(A pedagogical experiment in universal history) PART II

THE EGYPTIAN SCIENCE OF SHADOW RECKONING AND THE DOUBLING OF THE CUBE. BY CONIC FUNCTION

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2. SHADOW RECKONING AND THE DIFFERENCE BETWEEN MAN AND ANIMAL.

As I discussed with you two years ago, the very first scientific inquiry of the Egyptians originated with the science of shadow reckoning and the most important discovery of ancient times they had made, thanks to that method, was the discovery of the {*precession of the equinoxes*}. The very first steps into that discovery were realized when the priests of Amon in Thebes and Heliopolis, such as the great Imhotep, began to teach the science of projecting, among other things, a light source against a gnomon, an obelisk, or a column which served to discover the relationships between the heavens and the earth, {*by angular measurement alone*}. Then, during the third millennium BC, the same method of shadow reckoning was used to build the astronomical observatory of the Great Pyramid, as I showed in the 21st Century, Summer 2004.

In other words, the Egyptian political leaders, such as Imhotep, were not teaching some mumbo jumbo Masonic mystical secret knowledge to an elite priesthood, as was done to do later on. The initial school of the Great Pyramid was teaching the very first science of astrophysics based on an already ancient method of shadow projection out of which ultimately came a solar calendar that established the Sun's annual motion of the Ecliptic in coordination with the motion of the so-called fixed stars. The study of the Sun's motion was not merely done to mark the passing of time. The marking of time by shadows was a mere practical result in this process of discovery of what the Greeks called {*Sphaerics*}, and what Gauss later called the complex domain. Therefore, the intention or purpose that shadow reckoning was aimed at was to solve an epistemological inquiry. That is, how can man demonstrate that he is different from the animal? How can man access the hidden principle behind the ordering of the universe, and through the strength of that knowledge, act on the paradox of changing the universe as a whole? What was the nature of the power relationship between heaven and earth? This is how the doubling of the cube began to be developed as a means to express the power of the human mind even before it was discovered to express a universal physical principle.

Today, British freemasonry makes believe that the relationship between the pyramids of Egypt and the heavens is based on the sophistry of mapping the three pyramids to the stars of Orion, and correlating the overflowing waters of the Nile River with the Milky Way. Is that a lawful relationship between the Heavens and Earth, between God and Man? Isn't this what Plato denounced as the evil of imitation {*mimesis*} in the Republic? Should man imitate God in this fashion? How can such a relationship be measured? Can this relationship be measured by repeating what He says and by copying what He does? Take Lyn as an example. Lyn is the most God-like person on this planet and throughout all of history. Does being like Lyn mean to imitate him? Every member of this organization had to reflect on that question when he or she joined the movement, and still does, to this day.

Nicholas of Cusa also banned the practice of imitation and he likened the relationship between God and Man, i.e. truth, as the incommensurable relationship between the polygon and the circle. Is that incommensurable leap not a more proper relationship than imitation? But, you might ask, is there not a complete discontinuity between the two, no common measure at all between the two, except an incommensurable epistemological gap? Or, is the difference between the two so transcendental that it is beyond all intelligibility and that only a mystical experience can be established between them, as secret societies claim they do in their Masonic initiations? How can this difference be made intelligible as an incommensurable measure between two fundamentally different levels of power? How can incommensurability be a measure? Isn't that a great paradox? How can you make intelligible the incommensurable gap between doubling the square and doubling the cube? Isn't that a similar paradox? If you can construct and solve one of those paradoxes, aren't you also shedding some light on how to solve the other?

"YOU ARE MY CHOSEN PEOPLE!"

In his book, {*The Kuzari*}, Judah Halevi wrote that when Bulan, the King of the Khazars, converted to Judaism, he had asked the Rabbi why the Jewish people were the {*chosen people.*} The Rabbi answered that it was in order to make the difference between man and animal! He also showed that the distinction arises when someone recognizes that the {*chosen people*} are those who don't consider that God created the world for their own sake, but for the sake of all of mankind; then, the chosen ones become universal souls as opposed to predatory creatures. So, this is the way they resemble God, not from the standpoint of perception, or imitation, but from the standpoint of God's divine characteristics. How do you know that is true? You know it because every new paradox that you solve increases your power over the universe.

Therefore, the Mosaic principle of man created in the likeness of God is not based on religious faith at all, but on the scientific recognition of the difference between man and animal; and as a result, man should never be treated as an animal, because between them, there is the difference of an irony that animals cannot understand. It is precisely this type of irony that created the special relationship between the Prophet and the Israeli people in the first place, and between Israel and mankind afterwards. Let me give you an example of how this works. Let me tell you the story of the young Jewish schoolboy, Moishe.

In school, Moishe was always chosen by his blind teacher to do a public reading before the class, simply because he always did it so well. However, one day, Moishe got tired of being chosen and decided to go and hide in the back of the class. The teacher came in and said: "OK, open your book at page ten, and Moishe, please start reading.

