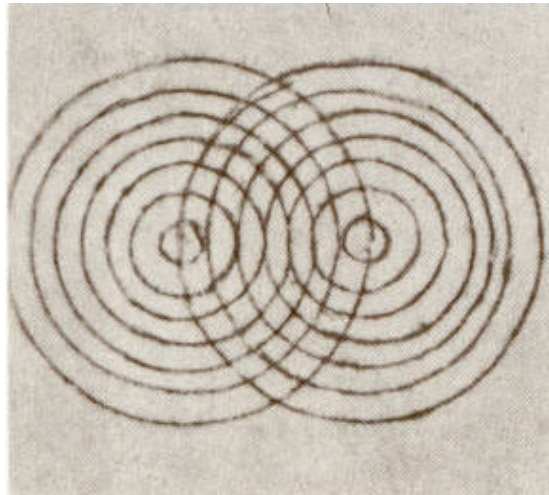


Figure 6. Leonardo drawing of two pebbles forming transversal waves in water. (Ms A, fol. 61 *recto*.)

For Leonardo, not only vibrations in water, sounds in the air, or light in space, but also every physical and mental phenomenon, including thinking processes, are propagated by the same universal law throughout the universe and cause everything to change in the universe. The most extraordinary part of this Leonardo discovery is his conception of radiation which he identified with Dante’s last vision in the *Divine Comedy*. As if to underscore the fact that he was identifying his principle with Dante’s discovery of a caustic image of God, Leonardo reproduced the last stanzas of the Dante poem in its entirety in *Cod. Atl.*, fol. 138.

*“Although my sight, as I gazed in this way,
Was growing stronger, a sole appearance
As I was changing, was varying with me:*

*In the deep and luminous subsistence,
From this higher light appeared three circles
Three colors in a single dimension.*

As rainbow is to rainbow, one circle

*Seemed reflected by a second, the third
A fire breathed by the other two.*

[...]

*As the geometer intently seeks
To square the circle, but he cannot obtain,
Through though on thought, the principle he needs,*

*So I investigated that strange sight:
I wished to see the way in which our own image
Suited the circle and found place in it.*

*But my sole wings could not have sufficed,
Unless my mind had been struck by lightning
That responded to what I wished to know.*

*Here, my elevated imagination failed me; but my
Desire and my will had already been moved
As if through a wheel revolving uniformly*

*By the love that moves the sun and the other stars.”
(Dante, *The Divine Comedy*.)*

Here, what is clear is that Leonardo was calling upon the strength of Dante to inspire him in reaching the highest form of generalization in the discovery of a universal physical principle. On the same page, Leonardo summoned the underlying principle behind all of the wave phenomena. And then, he went a step further, reaching beyond the sense-perception of vision into the physical phenomena accessed by other senses. He first went from water, to sound, to light, and then, as Dante did, to the domain of mind. Then, he came back, reflexively, to the idea of universal radiation, into the non-visible forms generated by heat, magnetism, planetary attraction, and even perfumes. Following the principle of Dante, Leonardo wrote:

“All bodies emit rays. The sun is the proof of that; it produces two types of phenomena: light and heat... The air attracts to itself, like a magnet, all of the similarities of the things that it envelops, and we see clearly in the Sun not only the form of the bodies, but also their nature. The entire atmosphere, which is its object, is completely impregnated with light and heat and receives, within itself, the form of the cause of heat and of the splendor (of light), and this, in its smallest parts. The tramontana demonstrates, with the magnet, that it produces the same thing, and each and every planet, without incurring any modification, produce the same thing. Among physical things, Muscat and other similar odors also do the same.” (Cod. Atl., fol. 138 verso-b)

