From the desk of Pierre Beaudry

A CONTRIBUTION TO THE PYTHAGOREAN QUADRIVIUM

by Pierre Beaudry 2/08/2011

A birthday special

1. The Banneker Puzzle

The challenge of the Banneker Puzzle is something that should be put on the school desk of every child in the world, because the underlying process of its discovery represents a unique function of the universal physical principle of metaphor or irony, which, when discovered, gives you the power to relive other processes of similar universal construction in both Classical artistic composition, as well as in physical science. This creative principle of metaphor, like the principle of credit upon which the American System of economics is based, looks to future production of mankind. That is the reason why the analog function by means of which the underlying proportionality of the Banneker Puzzle leads you to discover creative processes, pertains to the Pythagorean Quadrivium of arithmetic, astronomy, geometry, and music. That's the future.

Figure 1. Benjamin Banneker (1731-1806).

BANNEKER'S PUZZLE

Divide 60 into four such parts that when the first being increased by 4, the second decreased by 4, the third multiplied by 4, the fourth part divided by 4, that the sum, the difference, the product, and the quotient shall be one and the same number.

Ans first part **5.6** ***increased by*** **4 = 9.6**
second part **13.6** ***decreased by*** **4 = 9.6**
third part **2.4** ***multiplied by*** **4 = 9.6**
fourth part **38.4** ***divided by*** **4 = 9.6**
60.0

After studying carefully this Banneker solution, you can find how to construct it simply by reducing the elements of the puzzle to a minimum, and by applying the principle of inversion to whole numbers. Just remember what Leibniz said about the principle of discovery of the catenary and the tractrix curves: "***Given the property of the tangent, find the curve***." In other words, the way to discover the key to the solution is to proceed by inversion, as in timereversal.

Since number 4 is the only constant element in the Banneker Puzzle, and all four arithmetic operations are involved, the solution must have some quadratic form; that is to say, it must be found within the boundary condition set by number 4, and must follow the quadratic rule of adding, subtracting, multiplying, and dividing. So, look into the inversion of the first operation and ask yourself: "What if I start from the end of the process and work my way back? Is it not the case that to make any discovery, you must always start from the end? Only fools start anything from the beginning." So, apply the following inversion: 0 + 4 = 4 and proceed to discover the singularity that generates the next step in the process. From this, you can easily establish the following:

The sum is ……….. 0 + 4 = 4
The difference is ….8 – 4 = 4
The product is ….…1 x 4 = 4
The quotient is…...16 ÷ 4 = 4
Total ……………. 25

By adding the sum, the difference, the product and the quotient of the different parts of 25, you can establish the proportion by means of which Banneker made the discovery; which is: 25 is to 4 as 60 is to 9.6. This is the singularity which produces the proportional ratio of the process, 25/4 = 6.25. The key is in the analog function of ***this is to this as that is to that*** which is the underlying measure of any metaphorical process, the only bridge leaping over the Rubicon. However, that measure is not found by a mathematical deduction, but by inferential knowledge. This is what John Quincy Adams had proven to Jeremy Bentham during his Hyde Park discussion of 1816, when he demonstrated to him that the British liberal method of discovery was inferior to the American principle of discovery. This is the process of proportionality that every discoverer looks for in the course of his passionate search for truth. Once you understand that, then you can demonstrate that a metaphorical function can be applied universally to anything you wish to discover. It is the inferential proportionality within the metaphorical process that counts as opposed to sense-perception and mathematical deduction. This is also how a good musician is able to interpret a good composer.

After you have constructed a few more Banneker puzzles of this type, you will notice that, although the ratio could be applied to any number whatsoever, it appears to have some particular affinity with every quadratic number because, similar to the function of the musical well-tempered system of C = 256, for instance, all of the logarithmic intervals between the octaves are held together by the book ends of whole numbers. Similarly, their arithmetic mean intervals act like the geometric mean intervals of Bach's well-tempered Lydian divisions by half, and by half the halves of their spiral actions. Note the corresponding intervals of minor thirds that the Banneker Puzzle implied, and compare them with the Ricercar theme intervals developed by Bach, and later transformed by Mozart, Beethoven, Mendelssohn, et all. These composers have all explicitly used those dissonant intervals as measures of change inside of their respective compositions. Let me fill in a few holes, here, and you will see the amazing correspondence between the Banneker and the Bach quadratic dissonances:

4 x 6.25 = 25	128 = C
5 x 6.5 = 31.25	152.2185 = Eb
6 x 6.25 = 37.5	181.0193 = F#
7 x 6.25 = 43.75	215.2694 = A
8 x 6.25 = 50	256 = C
9 x 6.25 = 56.25	304.4370 = Eb
10 x 6.25 = 62.5	362.0386 = F#
11 x 6.26 = 68.75	430.5389 = A
12 x 6.25 = 75	512 = C
13 x 6.25 = 81.25	608.8740 = Eb
14 x 6.25 = 87.5	724.0773 = F#
15 x 6.25 = 93.75	861.0779 = A
16 x 6.25 = 100.	1024 = C
Etc.	Etc.

As you can see, the proportionality of Banneker applies only to a limited series of book end whole number ratios, 25/4, 50/8, 75/12, 100/16, etc., as if the octaves represented the peaking vibratos of a wave function underlying the process of construction of his Puzzle. The general rule is that all of the numbers of the whole number series, I, 2, 3, 4, 5, multiplied by 25 will yield all of the whole number solutions to the puzzle. Thus, Banneker's Puzzle is the arithmetical equivalent of Johann Sebastian Bach's Lydian division as he developed them in his Ricercar theme. As for the specific spiral action of the Lydian divisions indicated above, if you play that same series of intervals on the keyboard, but as a downward spiral starting from the end at A- C, F#-A, Eb-F#, C-Eb; A-C, F#-A, Eb-F#, C-Eb; A-C, F#-A, Eb-F#, C-Eb, you will ease nicely into the adagio movement of the Beethoven ***Moonlight Sonata***!

