From the desk of Pierre Beaudry

THE LEIBNIZ METHOD Of INVERSION OF TANGENTS AND OSCULATION FOR LOCATING THE CATENARY AND THE TRACTRIX CURVES.

[For Natalie Lovegren: a geometric construction in the spirit of Leonardo's {*Virgin of the Rocks*} and of the Huygens-Leibniz correspondence of the 1692-1695 period.]

by Pierre Beaudry 10/25/2007

Here is a little construction of mine that might help you elucidate the Leibniz dynamic of inversion of tangents and osculation in the gravitational field of Keplerian astrophysics. It should also help you understand the axiomatic difference between the formal geometry discoveries of Huygens and the physical geometry discoveries of Leibniz with respect to the catenary principle.

When you are looking for a creative idea, or better, a universal physical principle, like Leibniz was investigating with the question of the catenary and the gravitational principle of the solar system, you always begin with shits and farts. And the reason it is like that is because you are confronted with the unknown. This is what Leibniz was doing with the idea of osculation as a process of measuring transcendental change in the universe, and in attempting to get Huygens to collaborate with him on this difficult subject. He was having difficulties with Huygens not only because of Huygens's advanced age, but also because Leibniz was projecting into the unknown. So, in a way, this is very appropriate for you, as well, because that is also the condition of youth. Since you don't have a lot of baggage from the past holding you down, you can naturally soar freely to the unknown future!

So, take this little experiment with the catenary and tractrix and look at it from the standpoint of the Leibniz method of inversion of tangents. How does that work? Well, the first part is actually very simple. "Given a curve, find a tangent." If you are given a curve, say a circle, and you are also given the property of the tangent, which is to be at right angle to the radius of curvature of that circle, or the normal of any given curve, then, it is easy to find a tangent at any point of that given circle, because you start with a known curve and its radius.

However, what happens when you inverse that process of construction? You become perplexed, because it is a much more difficult task to inverse that process and assign the tangent to a curve that isn't there. It's like the riddle of the man who wasn't there: "As I went up the stairs, I found a curve that wasn't there. It wasn't there again today, I wish, I wish it stayed away." Yet, this is what Leibniz is asking you to do, when he says: "Given the property of the tangent, find the curve." How can you do that? Where do you start looking for a curve that isn't there?

What Leibniz did is he knocked at the door of Nicholas of Cusa. You have to do the same. Go to Cusa and use his method of traveling to infinity and back! The answer lies in the infinite. Leibniz solved that problem by using Cusa's idea of transformation at infinity. This is also where he got his {*principle of continuity*}, that Kepler had also acquired from Cusa, because the infinite is the place where everything has to change axiomatically. Therefore, how do you apply the method of inversion of tangents and of osculation to locate and measure the change of curvature in the catenary and tractrix curves from infinity? See **Figure 1.**