Then, there was a heavy pause of silence. "Where is Moishe?" The teacher asked. "Why aren't you reading?" After a second pause, Moishe answered in a high pitch voice from the back of the room: "Moishe is not in school today!" - "All right," said the teacher, "then, you do the reading!" You see, that is how the chosen people are chosen: God chose them because they have a talent, and they have a talent because God chose to give it to them and not to animals.

In Part I, Section 102 of {*The Kuzari*}, Bulan-Khazari asked the Rabbi: "{*Would it not have been better or more commensurate with divine wisdom, if all mankind had been guided in the true path?*}" The Rabbi replied by going directly to the axiomatic issue and answered back by asking the totally provocative question: "{*Or would it not have been best for all animals to have been reasonable beings?*} Thus, the chosen people of Israel became the instrumental cause of the good in the universe; by choosing to act differently from predatory animals, and only to the extent that they acted in accordance with the axiomatic difference between man and animal, which is to act as a historical being, as an immortal being, out of love for mankind. This is what the Khazar Kingdom was all about during their ecumenical alliance with Charlemagne and Harun al-Rashid, during the first part of the 9th century. The Khazars represented the first international application of the principle the {*Advantage of the other*} of the Peace of Westphalia, the germ of the American System of Political Economy.

Now, consider the following form of measurement between animal, man, and God. What if I establish an incommensurable difference of species that says: {*animal is to man as man is to God*}. Does that make sense? Is that an appropriate and intelligible proportionality? What if I expand this into a double proportional relationship such that {*The Abiotic is to the Biotic as the Biotic is to the Noetic in the same proportion as the Noetic is to the Divine*.} Is that a Divine Proportion in which the "fourth domain" that Lyn speaks about subsumes the other three domains proportionately? As Lyn indicated, the chemistry of the living cannot be found in the non-living. Similarly, the chemistry of human cognition cannot be found in animals, yet, all four domains interact with each other. That is what civilization started from. This is what Thales, Pythagoras, and Plato called {*Hylozoic Monism*} in which the intention of the universe as a whole was expressed as a {*single living principle of change*}. "You never bathe twice in the same river," said Heraclites.

SHADOWS, SHADOWS, AND SHADOWS.

So, how can this be expressed as a dynamic incommensurable relationship? How can such differences of power be expressed as the difference of powers between the line the surface and the volume? How can we find such a proportion simply by looking at these primary Egyptian shadows? How can these simple angular forms act as appropriate incommensurable metaphors for what we are trying to measure? First, take the illustration I sent you a few weeks ago on the different shadows projected in Egypt, at the latitude of 30 degrees, and relate them to this question of incommensurability between the Heavens and Earth. What do these shadows tell you? Establish the boundary conditions for your observation and apply to them the Dirichlet Principle.

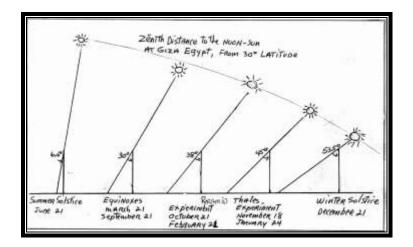


Figure 1. [Shadows projected in Egypt at the latitude of 30 degrees.]

One of the first crucial steps the Egyptians took in the investigation of the heavens consisted in using angular measurements of celestial phenomena and they reflected on the nature of their relationships between human intelligence and the intelligence in the heavens. They began with relating the fixed stars with the axis of the celestial sphere, the apparent movement of the Sun during the year, and the relative position of the Pyramid of Egypt on Earth, vis-a-vis the Sun and the North Star. This was an early form of seeking to discover a solar calendar, otherwise known as the astrolabe, but without making its actual discovery.

So, with latitude at 30 degrees, the positioning of the Great Pyramid of Egypt was determined from the Pole Star, and then the range of the Sun's apparent motion was between 6.5 degrees at Summer Solstice (June 21) and 53.5 degrees at Winter Solstice (December 21). How did they arrive at that conclusion? What determined those angles and that range? The most important discovery was that of the Zenith Function, that is, the determination of a non-existent point above your head, located on the heavenly sphere, and which represents your location with respect to the entire universe as a whole. That is the position of the scientist on the surface of the heavenly sphere. Now, if you project a ray from that Zenith point over you head perpendicular to the site of the Great Pyramid in Egypt, where you stand, your Zenith distance to the North Pole forms a right angle with the position of the Sun at noon, then, you are on the day of the Equinox, and the Sun is crossing the equatorial circle of the celestial sphere.

In **Figure 1**., the Zenith distance to the Sun is chosen to determine the angles at noon. The point to be made here is that if you know your Zenith distance to anything in the universe, at any time night or day, you can never be lost! Secondly, establishing the minimum and maximum angles between the Zenith distances to the sun at noon on the days of summer solstice and winter solstice, and comparing them with the Zenith angles

to the Sun at the equinoxes, determines and locks in the range of the yearly cycle of the Sun.