It then becomes clear that the arithmetical underlying ordering of the Banneker Puzzle and the geometrical underlying ordering of the Bach Lydian musical division pertain to the same underlying quadratic proportionality. However, aside from holding together the four elementary arithmetic operations in a quadratic form, as if it were their common denominator, the question is: can such small inferential angles of dissonance within quadratic proportionality have other applications in the universe we live in? The answer is yes, and notably, in the domain of political economy, but as long as this "petite voie" comes from above and not from below. A case in point is exemplified by the 1791 Banneker letter to Jefferson in which Banneker proved the strength of this dissonant weak force by demonstrating the "***Self evident truth that all men are created equal.***"

2. A Biquadratic Wave Function

The study of galactic evolution, and of the periodical star motions within it, requires that special consideration be given to elliptical modular wave functions, as Riemann investigated them in such doubly-connected periodic functions as he found as the motor of the torus. Additional attention should also be given to the fact that such elliptic functions also work like memory modular wave functions. In fact, individual memory reflects on itself at the same time that it reflects on the collective memory of discoveries of principle that has been produced by the human species as a whole for thousands of years. In that sense, as periodical functions, both the galaxy and the human mind reflect the same principle of spacetime motions which pertain to similar geometric processes of ***analysis situs***. The simplest way of developing this is by constructing a doubly-connected elliptical function whose complex spacetime motion is expressed by the dual Poloidal/Toroidal motion of physical spacetime fused into one. This is also the simplest expression of nuclear fusion processes. (See Pierre Beaudry, ***Fusion Power is not Democratic***, 2/16/2010, and ***Timereversal in the simultaneity of eternity,*** 5/19/2010.) And the question is the same as in the Banneker Puzzle: What is the underlying proportionality of that ratio? Take the case of the solar system.

The reason why the solar system knows where it is going is because it is moving inside of a galaxy. Without the galaxy, the solar system would have no idea where to go next. It works the same way for state governments within a nation: a state government has no idea where to go, unless its function is moving with respect to an appropriate federal constitutional governing body. From that vantage point, this dual process of development of a solar system within the galaxy indicates that the domain of the solar system is not limited to what goes on inside of that system, but reflects also what goes on outside of it as well. The pathway of the sun is affected by the future of what is going on inside of the galaxy as a whole, in the same way that a state is affected by the future orientation of the sovereign nation it belongs to. In other words, the solar system is like a Hamiltonian credit system; it depends primarily on its future investments in order to exist and be creative.

Thinking processes work in the same way, from the standpoint of universal history of the future. Human thinking does not exist outside of history. Not only does the mental activity of an individual human being reflect what goes on within the domain of his or her own mind, but its development is also completely determined by the universal history of mankind as a whole, and depends for its future development on the discoveries made by other creative individual minds in the past, as if their connecting point were "presently" orthogonal to one another. Isn't it an obvious fact that the only way to look at the past is from the future?

Now, at the same time, imagine that you are attempting to solve a galaxy problem which involves such spacetime motions as two orthogonal directions intertwined into a single process, and in a manner such that, between them, they involve universal change in living processes by means of cosmic radiation. The changes in the P/T ratio of the earth's 60/140 million-year cycles through the galaxy will reflect both the direction of the mitogenic radiation of Gurwitsch and the direction of the biospheric field of Vernadsky. How do you construct an ***analysis situs elliptical*** process that describes such a higher form of integration? That is the efficient intention this report wishes to contribute to. For example, think of a number as the shadow of an economic investment that takes you to the next step of development into the future to such an effect that you are living in a future that always was present, because you have entered into a process that never dies. As Lyn put it: "***What is right, is right forever***."

The point to be made, therefore, is that we are not looking for a geometric model based on sense perception, but for a geometric method that must express the limitations and the failures of our sense perception apparatus. As Leibniz remarked to Huygens: "***Nature always produces more than what your geometry can handle***." The nature of universal change itself implies, therefore, that nothing in the universe is ever completed, as Kepler showed about the orbits of the Earth and Mars. So, what we are looking for, instead, is the least imperfect form of geometry which includes within its construction the failure to represent completion within living or cognitive events in the galaxy. In other words, we must discover ways of demonstrating how inferential knowledge proves the ineptitudes of empirical and positivist knowledge.

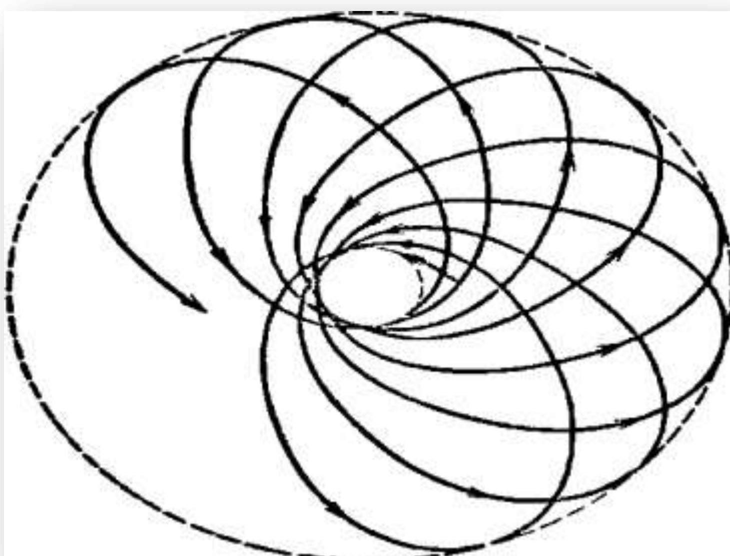


Figure 2. An elliptical modular wave function representing the P/T spacetime motion of a star travelling through a galaxy forms an ellipsoidal torus inside of which a rotating elliptical coil represents the Poloidal spacetime motion of its solar system, while the Toroidal motion of the ellipsoid represents the effect of the rotation of the galaxy as a whole.

The importance of such geometric demonstrations of imperfection lies not in showing what is lacking in the visual process under observation, but rather in revealing the inner workings of the limitations that our geometry fails to explain during the process of improving human knowledge. If the message doesn't get to its destination, don't blame the messenger and his visual apparatus, blame the faulty mailing system. Moreover, don't think of this as a mathematical model, think of it, instead, as an Abelian elliptic function on crutches. As I will show below with the Archytas construction for duplicating the cube, the least action form of ***analysis situs*** geometry does not represent the visual shapes of physical processes, but rather, the least defective process of the human mind in conceiving the future through the small angular path of its coming into being.

For instance, the first thing that this present geometric exercise is not able to explain is the practical intention for such an exercise. If you were to ask me: "What is the usefulness of this report?" I could only reply as Leibniz did to Montfort, when he asked him about the usefulness of games in ***analysis situs***: the purpose is to ***develop the powers of creative imagination by way of the principle of continuity.*** As Riemann demonstrated in the footsteps of Leibniz, whenever you must displace yourself in physical spacetime, the human mind is always capable of going from simple circular action to doubly-connected circular action in a continuous fashion and in a perfect self-connected manner. However, it is not enough to simply understand this "visual" continuity. Every time you are asked to reproduce this geometrically, your geometry fails to do that, and you require going through a discontinuity to demonstrate the "jump" that sense perception must make, in order to get a truthful picture. And, the truthful picture is always limping with imperfections. The reason this is the case is because you can only tell the truth by showing the fallacy of the mistake. This is why you cannot look someone in the eye and kiss his ass at the same time.