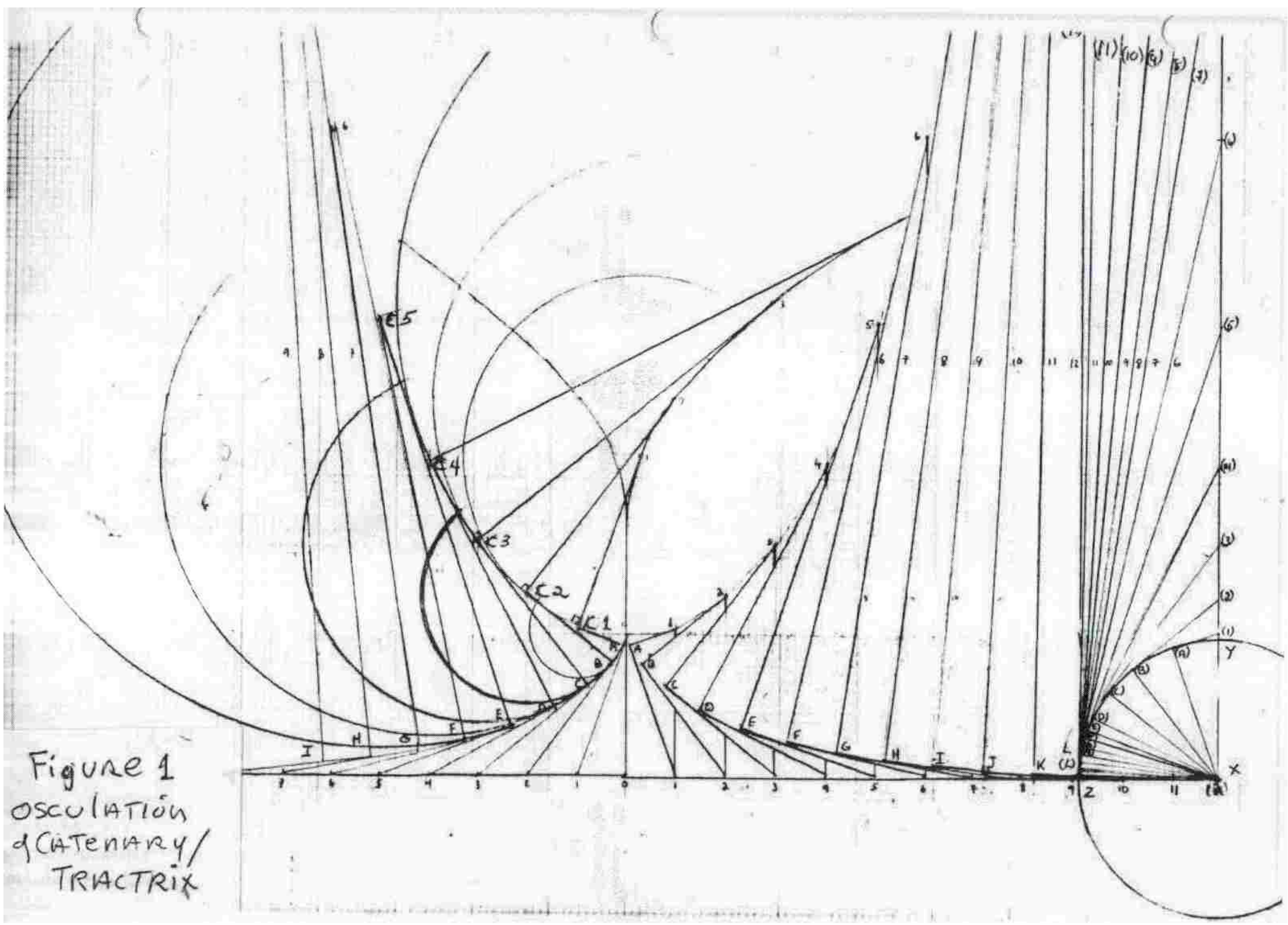

Figure 1. Locating and measuring change in the Catenary and Tractrix curves by inversion of tangents and osculation.

(1) First of all, draw on the right bottom side of a large sheet of paper, and in a decreasing manner, 12 radii inside of a quarter circle followed by 12 tangents at right angle to them. Connect the tangents to a vertical line extended from the center of the circle and identify them as characteristic lines by some number (1),(2),(3),(4), etc. Extend the last circle tangent (12) (at 9 o'clock), vertically to infinity. Set the horizontal distance of the catenary and tractrix vertices at the same height as the quarter circle and at twice the distance of the diameter from that circle's center.

(2) Next, go to the other end of that tangent (12), at infinity, and transform, by inversion, that last circle tangent into the last tangent 12 to a catenary curve! Now, don't freak out. Follow me closely. I know you are far from home, but you are not lost. You are simply creating a completely new transcendental curve from infinity, out of the physical and ethereal principle of proportionality, that is, out of the proverbial, but proportional, thin air. If you can't see what you are doing, don't worry. It is because infinity appears to be very far away. So, just hang that invisible catenary tangent point (12) to one of Cusa's skyhooks, and work your way back down and leftward from there and determine all of the tangents in inverse manner from 12,11,10,9,8,7, etc. Notice that the curves you are looking for are not there, but their construction principle is at work. It is the construction principle that counts, not the curves in and of themselves. You are merely constructing the enveloping location where the curves should be, not the curves themselves. Like in a musical composition, you are playing the harmonic intervals of action not the notes themselves. As Lyn keeps reminding us, the ordering principle of musical composition is the action between the notes.