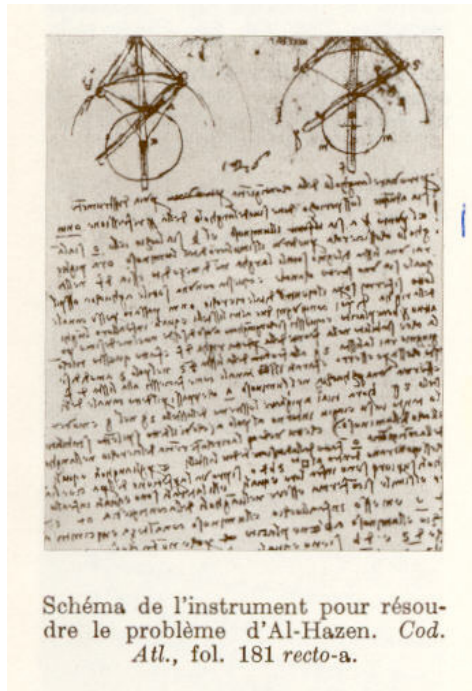
These ideas had a tremendous impact on Christian Huygens over two hundred years later. In a letter to his brother Christian, dated Kinsington March 8, 1690, the year that *The Treatise on Light* was first published, Constantin Huygens told his brother that he had just bought a manuscript of Leonardo da Vinci for the ridiculous price of three guineas and that it contained a discussion on perspective, that is to say, on optics. Italian art historian, Domenico Argentieri, who had dug out this letter from the complete works of Huygens published by the Dutch Science Society, noted that not only Huygens, but also Fermat, had developed his principle of least-time from the ideas of Leonardo on optics. Argentieri wrote:

“About two centuries before Fermat, Leonardo had already stated in precise terms: ‘Each natural phenomenon is generated by the shortest pathways.’ (Anatomy Notebook, IV, fol., 16 recto). See also Cod. Arundel, (fol. 85 verso) The precise words of Fermat are as follows: ‘Nature always acts by the shortest paths.’”(Domenico Argentieri, LEONARD DE VINCI, Editions Atlas, Paris, 1975, p. 410.)

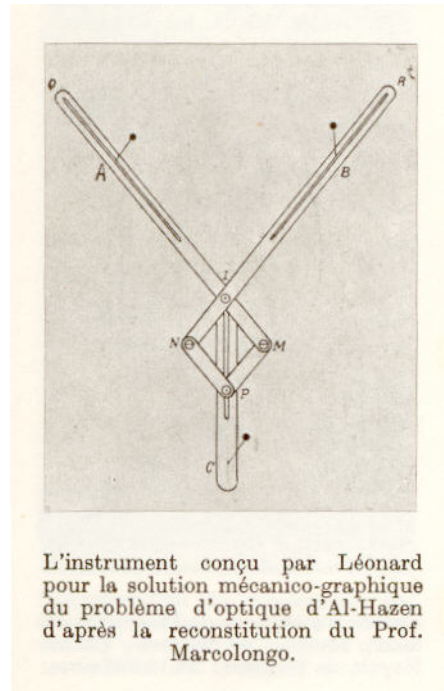
Thus, the historical record shows the connection among Leonardo, Huygens and Fermat. Leonardo also directly influenced Leibniz and Bernoulli. And, just as Leonardo designed his least-time principle with the enveloping/developing characteristic of the catenary/tratrix caustics, so did Leibniz after him. Leonardo applied the same principle of enveloping development to a number of experiments, notably one that he investigated from Alhazen Ibn al-Haytham, the Arab father of modern optics. Like the Archytas doubling of the cube, the Alhazen problem is a Sphaerics problem that requires to be constructed from the principle of a stereo-physical-geometry. The problem is posed as follows:

“Given a source of light A and a spherical mirror C, determine the point where a ray of light must strike on the spherical mirror in order to be reflected to the pre-determined position of an observer at B.” (See Figure 7a,b.)

This problem was reportedly solved geometrically during the 11th century by Alhazen Ibn al-Haytham in the fifth book of his *Opticae Thesaurus*; solved mechanically by Leonardo during the 1480's; solved geometrically and analytically by Huygens in 1672; and finally solved mathematically with an equation of fourth degree by Abraham Kästner in 1776. Like the doubling of the cube, the solution to the Alhazen problem required a cubic root.



a



b

Figure 7a shows Leonardo's instrument for solving the Alhazen optical problem. **Figure 7b** shows the reconstruction of Leonardo's mechanical device by mathematic Professor Roberto Marcolongo of the University of Naples.