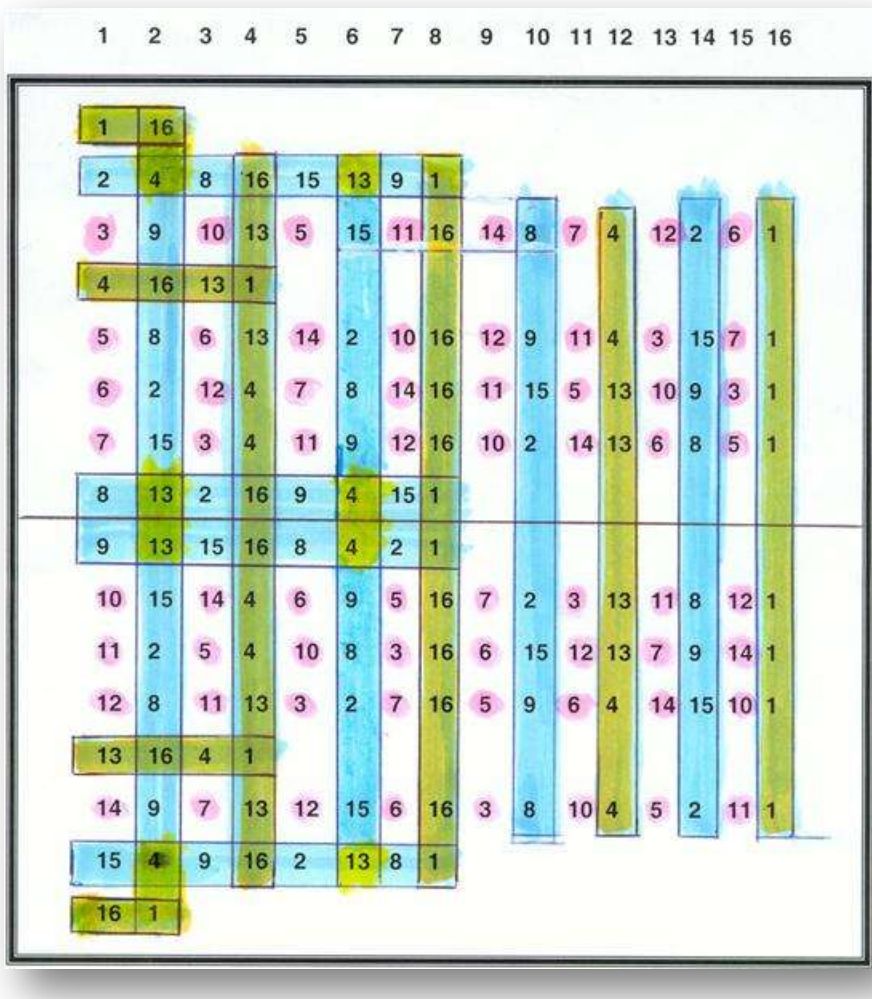
Take, for instance, the existence of several different motions, the first of which is either included entirely in the whole, or is partially included as a one of several moving parts incorporated into the same process as a whole. How can you demonstrate that? As you go along, note how much your mind has to work in order to execute this simple task. The underlying truth of it is that basic arithmetic works like that inside of your mind.

For example, work out this following problem of number theory where 6 is a primitive root (mod 17), and discover what sort of work its process of development accomplishes in your mind. The simplest definition of a primitive root was given by Gauss when he wrote in ***Disquisitiones Arithmeticae***, Article 55: ***"There always exist numbers with the property that no power less than p – 1st is congruent to unity, and there are as many of them between 1 and p – 1 as there are numbers less than p – 1."*** From this, we can establish in the spirit of Louis Poinsot, who was the first to discover the higher ordering principle of such powers, that the ***analysis situs*** of primitive roots represents the fundamental theorem in the geometry of the torus. The physical geometric process to compute primitive roots is the following:

If you have P poloidal wave intervals arranged in a torus, and you join them into a continuous motion from P to P, P being a primitive root of T, you will necessarily pass through all of the intervals of T before returning to your starting point, and you will necessarily have covered P($\frac{T-1}{2}$) times the complete toroidal circumference of the torus.

Compare how empirical sense-perception proceeds long hand, and how inferential sense-conception does it in short hand and you will understand how this method is the only least action method to use in order to establish the geometry of primitive roots. The problem is not as complicated as number theory makes it appear to be, so don't waste time looking for the solution of this problem in number theory books, you won't find the solution there. You can only find it in your mind. It was the French geometer, Poinsot, who discovered that primitive roots were comparable to prime numbers, but from the higher standpoint of their exponents. Contrary to what Euler had claimed, Poinsot discovered the solution by looking for what was not there. He considered that a primitive root was any number less than modulus p – 1 whose power exponents were not a square, a cube, or a fifth power. As Poinsot put it:

> "***When you wish to find them (*prime numbers*), one considers all of the simple factors of a given number; and from the natural series 1, 2, 3, 4, 5, etc., one excludes all of the multiples of these simple factors. Here (*with primitive roots*), instead of those multiples, it is necessary to exclude all of the powers of exponents identified by these factors: the result, as one can see, is an operation of the same type, except from a higher level.***" (Louis Poinsot, ***Reflexions sur les principes fondamentaux de la théorie des nombres***, Paris, Bachelier, Imprimeur-libraire, 1845, p.75.)



1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
1	16														
2	4	8	16	15	13	9	1								
3	9	10	13	5	15	11	16	14	8	7	4	12	2	6	1
4	16	13	1												
5	8	6	13	14	2	10	16	12	9	11	4	3	15	7	1
6	2	12	4	7	8	14	16	11	15	5	13	10	9	3	1
7	15	3	4	11	9	12	16	10	2	14	13	6	8	5	1
8	13	2	16	9	4	15	1								
9	13	15	16	8	4	2	1								
10	15	14	4	6	9	5	16	7	2	3	13	11	8	12	1
11	2	5	4	10	8	3	16	6	15	12	13	7	9	14	1
12	8	11	13	3	2	7	16	5	9	6	4	14	15	10	1
13	16	4	1												
14	9	7	13	12	15	6	16	3	8	10	4	5	2	11	1
15	4	9	16	2	13	8	1								
16	1														

Figure 3. Table of remainders for module 17. All of the powers are connected together by way of the weak force of their remainders (mod 17) which are arrayed in a mirror image reciprocity that always total 17. What holds them together is a special reciprocity relationship between the biquadratic and quadratic

residues, and the primitive roots. The blue, 2, 8, 9, 15 are the quadratic residues, the green 1, 4, 13, 16 are the biquadratic residues, and the two sets of pink 5, 3, 12, 14 and 6, 7, 11, 10 are primitive roots.