(3) Therefore, starting from the singularity at infinity, bring this tangent 12 perpendicularly down to the ground level and connect it, at right angle, with the horizontal radius of curvature of the circle. However, since that tangent 12 is now the tangent to the catenary, the radius (L) of the circle has, consequently, been transformed into a tangent to the tractrix curve, at the contracted infinite point L, also touching the same circle at 9 o'clock. From this point on, all of the tangents of the tractrix A,B, C,D,E,F,G,H,I,J,K,L, are going to be the same as the radii (A),(B),(C),(D),(E),(F),(G), (H),(I),(J),(K),(L), of the hereditary circle, and will be parallel to them. In other words, you are merely translating parallel tangents and radii from the circle to the catenary, to the tractrix, and back to the home base circle again. Leibniz describes this tractrix process briefly in his letter to Huygens of October 1/11, 1693.

(4) Repeat the same blind process by drawing corresponding parallel tangents between the circle and the catenary until you get to tangent 6 on the catenary side of the infinite discontinuity. Note how tangent 6 of the catenary, parallel to tangent (6) of the circle, connects at right angle with tangent F of the tractrix, which is also parallel to radius (F) of the circle, connecting to its tangent (6) at right angle. As the smallest interval of action changes, so does the whole. The entire dynamic system of the circle, catenary, tractrix, and back to the circle again is like a metaphor of the universe as a whole, or in part, finite and self-bounded. The universe is bounded by and depending on universal physical principles in such a way that if a warping accident were to occur to the orbital pathway of a planet in some remote galaxy, the dynamic system would correct the

error harmonically and readjust itself as a whole. This is how I see, in my mind, the method by means of which Leibniz saw, in his own mind, how the harmonic gravitational field of Kepler's solar system was developing least action surface pathways of {*ambient deferent rays*} in astrophysical space-time.

(5) The way to measure the {*corresponding parallel tangents*} of the circle-catenary connections is by correlating them harmonically with their {*radii of curvature*} from the circle-tractrix connections, which are also parallel to each other by virtue of the property of the tangent. Note that by this inversion process, the tangents of the catenary curve have become the radii of curvature of the tractrix curve, and the tangents of the tractrix curve, from those right angle connections to the catenary tangents, have become the radii of curvature of the circle. Everything is connected together nicely and forms a field of self-transformation, which is typical of gravitational transformations and adjustments inside of the universe as a whole.

(6) Next, transfer yourself to the left side of the catenary construction and apply the same principle of construction in reverse, that is, from left to right. Then, identify how each tangent of the catenary curve is, in reality, nothing else but a radius of curvature of an osculating circle embracing the tractrix curve. For Huygens, it was the mechanical geometry of the evolute that generated the catenary curve from the vertex caustic curve along the axis of the catenary, because he was attempting to measure the length of the catenary and the quadratures of its area, not the physical cause of change. However, for my Leibnizian construction, the catenary curve is generated by the dynamical physical geometry of osculation by inversion of tangents, with the purpose of measuring the physical action of change, not lengths. In that sense, osculation is not a formal geometric process measuring extension. Osculation is a physical process that is represented geometrically for the purpose of adjusting infinitesimal increments of change in the curvature of physical space-time. There is, therefore, an axiomatic difference, here, between the Huygens and Leibniz methods.

(7) What is implied, here, is the application of the Leibniz least action principle. The question is not how to apply measure to determine extension like the Cartesians and Newtonians do, but to determine change. The problem with mathematics today, and you will find this on every university campus around the world, is that students are taught to measure a displacement in terms of extension and not in terms of change produced by a certain amount of work. As Lyn has shown, the measure of change is based on the density of singularity per small interval of action, not on size or length. This is why Lyn de-emphasized the measuring of quadratures. In economics, measuring lengths and sizes is a fallacy of composition. An economist is not a tailor.