The discovery of a pathway of light – refracted or reflected in the solid domain – is always the clearest expression of an insightful curiosity of the human mind in his quest for understanding the fundamental laws of the universe. In this regard, note how, with one of his most extraordinary insights into the nature of the curvature of light, Leonardo went about shaping a solution to problems such as Alhazen's problem. In point of fact, Leonardo was the first to explain scientifically how the pathway of a light ray was non-linear when it propagated through the atmosphere. Don't forget that the introduction of a spherical mirror is to show how light changes curvature all the time and in accordance with a universal law. Leonardo used different concentric spheres to show the curvature. Using the same principle, he also explained that the atmosphere had different shades of blue during the day for the same reason. Leonardo's two statements to those effects are revealing. The first statement was:

"All of the light rays that go through a homogeneous medium, do it in a straight line."
(Cod. Atl., fol. 150 recto-a)

Here, Leonardo is being very provocative, and very Leibnizian, in his creative method, because he is creating a ray of light out of thin air in the same way that Leibniz later created a tangent by inversion. Knowing that the atmosphere is never a homogeneous medium, what Leonardo meant to say, by inversion, was that since the atmosphere always changes in density, the stream of light rays and shadow rays that

travel through it could never travel in straight lines, but only in the non-linear form of caustic envelopes, as from a stereographic field-perspective of catenoid/tractroid curvature.

Therefore, as in the Leibnizian anti-Euclidean method, Leonardo reasoned in the following manner: “***Given the straightness of a ray in a homogeneous medium, find the curvature of that light ray in a real physical atmosphere.***” This meant that in the real atmosphere of relative change in density, refracted rays always express some sort of logarithmic curvature in a stereographic manner, as Archytas had originally developed in a conical function for the doubling of the cube. Leonardo never stated that explicitly, but his method of artistic composition shows that he clearly understood this principle of composition to be true. Thus, the art of painting became the art of reproducing light and shadow envelopment and development in accordance with the scientific principle of least action of light propagation in stereographic physical space-time.

Next, Leonardo applied the same principle to the different shades of blue of the day sky. The second statement he made to that effect was found in his notebook:

“I say, the azure that we see in the atmosphere is never its specific color. What causes it is the warm humidity, evaporated in minuscule and invisible particles, which the sun rays attract and make appear illuminated when they are separated from the intense and deep darkness of the solar region that forms a lid over them.” (Cod. Leicester, fol. 4 recto)

These two statements, taken together, are direct expressions of Leonardo’s catenary/tractrix principle; that is to say, the principle of the ***catenary-enveloping with the tractrix-developing process***, to use the language of Monge. Here, don’t forget that it is the process that generates the particular, not the particular that generates the process. Not only is Leonardo’s conception of the atmosphere filled with water molecules (“***minuscule and invisible particles***”), as confirmed by modern physics, but these ***invisible particles*** are filled with rays of light and darkness forming invisible wavicles that Leonardo has defined as ***reflexive streams***. Such ***reflexive streams*** rotate and penetrate each other without interfering with each other as inversed tangents of light do in generating the double curvature of light propagation in a changing medium, and thus, they produce a constantly changing region of caustics of lights, shadows, and colors defining the fundamental palette of Leonardo.

Imagine then, that the universe as a whole were a multi-layered finite region of constantly changing flux-density of the medium in which ***reflexive streams*** propagate in such a manner that while their inversed tangency forms a family of catenary curves, on the one hand, they also form a family of tractrix curves, on the other. The traces that those ***reflexive streams*** make reflect the pathways of least action. That is the simplest expression of Leonardo’s conception of double curvature process of self-development in the universe as a whole.

Furthermore, to conceive of a “*deep darkness*” beyond the lid of the atmosphere is an extraordinary insight into what could only have been observed by stratospheric flights during the recent decades of the second half of the twentieth century. How did Leonardo know that? This gives you an idea of the power of insight of Leonardo who could only have hypothesized this idea by the same Leibnizian method of inversion of tangents, the method for generating the catenary/tractrix function. That is the kind of thinking that will be required for solar system traveling in the next 50 to a 100 years.

Leonardo also developed this idea as a method of well-tempering in painting, especially his method applied to caustic pyramids and his descriptions of the atmosphere as a reflexive envelope of light and shadows. (See **Figure 8.**) Consider Leonardo’s paintings as compositions of what he called “*solid bodies being surrounded and dressed with light and shadows;*” in a manner such that light and shadow propagations proceed in accordance with the least action principle of a catenary/tractrix enveloping and developing process. If you observe closely the following caustic pyramid by Leonardo, you will discover how, in Dante’s vision, for example, rays of light, themselves, paradoxically generate shadows by traversing and overlapping each other.