In order to eliminate the powers of factor exponents, do as follows: If T = 17, and T – 1 = 16, then, eliminate 2 of them that are of the squared power (1, 16), 2 of them that are of the forth power (1, 4, 16, 13), and 4 of them that are of the eight power (2, 8, 15, 9). As a result, there must be left two sets of 4 primitive roots each, (6, 7, 11, 10) and (5, 3, 12, 14). See the horizontal series starting from the left column in the table of **Figure 3**.

Here, anyone who is a fanatic of number theory will throw this report out the window, unless he or she adopts a constructive geometric approach for understanding the weak force of the remainders of quadratic residues and of primitive roots. Otherwise, note that all of the remainders are reciprocals of one another, within their power groups, including primitive roots. This is worth reflecting on, because something quite unique is happening, here, within the underlying process of whole numbers that Gauss had identified as the most fundamental theorem of arithmetic, and which he identified with the ***principle of reciprocity***. Indeed, the process of generating whole numbers in this manner causes their actions to form closed cycles of quadratic and biquadratic numbers in the company of primitive root cycles. This is how different cultures create different nation-states. That sort of self-reflective anomaly should tell us something about how the universe works. How can you have so many different processes operating autonomously within the same unified system? Once you eliminate the underlying assumption that numbers represent things, and understand them as intervals of action, you begin to discover what they really represent: ***intervals of self-reflexive universal reciprocity***!

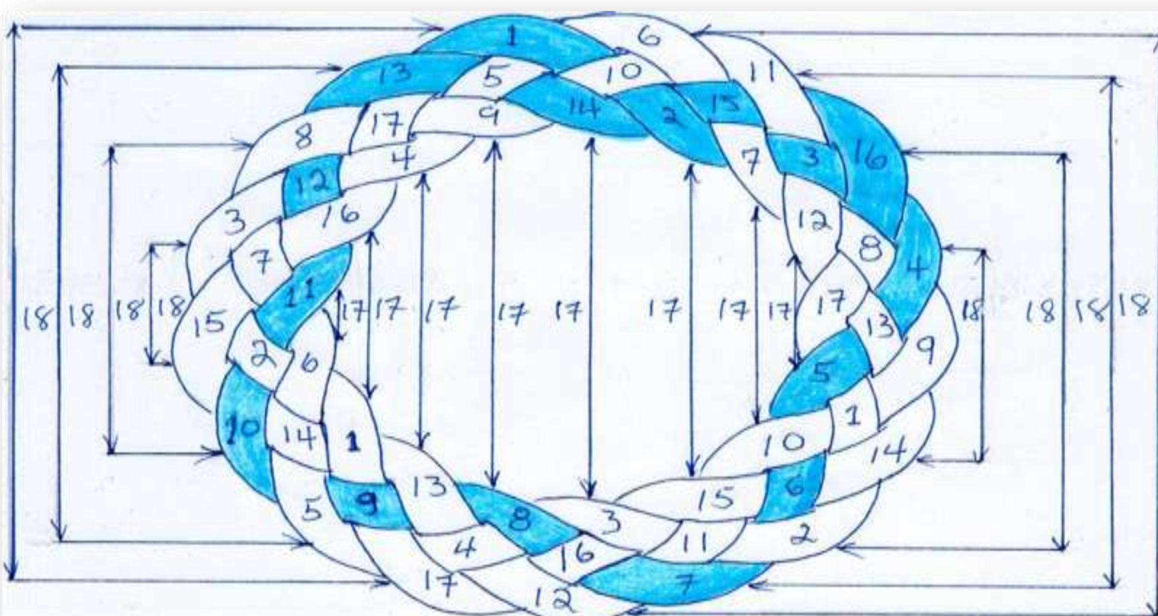


Figure 4. When 4 is a biquadratic residue (mod 17), there are 8 reciprocals of 18 on the outside of the torus, and 8 reciprocals of 17 on the inside. The Poloidal circumference is composed of 4 units of action and the Toroidal circumference is composed of 17 poloidal waves, for a P/T ratio of 4/17.

For example, when you generate 4 as a biquadratic residue (mod 17) inside of a torus, the system expresses an amazing form of reciprocity both on the inside and on the outside surface of the torus, creating a constant dissymmetrical balance within the dynamic field of the system. Thus, as in the Peace of Westphalia, the principle of reciprocity becomes the keeper of the golden rule of the advantage of the other: ***Do onto others, as you would have done onto you***.

The fascinating feature of this biquadratic residue modulus is that it locates all of the 4 biquadratics residues, the 4 quadratic residues, and the 8 primitive roots (mod 17), each and all in their appropriate position of reciprocity, merely by means of the well ordered remainders of their powers. If you are not perplexed by the effects of this ***reciprocity principle***, then, you should take a cold shower. Just study, for example, one of those reciprocal processes. First, rotate your curser, continuously through the first wave of 1-2-3-4 clockwise (**Figure 4**), then follow the same process four times, then again sixteen times, and lastly thirteen times, in the same direction, before returning to your starting point. What happened? You have developed the entire cycle of the four biquadratic residues (mod 17) by completing 8 times the great circumference of the Torus, without disturbing the quadratics and the primitive roots. Moreover, you have discovered that biquadratic residues are residues of each other: they manifest the principle of reciprocity. Generate similar clockwise rotations, starting from 2, from 6, and then from 5, on the external surface of the torus, and you will get the same result. In other words, all of the integers of the system will be generated into four sets of reciprocal pairs of 17 rotating inside of four independent periodical cycles. Each period is generated from one another's reciprocal remainders with the same total of 8 times the toroidal circumference; that is to say, 4 x 8 = 48, the same total as 6 primitive root of 17, or P(T – 1)/2 = 48.

biquadratics	quadratics	primitive roots	primitive roots
1, 4, 16, 13	2, 8, 15, 9	6, 7, 11, 10	5, 3, 12, 14

Again, I repeat: To locate the four biquadratic residues, namely, 1, 4, 16, 13, all you have to do is to rotate your curser clockwise, starting from 1, and count each poloidal wave as an interval of four units of action. The number that you start from indicates the number of poloidal waves you have to count in order to reach the next remainder in the series. Thus, the first wave takes you to 4, four waves takes you to 16, sixteen waves takes you to 13, and thirteen waves takes you back to 1, inside of a complete and closed circuit. This generates 34 x 4 = 136 units of action per circuit and corresponds to the total of (T-1)/2 times 17. You can find the 4 quadratics and the two sets of 4 primitive roots circuits in the same fashion, starting from any one of them. All of the 8 primitive roots can also be derived as one set by inverting the motion counterclockwise.