(8) As I have shown in the Fidelio, Spring 2001, the Leibniz catenary is not mechanically generated by an evolute like in the Huygens construction. It is generated by a harmonic principle of proportionality showing the change between combined arithmetic and geometric progressions initiated from a given projective common ratio between the logarithmic curve and the catenary curve. Therefore, we are dealing, here, with a transcendental change. And, those two curves are generated directly from the universal

physical principle of proportionality that Thales of Miletus had first established for Pythagorean {*Sphaerics*}. For the determination of such a projective function, see my BOGOTA LYM CLASS # 5. {*Thales Theorem and the Archytas Model*}, at ftp.ljcentral.net/unpublished/Pierre_Beaudry/ As for the tractrix, Leibniz constructed it separately with the sliding ruler idea of Dr. Perraut and discovered, as shown here, that the right angle tangents and radii of the hereditary circle generated the envelope of the curve. (Letter to Huygens, October 1/11, 1693) However, even though this letter shows that he knew this construction perfectly well, Leibniz did not, as far as I am aware, discuss the explicit connection between the tractrix and the catenary curves.

(9) The key idea, here, is that the method of osculation is a process of measuring the curvature of change in the transcendental domain: the change of change, the variation in the direction of change. The tangent no longer has the function of giving the direction to a curve. Leibniz has obliterated that fallacy of composition and superseded it with the process of osculation. It is the osculating process, which represents the higher transcendental form of measuring change and of determining the direction of changing curvature. The implications for the Leibniz calculus applied to astrophysics are, of course, considerable, as shown later by Gauss.

(10) Note that when you consider this osculating process by inversion of tangents as a change function, it generates an axiomatic change between two domains, at the two ends of the change function. At one end, the process generates a change of direction of the (*développante*} of positive curvature (involute curve) and, at the other end; the same process generates the cusp of a {*développée of retrogression*} of negative curvature (evolute curve).

(11) Consequently, if one were to rotate the catenary and the tractrix curves along their respective axis, in opposite but complementary directions, the function would produce a surface of positive curvature at one end (an isoperimetric Catenoid), and a surface of negative curvature at the other end (an isochronic Pseudosphere). This is the harmonic curvature of physical space-time, which determines gravitation, not attraction at a distance. As Leibniz put it, this works as an irony, like the metaphor of a burning caustic mirror in your mind because you are created in the image of God. Therefore, each cusp of caustic discontinuity represents an axiomatic change in the development of the human mind. This is how I envision the anti-entropic Riemannian change of phase-space, which occurs in the register shift between the chest register and the head register in Bel Canto singing or the register shift of the solar system located at the asteroid belt.

(12) Lastly, this is how I see the Leibniz connection between the two separate, but congruent movements of the planets with respect to the solar system and the precession of the equinoxes. The connections where one principle of motion is the cause of the gravitation field of {*ambient deferent rays*} of each and all of the planets in the solar system are congruent with those connections of the other principle of motion that causes the magnetic field of {*parallel alignments*} in each and all of the planets with respect to the celestial north pole. Both processes require a higher principle of harmonic proportionately across the range of the galaxies of the universe as a whole. It was Monge

who, in the footsteps of Kepler and Leibniz, later identified these changes in minimal surfaces as characteristic surface envelopes. I think that this might be where the expression "{*pushing the envelope*}" has been derived from. But I am not sure. At any rate, here is how it works. See **Figure 2**.