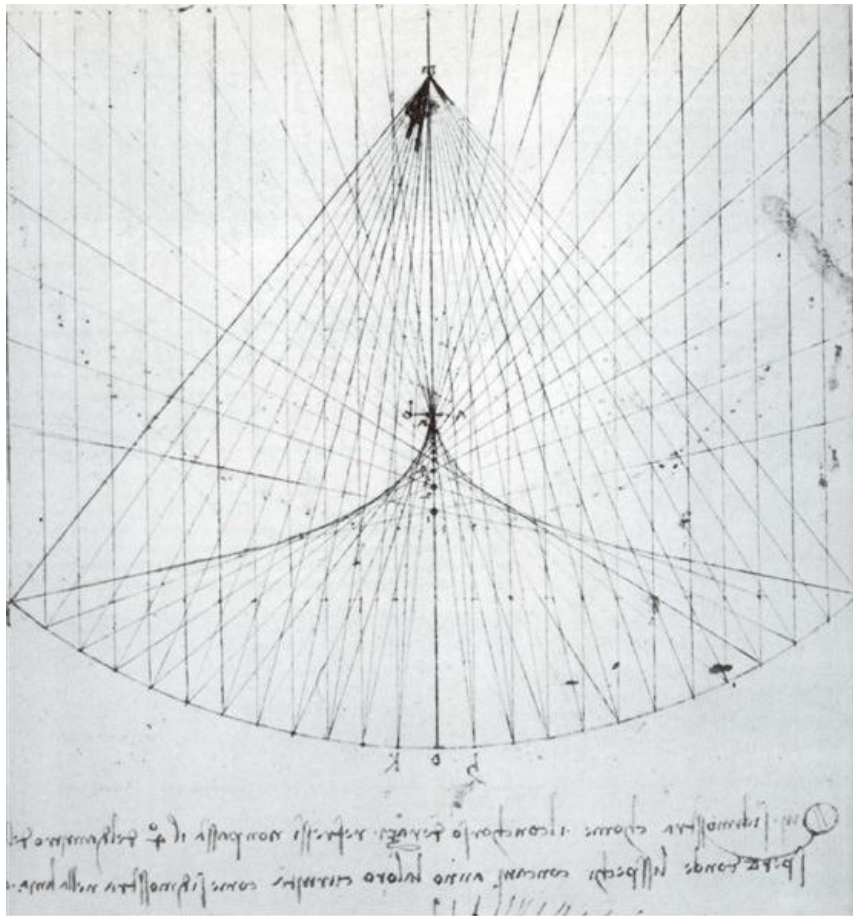


Figure 8. Leonardo da Vinci. Caustic of light producing shadows by a reflex stream as does the catenary/tractrix function by tangent envelopment/development.

Leonardo wrote:

"Every body is surrounded by a limiting surface. Every surface is full of infinite points. Every point makes a ray. The ray is made up of infinite separating lines.

In each point of any line, there intersect lines proceeding from the points on the surface of bodies, and they form pyramids. At the apex of each pyramid there intersect lines proceeding from the whole, and from the parts of the bodies, so that from this apex one can see the whole and the parts. The air that is between bodies is full of the intersections formed by the radiating images of these bodies.

The images of the figures and their colors are transferred from one to the other by a pyramid. Each body fills the surrounding air with its infinite images by means of these rays. The image of each point is in the whole and in each part of the line caused by this point. Each point of the one object is, by analogy, capable of uniting the whole base of the other. Each body becomes the base of innumerable and infinite pyramids. One and the same base serves as the cause of innumerable and infinite pyramids turned in various directions, and of various degrees of length. The point of each pyramid has in itself the whole image of its base. The centerline of each pyramid is full of an infinite number of points of other pyramids. One pyramid passes through the other without confusion..."