At any rate, this is the way you want people to think about the galaxy of nation-states in the future. It is the same congruence of reciprocity that is represented by the weak force of the Peace of Westphalia and the Hamiltonian credit system of the American economy, which reflects the same lawful ordering of the universe, in opposition to the monetarist power of British imperialism and their derivative mathematics. Living processes generate themselves in a similar manner whereby each previous stage generates, self-reflectively, the preconditions for the existence of the next step in the process. However, be careful how you tell that discovery to a positivist or a Wall Street investor, because he might want to kill you for taking his fire away from him.

There is, in **Figure 4**, an underlying process that number theory calls primitive roots, but the real name for this type of process is ***perpetual change***. In other words, primitive roots work like a credit system in a physical economy, or like cosmic radiation in a galactic system; that is to say, they reflect a principle of congruence by means of which you can go into debt provided you are able to produce more than what is necessary to account for your initial investment. The investment is not the principle, but merely the arithmetical resonance of such a process which relates to the universal physical principle of anti-entropic reciprocity through which ***the measure of change in the universe has become the change of measure***. It is in that sense that the system is finite, but not bounded. However, the perpetual motion will only work provided you increase the energy-flux density of the system throughout the continuous process.

Furthermore, the spacetime feature of this process is an expression of timereversal, which proceeds from the future toward the past. In plain language, the least action form of the poloidal wave action knows how to get to the next phase of work by projecting into the future from the preceding past remainder as credit, but, the question is: how does the least action know where to stop and change its specified amount of motion? How does light know where to change direction in refraction?

The difficulty, here, is to determine what the next step will be. You know how to get to the future from the past, because you are being pulled by it all the time, no matter what you do, or don't do. However, you don't know where you are going to end up, because you don't know what circumstances nature can bring that might change this course of progress. This is like a living process in which the next stage of development is pregnant with the preceding step in a manner such that the previous step knows what the next step will be, because it already contains the substance of the next moment of growth within itself, but, at the same time, it must protect itself from changes that might come from an aversive environment that might stunt its growth.

As Rabelais said: "***Destiny leads the willing, but drags the unwilling***."You don't know what the future will bring, because the future is never given in advance of time, unless you start from the end of the process and you determine what the future will be. Aha! That is the key. If the future remainder is reached from the known value of the previous remainder, you know where the next stop will be, but you won't know where to go from there. How will you know without doing the calculation long hand from the previous step? That is the problem. You can no longer count on the past, because everything from the past has failed to lead you where you needed to go. Therefore, how do you discover the process of least action that will determine what the remainder of your next step will be? Do you see the problem? The point is that you only know where you are going if you start from the end; otherwise you are at the mercy of destiny's catenary pull. But, what is the method by means of which you will establish what this end will be? This is what we must now examine with Archytas.

3. The Archytas Doubling of the Cube and the discovery of the Astrolabe

I have to warn you about a possible misunderstanding, here. This report is not promoting a new recipe, or some new formula, for developing number theory. The purpose is to develop a metaphorical process for generating the higher energy-flux-densities required for making discoveries of principle in the minds of others. The same intention can be applied to Archytas. His intention was not to discover the duplication of the cube. The Archytas intention was to show that the only way you could discover a universal physical principle was by completely disregarding the cultural domain of mathematics and of the insidious trappings of sense-perception that go along with it. I don't know how the problem of doubling the cube really started, historically, but it rapidly became the test case which, very early on, kept the priesthood of the Oracle of Delphi and its retinue of early Greek mathematicians, so completely baffled and furious that they declared the problem to be impossible to solve. That was not the case, however, because it was properly treated by the Pythagoreans as a crucial test for solving the most important existential problem of creativity. It became known as the Delian challenge which demonstrated that the future of mankind utterly depended on being able to solve this type of construction problem, if mankind were to grow out of the Dark Age that Greece had fallen into at that time. So, I bring this up, again, because the crisis of civilization has come to a similar situation today.

The problem, as it was formulated before Archytas, probably started as a fairly innocent problem such as the derivation of the cubic content of a simple cube container. However, as soon as the double of the initial cubic container had to be constructed, physically, the problem rapidly turned out to be a mathematical impossibility, because it defied the domain of the commonly accepted experience of simply-extended sense perception of ***this is equal to this***. Archytas proved that the apparent truth of ***this is equal to this*** was not valid a priori requirement as a matter of principle, because equality implied no-change. On the other hand, the necessity of change implied that one had to go to the higher domain of ***this is to this as that is to that***. In other words, the main difficulty resided in the fact that the physical domain in which the problem required to be solved could not be done by a culture of mathematical deduction, but only by a culture of inferential knowledge. And mathematicians, as well as political intelligence officials, are known to be blind to this sort of problem. Their attempts at using equations by means of a calculator are the equivalent of trying to fit a square piece of wood into a circular hole of the same area. Equalizing simply does not compute properly, because it can only deal with an unchanging world.

In fact, the problem points to the failure of the deductive process of positivist and empiricism more generally. But, the point of interest I wish to bring your attention to, here, is the idea of doubling as a form of stereographic incommensurable power, as opposed to some mathematical quest of some exponential function. The problem is crucial because we know that nature doubles the size of things all the time. However, everybody takes it for granted that the universe is capable of doing it, but no one can tell you how it is done. And those who believe that mathematics can give you the solution are nothing but fools, because they are attempting to sell you a bill of goods based on generally accepted, but unproven assumptions. However, some people in ancient Greece thought this was an unfair situation, since man is creative; so they decided to solve the problem by means other than mathematics, and most emphatically, by inferential projection. And, the idea was not to make visible at a higher level, something that was not visible at a lower one. That would have been sophistry. The idea was to discover a principle from the corner of your mind's eye.

For example, the point that Plato made in his ***Timaeus*** was that the solution to the problem of doubling the cube required the principle of proportionality. He was right, and it was his friend the Pythagorean, Archytas (428-347), who discovered that the solution required two mean proportionals between two extremes that were given in the ratio of 2/1. In his proof by construction, Archytas demonstrated that in its creative doubling powers, the universe did not function like a deductive system, but rather like an inferential ironical process. The most important aspect of his discovery was to establish that the natural growing processes of physical spacetime in the universe required a principle of credit which always reflected a process that never was an extension of sense-perception, but an incommensurable extension into the unknown, an extension of what has not yet come into existence, but was on the verge of coming into being. So, they needed to know what the solution to the problem was before they started. They needed a sixth sense, that is, a sense of what the future could bring.