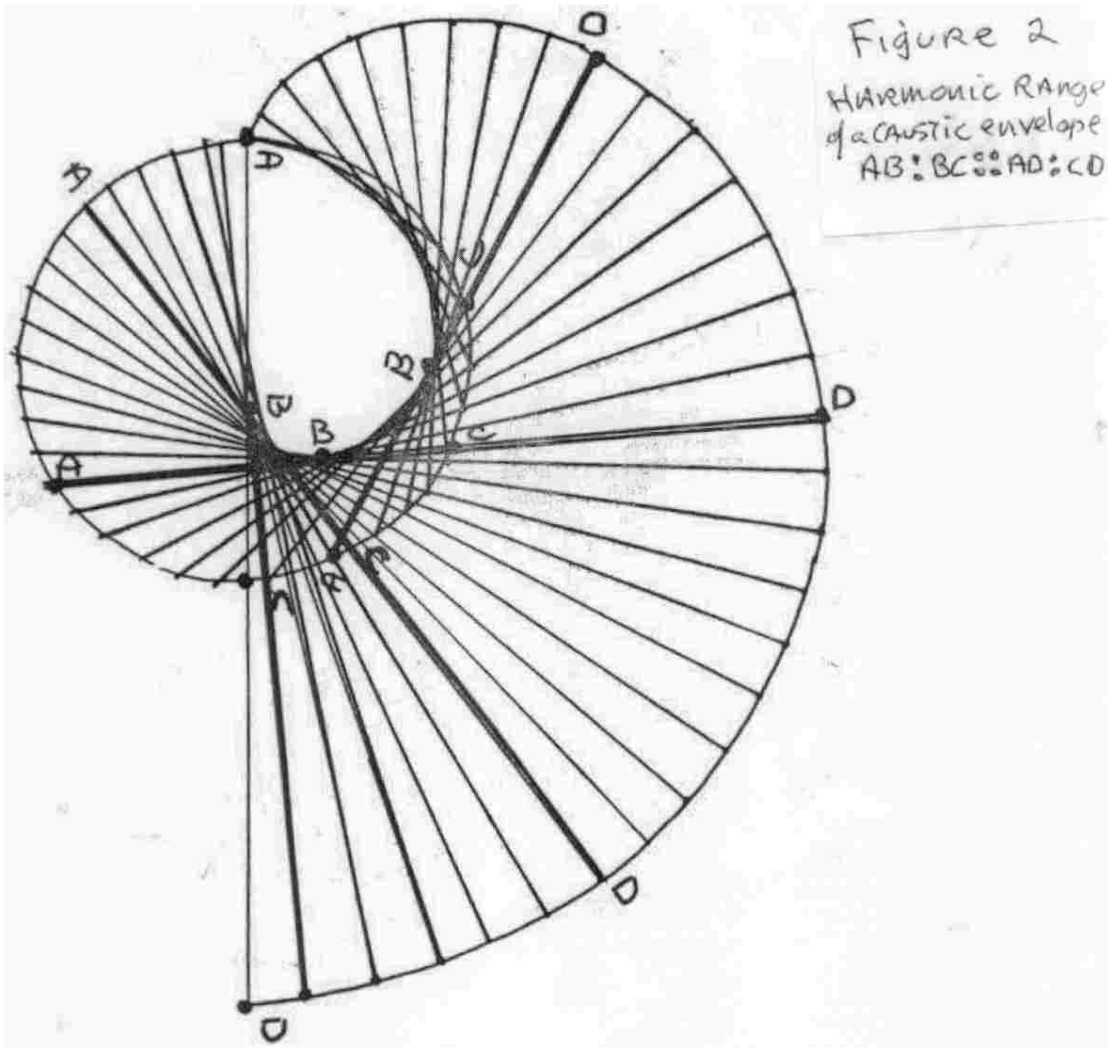

Figure 2. This harmonic Range of a caustic wavicle envelope can serve as an example of how a voice register shift singularity occurs in Bel Canto.

Ask yourself: how can you bring together seeing what you hear into a higher manifold? Look at the caustic in **Figure 2**. The singularity of the wavicle envelope in the Bel Canto register shift of the human voice always has the same well-tempered characteristic of change of curvature as in a caustic of light. Thus, we have an irony, because we have an ambiguous caustic singularity that changes registers by intersecting positive and negative curvatures through a spherical great circle. Project a ray of light through a spherical glass of red wine and you will generate a similar caustic onto a white tablecloth. In our example, it is the harmonic range voicing of that envelope of proportionality which bridges the change between two inverse curvatures; that is to say, where the radii of osculation of the {*développante*} at points **D** are all tangents to the {*développée of retrogression*} at points **B;** thus, forming everywhere the Lydian harmonic field where **AB : BC :: AD : CD.** As you can see, I prefer to use the French identification of those curves borrowed from Monge, instead of the confusing English names of evolute and involute, because it is the development process of osculating envelopment that counts, not the curves. That is, if you wish to see better what you hear between the notes.

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