Now, Leonardo used the very same method as that of the Leibniz differential calculus and applied it to perceived shadows with respect to the mind, as if he was defining an artistic composition as a form of infinitesimal differentiations of secondary derived light and shadow reflections from primary lights and primary shadows. He described the method of composition in the following remarkable manner as he made plans to write seven books about this subject of non-linear caustic-field-perspective:

"The scientific and true principles of painting first determine what is a shaded object, what is direct shadow, and what is light, that is to say, darkness, light, colour, body, figure, position, distance, nearness, motion, and rest. These are understood by the mind alone, and do not entail manual operations; and they constitute the science of painting which remains in the mind of its contemplators; and from it, is born the actual creation, which is far superior in dignity to the contemplation or science which precedes it.

In the practice of perspective, the same rules apply to light and to the eye.

Shadow is the obstruction of light. Shadows appear to me to be of supreme importance in perspective, because without them, opaque and solid bodies will be ill defined; that which is contained within its outlines and the

outlines themselves will be ill understood unless it is shown against a background of a different tone. Therefore, I state as my first proposition concerning shadows that every opaque body is surrounded and its whole surface enveloped in shadow and light. And to this I shall devote the first book.

Moreover, these shadows are of varying degrees of darkness because they have been abandoned by a varying quantity of luminous rays; and these I call primary shadows because they are the first shadows to form a covering to the bodies concerned. And to this I shall devote the second book.

From these primary shadows there issue certain dark rays, which are diffused through the air and vary in intensity according to the density of the primary shadows from which they are derived; and consequently I shall call these shadows derived shadows, because they have their origin in other shadows. And of this I shall make the third book.

Moreover these derived shadows in striking upon anything create as many different effects as there are different places where they strike; and of this I will make the fourth book.

And since where the derived shadow strikes, it is always surrounded by the striking of the luminous rays, it leaps back with these in a reflex stream towards its source and mingles with and becomes changed into it, altering thereby somewhat of its nature; and to this I shall devote the fifth book.

In addition to this, I will make a sixth book to contain an investigation of the many different varieties of the rebound of the reflected rays, which modify the primary shadow by as many different colors as there are different points from whence these luminous reflected rays proceed.

Furthermore, I will make the seventh book treat of the various distances that may exist between the point where each reflected ray strikes and the point whence it proceeds, and of the various different shades of color which it acquires in striking against opaque bodies." (The Notebooks of Leonardo Da Vinci, Oxford University Press, 1980, p. 128-130)

Leibniz addressed this very same question of least action of light propagation two hundred years later in a letter to Huygens, in which he stated his implicit agreement with Leonardo. He said: "*The whole question lies in the manner with which you have yourself considered that each point is itself radiating, and how you have composed a general wave for all of these auxiliary waves.*" (Leibniz, Letter, June 12-22, 1694) Thus, Leonardo, Huygens, Fermat, Leibniz and Bernoulli had the same understanding of a wave-particle or wavicle function of the catenary/tractrix, as a differential enveloping/developing process of change in light propagation. Remember also that Leibniz had access to some of Leonardo's work through the *Huygens Codex* that Huygens's brother, Constantine, had brought from Italy at the time. Thus, with the

multiple light/shadow palette of *The Last Super*, Leonardo applied to this fresco painting the discovery of the very idea that he had been developing about the curvature of light in the spirit of Dante, that is, the method of inversion of tangents that generates the “*reflex streams*” of a field-perspective as opposed to a linear central-perspective. This is the most revolutionary discovery of Leonardo that is entirely in keeping with the Einstein and Vernadsky conception of relative physical space-time which is the key to understanding the next step in the science of sub-atomic physics today.



Figure 9. The four different shadows of a body that is illuminated by two sources of light. (Windsor Royal Collection. N° 19.149 verso.)

The point to be made, in conclusion, is to realize that Leonardo's insights into classical artistic composition is a crucial form of technology that must be spread throughout the world if we are to colonize the solar system in the years ahead. It is that gifted quality of Leonardo's insights that needs to be replicated for humanity. All you need is a few individuals carrying forward such dynamics as the seed crystals necessary for securing the next step in advancing civilization. However, it is not just Leonardo's ideas that are important; it is the quality of cognitive power that he was able to generate in those ideas as a method that is crucial to transmit for the future. This is the typical way in which you want people to think 50 to a 100 years from now. It is only by spreading such insights around the world that humanity is going to secure civilization in this new millennium.

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