As Lyn demonstrated, the sixth sense consists in seeing what is not there before it comes to be, and most dramatically, that which is coming from the future. When a fox is looking for his next lunch in the middle of winter, and it cannot find it anywhere, it must turn around until it comes to be in line with the magnetic lines that flow in the direction of the north-south axis of the planet, and then, with its back to the Magnetic North Pole, it pounces on it! Even though it cannot see it, it knows it is there because it senses it resonating along the magnetic lines connected with its sixth sense. Fox Snow Dive. Now, that is what I call fox hunting.

In the same sense, the universe does not know where it is going, but it knows how to get there, because it has a preestablished harmony within which it has been enfolding since time immemorial, and its future oriented compass is constantly improving as new singularities of physical spacetime come to be generated. But now, man not only has a chance to be such a singularity, but has a duty to change the course of the universe as a whole for the better, and he must, therefore, develop that sixth sense, if he wishes to survive. Yes, we know where we want the universe to go, but it is as if we didn't know when to pounce. We may not know where we are going to end up, but, for the first time in history, the human species knows how to get into proper alignment and leap forward.

The difference between us and the fox, therefore, is that, for the human mind, the course we are engaged in is a conscious one. The fox is not creative, it is simply hungry. We know what the creative intention is, and the fox doesn't. From that vantage point, therefore, we must willfully discover the pathways that universal physical principles take as extensions of our inferential knowledge, as opposed to simple extensions of our deceiving sense-perception. This should not be too difficult to envision, if we understand that the universe as a whole is also literally creative and metaphorical in nature. So, as our own old fox put it, man must constantly look for new ways of expressing creativity in the universe:

> ***"Mankind creates, and that willfully, specific types of higher states of organization in the universe than would be generated by other means. It is such human activity, whether by some original inventor of the discovered principle, or by assimilating some broader implication of an already discovered such principle, that the essential changes of the human race to higher qualities of states of existence within the universe are accomplished in a willful mode."***

(Lyndon LaRouche, ***THE DEATH OF LONDON'S ROMAN EMPIRE,*** Morning Briefing, January 10, 2011.)

So, approach the Archytas discovery, by asking yourself: "***How do you make what does not yet exist, come into being?***" The idea is to look ahead, but to look ahead with the end product already existing in your mind. So, once you have the end in mind, you look for is a special sort of singularity, an anomaly that comes from a universal physical principle that is outside of you, and which is created to cause a change inside of your mind through the filtering screen of your sense-perception. For example, take the idea of beauty in Classical artistic composition. Beauty in art is not something that is nice to hear or nice to look at. Beauty is like a blemish, like an effect which offends people's beliefs and prejudices, and puts into question what they think should be nice. Classical beauty is always a shock to prejudices, an effect that is generally misunderstood because it is a truthful axiomatic disturbance.

In that sense, the beauty of artistic composition is something that is truthful rather than nice or pleasant. It is not an effect of sense-perception, but an effect that comes from a universal physical principle that uses sense-perception as a means to reach your mind in order to connect with the same principle located inside of you. This is what Plato called pre-existing knowledge. Such singularities are generally transmitted inside of an image, or inside of a poetic or musical phrase, for the purpose of calling your attention to something that must be changed, but which can only be caught by insight by looking into your own soul.

Singularities lodge themselves in the footprints of a principle as something that should not be there, something extraneous, like a pebble in your shoe. Most people will try to wiggle the pebble out of the way, but it will always come back annoying you, as soon as you start walking again. Unless you grow out of those shoes, you can wiggle your toes all you want; you cannot get rid of it. If the shoe fits, grow out of it. That is why only people with insight, like Archytas, will discover those singularities for what they are, and will deal with them as agents of change. Take the singularity of the intersection point between the cone, the torus, and the cylinder in the Archytas doubling of the cube. The doubling of the cube is a very annoying pebble. How do you deal with that?

The secret is not to focus on the physical objects of the cone, the torus, or the cylinder. Focus, instead, of the intervals of interaction represented by three different motions, a cylindrical/toroidal motion, a conical/toroidal motion, and a cylindrical/conical motion. It is the harmonic proportionality among those three interacting motions that generates the solution for the duplication of the cube, not the static sense-perception of three solids intersecting each other at one point.

Therefore, it is from the vantage point of such interrelated doubly-connected motions that you want to look for singularities; that is, limitations or interruptions in the boundary conditions relating to those processes. And, since everything that moves has to change, you look for the effects of such changing motions. And, the first change you discover with this method is that each doubly-connected motion traces a double curve: the toroidal/cylindrical curve, the toroidal/conical curve, and a cylindrical/conical curve. Your mind, once again, must focus on a process of double-curvature. With this in mind, you can then begin to investigate what real process of change, if any, you might relate these processes of double-curvature to. You then begin to see in your mind's eye that the dual motion of the moving cone with the moving torus, for instance, can be construed as a metaphor of some universal physical process. But what process?

These doubly curved footprints do not only generate traces that a cylinder, a torus, and a cone leave on each other by rotating against each other. That is another form of reductionism. Think of them as metaphors which act in cyclical fashion. (**Figure 5**, section 2 Archytas Model.) However, this is not the cyclical hand waving of a New York policeman during rush hour. It is closer to the baton waving of a Furtwangler, because it is the conical quadratic division of musical intervals within an octave that generates the doubling of the cube. Within the logarithmic values of our well-tempered tuning system, it is the appropriate octave intervals among C, E, Ab, and C that determine the musical function of doubling the cube.

Again, imagine that what Archytas is doing is not simply looking for a practical way of doubling the cube, and that his primary concern was to develop a method of seeing into the future. He was devising a method of forecasting future events. Now, take the following bold step, and imagine further that the connections among the multiply-connected motions of the torus, the cylinder, and the cone are shadows of the precession of the equinoxes! This is a shocking idea, indeed.

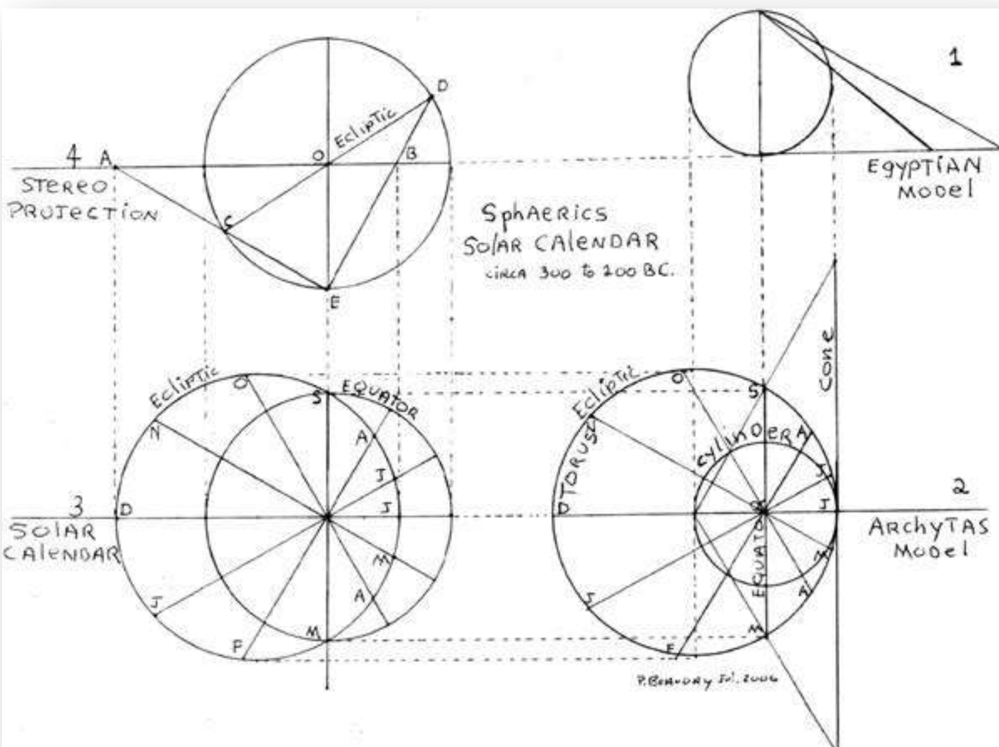


Figure 5. Footprints of the two-way process of the Archytas and Hipparchus discoveries: The Solar Calendar caught in between the shadow metaphors of the Archytas construction for the duplication of the cube and the Hipparchus stereographic projection for the precession of the equinoxes.

Take the intersection of the cone and the torus from section ***2 Archytas Model*** and connect them together as in section ***3 Solar Calendar*** of **Figure 5**. What you get is a solar calendar with the intersection of the equatorial circle and the ecliptic circle at the equinoxes of March and September. This is the key connection between the earth and the heavens that must be discovered in order to build the astrolabe. This is a doubly-connected manifold on the basis of which everything in the heavens can be made proportional with the orbs of reason in the human mind. Without this crucial connection, the astrolabe could not have been conceived, or constructed. The only thing that is missing to complete the idea of an astrolabe, however, is the practical application of the stereographic projection.

In other words, within these two intervals of discovery, there is an ironical anomaly of about 300 years which emerges inside of segment ***2 Archytas Model***. It is located in the equatorial line MS which is the shadow singularity of the conical elliptical function of our solar system moving around the inside of a galaxy represented by the torus, and which is located in the position where the three rotating Archytas solids come together to intersect in the process of duplicating the cube. That intersection is the kind of metaphorical branching point that you look for in a discovery of principle. One branch leads you to the doubling of the cube; the other branch leads you to building the astrolabe. Together, they reflect the creative principle of irony. Once your mind makes that connection, you have a metaphor of creative insight. As the famous American philosopher, Yogi Berra, put it: "***When you come to a fork in the road, don't hesitate, take it.***" This elliptic intersection MS is, therefore, like the perplexed prescient state of ambiguity of what is about to be discovered: either the proportionality leading to the duplication of the cube, or the proportionality leading to the precession of the equinoxes. Archytas discovered the first and Hipparchus discovered the second. See the main shadows of the Hipparchus discovery in **Figure 6**.

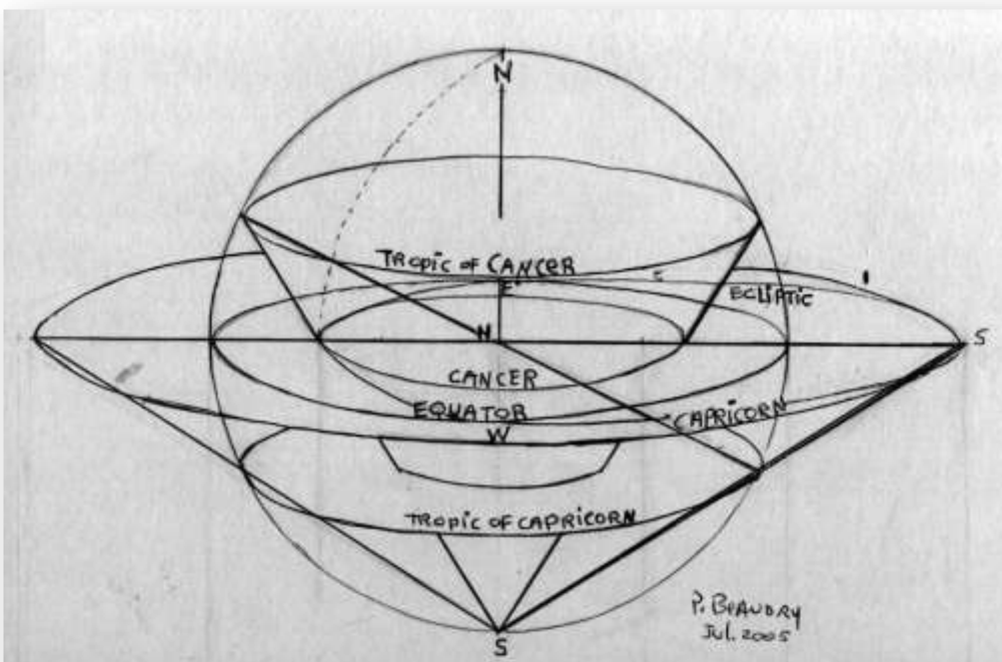


Figure 6. The Hipparchus stereographic projection of the universe rotating like a spinning top. All projections are made from the South Pole (S). Note that any star on the northern half of the celestial sphere may be mapped on the equatorial circle of that sphere, and every star on the southern half of the sphere may be projected, paradoxically, onto the extension of the same equatorial circle, but outside of the sphere of the universe.

The Archytas construction was a discovery made from the future, as if from the prescience of what was to come, but which had not yet been stated, and could not be stated until the stereographic principle had been discovered. As Lyn put it with reference to the function of the principle of metaphor:

> ***"The function of the great principle of metaphor, as in Classical poetry, and as in musical counterpoint composed and performed according to the principle of Johan Sebastian Bach, is to provide mankind with a prescience of the approach of a discovery of a great principle. We call this "Classical artistic" composition, because it is presented to us as if a voice of truth spoken from the future, not yet the present. It is the shadow cast by a principle of the universe which is yet to be spoken; thus, properly uttered, or performed as music, it is the idea which can never be deduced, since it is the prescience of that reality which is still waiting impatiently to become discovered.***
>
> ***"Those who do not serve that principle should – please – never compose Classical poetry or music. All competent practice of scientific discovery depends upon exactly that same principle of the prescience of a hand from the future reaching in to touch one's soul with a discovered principle of nature yet to be born. The true mission of the human individual is to feel the prescience of the principle which is about to be born.***
>
> ***Such is the true content of the much abused word named for a prescience of immortality, a prescience which you might wish to call "love" of being, in that moment, truly human."*** (Lyndon LaRouche, ***THE DEATH OF LONDON'S ROMAN EMPIRE,*** January 10, 2011.)

And, that was also the only way that Archytas could conceive of his own discovery as the prescience of the future discovery of the astrolabe by Hipparchus. What he had to do was to imagine himself taking the impossible bold step outside of his own universe, and look at himself from the vantage point of an external point of view of what he wanted the universe to become. The discovery of the astrolabe was developed with this same idea in mind, following the pathway set by Archytas, and by Hipparchus applying the paradox of being, in the simultaneity of eternity, both inside and outside of his observation of the universe, at the same time, as defined by spherical angular differences between stars. Naturally, one way or another, such discoveries were always ahead of their times.

The invention of the astrolabe by Hipparchus (active between 161–126 BC) was one of the most extraordinary discoveries of all times, precisely because it reflected the footprints of the principle of creativity located in the heavens, and as first devised by Archytas for the duplication of the cube. The Pythagoreans and the Platonic philosophers understood this principle, because they had a profound understanding of the paradoxes involved in the science of ***Sphaerics***. They understood that the orbs of the heavens had to be incommensurably proportional with the orbs of human reason, without which humanity would be driven to madness.

From that standpoint, therefore, the greatest usefulness of the astrolabe is not its practicality of locating the position of a star on any day of the year with respect to the position of any observer in the universe, although it could do that with extreme precision, but the seemingly impractical design of developing the creative weak forces of the human mind. Like the Archytas duplicating of the cube or the solution of the Banneker puzzle, the construction of the astrolabe was one of the most important inventions for the development of human creativity, because it was meant to break the human chains of bad habits and save civilization from being destroyed.

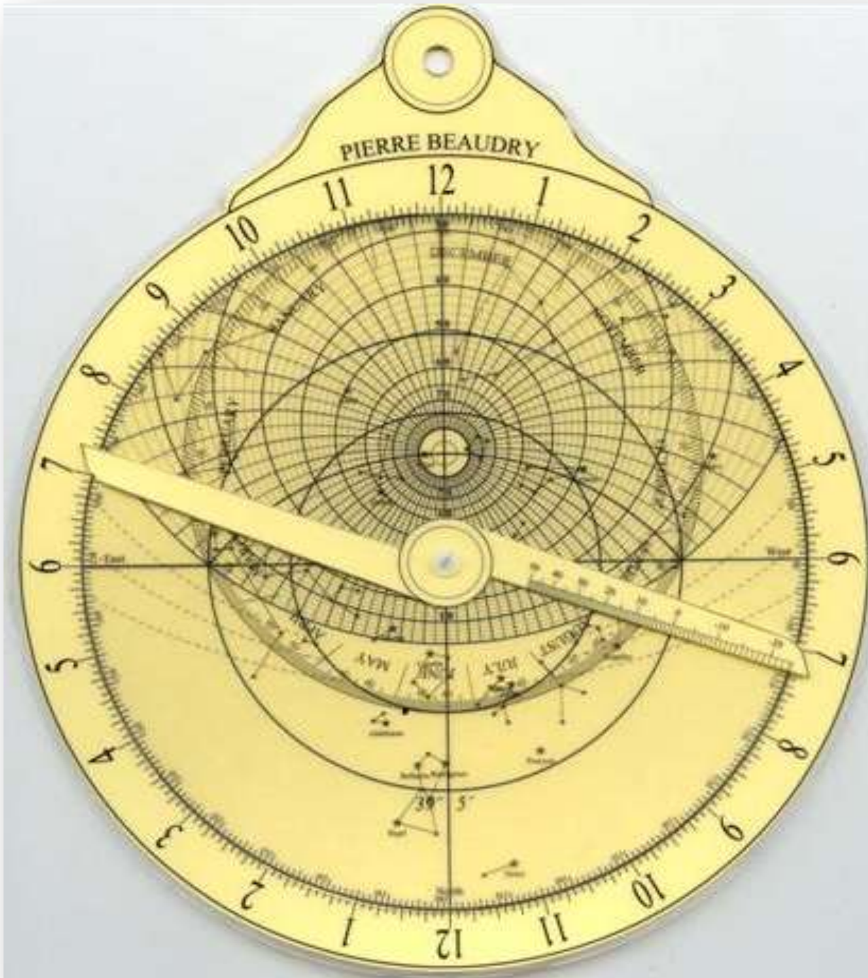


Figure 7. Astrolabe for the location of Leesburg Virginia, 2010. This astrolabe was built by James E. Morrison for Pierre Beaudry from a non existing Zenith point outside of the universe which corresponds to Earth Latitude 39° 5' and Longitude 77°35' west. Longitude correction is -10 min. 20 sec. If you wish to construct an astrolabe for your own location, see my report ***MIND AS A MATTER OF POWER OVER THE UNIVERSE***, Montreal LYM class, 6/5/2009, or write to me at pierrebeaudry@larouchepub.com

In conclusion, let me focus your attention on the two fundamental paradoxes that are required to be solved in order to properly relive the discovery of principle of the astrolabe. First, you have the paradox of stepping outside of the universe in order to determine its boundary conditions, and second, you have to resolve the impossible paradox of flattening the sphere onto a plane by maintaining the same angular determinations.

The first paradox is the Archytas Paradox, because Archytas was the first to explicitly challenge man with the idea of imagining himself looking outside the limit of the universe. By doing this, he was able to find the solution for doubling the cube, which was a first step in establishing the discovery of the astrolabe. The second paradox is the Hipparchus Paradox, because he was the first astronomer to discover, and to successfully apply stereographic projection to astronomy by solving the paradox of mapping the sphere onto a plane by inventing stereographic projection, and establishing the precession of the equinoxes. In both cases, the solutions of the paradoxes required that the observer step outside of the universe and set his observation from the top down, looking in from that higher vantage point, as opposed to looking out from the bottom up. In other words, the power of reason that rotates over your head must be made proportional to what rotates inside of your head, and not the other way around.

Figure 8. *The Flammarion Woodcut*, Archytas discovering that the boundary singularity of the universe is "***Here and everywhere!*"(*Urbi et Orbi!*)** Cf. Pierre Beaudry, ***Timereversal in the Simultaneity of Eternity,*** 5/19/2010.

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