That also gave the Egyptians the four markers for the orientation of the Great Pyramid, and, to their surprise, the difference, year after year, was, in each case, always the same angle. Thus, the Egyptians had found a normalizing mean of establishing an astrophysical solar calendar. The year cycle was of 360 days (or 360 degrees), that is, twelve months of 30 days each, and three seasons of 4 months each. The added 5 and ¹/₄ days remaining were gifts from the gods that were not accounted for in their common calendar.

Now, let's look at the significance of those shadows. No matter what year it is, the angle between the maximum and the minimum is always, invariably, 47 degrees. One interesting feature of this is that on the days of the Equinoxes, on the plane of Giza, the Zenith distance to the Sun at noon is 30 degrees, while the Zenith distance to the North Pole is 60 degrees, which means that, at the location of the Great Pyramid on the days of the Equinoxes, all of the gnomons form shadows that relate to the equilateral triangle as well as the scalene 90, 60, 30 degrees right triangle. This shadow reckoning should also be experimented by the LYM of Houston, because their latitude is also 30 degrees (or to be more precise, 29.97 degrees). How does that relate to the so-called Platonic solids and to the Great Pyramid? Also, how does that relate to the boundary conditions of the Sun's yearly cycle of the Ecliptic? The question is, how do you find the location of the Ecliptic in the heavens? How can you find that pathway of the Sun during the entire year, when all you see are the time lapse snap shots of the Sun following a straight line up and down in the blue sky above?

What is the significance of this sort of animation? What does it tell you about the motion of the Sun around the universe? This locates the second most important non-existing point in the heavens after the Zenith Function, that is the crossing of the Celestial Equator by the Sun at noon on the day of the Equinox. Now, what is the significance of this intersection? Remember, you cannot see the pathway of the Ecliptic and you cannot see the pathway of the Celestial Equator, yet you can know the angle between the two. How can you do that if you can never see them? How can you determine the angle between two things you don't even see? You don't see those two curves, yet you know that the angle of their intersection is 23.5 degrees!! In other words, the angle between the extremes of 6.5 degrees on June 21, and of 53.5 degrees on December 21, is always 47 degrees and you know that half of that angle is 23.5 degrees, which establishes the angle of the two Equinoxes at 30 degrees at the Great Pyramid.

So, now that you have established the angle between two invisible things, you have now two new markers, representing the days of this special invisible intersection, March 21st, and September 21^{st,} the days of the Equinox. This is the way the Egyptians lockedin the astrophysical pathway of the Sun during their calendar year of 360 days. This is a major astrophysical discovery, because they were able to create a {*stereographic mental mapping of the motion of the Sun*} moving around the universe as a whole. Then, all they had to do was to find the right projection from the sphere of the heavens and connect it with the plane. The great incommensurable relationship between the sphere and the plane began to be resolved. Again, the celestial sphere had to be projected against the wall of Plato's cave, that is to say, onto the plane of the visible domain?

But it gets more complicated. Just when they thought they had pinned down the Equinoxes and the Solstices, the Egyptians discovered that these points were also moving. They noticed that the equinoxes were moving, year after year, in an opposite direction, that is, from East to West during a very long period of about 25,920 years (one degree every 72 years), which they called the {*Precession of the Equinoxes*}. That was another crucial non-visible and incommensurable Riemannian type of relationship. The {*Precession of the Equinoxes*} represented a similar higher dimensionality as did the doubling of the cube with respect to the doubling of the square; which is why it is entirely feasible that the doubling of the cube was first discovered in ancient Egypt, before it was discovered in Greece.

Indeed, not only the Egyptians had to account for the time of the apparent motion of the Sun during the year, but also they had to account for the time that the Sun traveled around the Universe during a period of 25,920 years. This created an infinitesimal difference between sidereal time and earth time for every second of the day. It appeared that there were no longer any means of establishing any fixed parameters outside of discovering what the ordering principle of change in the infinitesimally small higher dimensionality was all about. This became the Kepler challenge that was later taken by Leibniz and his calculus. As a result, the Egyptians began to realize that the sidereal day was shorter than any infinitesimal moment of the smallest part of each second during one revolution of the Earth.

So, to sum up what I have said, those shadow angles of Figure 1., are not merely fixed reflections, or traces of the daily positioning of the Sun. They also reflect the limit of the year and the boundary condition for the {*Precession of the Equinoxes*}; neither of which have visible curvatures! So, here is the problem: how can you map the curvature of the Ecliptic, that is, the pathway of the Sun during one full year? All that we see are the daily-snapshots of the yearly traveling of the sun, at noon, but these snapshots are pointing to its curvature like the slow time-lapse sequence of an animation. So, we get into a nice little ambiguity here, because of the Ecliptic and because of the {Precession of the Equinoxes }. The next question has to be: how did they determine the curvature of the Ecliptic cycle for an entire year? How did they express the non-visible curvature of the Sun's yearly cycle by means of {*Sphaerics*}? This is not just the apparent motion of the Sun going around the Earth. This is the non-visible motion of the Sun going around the Universe as a whole in coordination with the motion of the pole star because of the rotating axis of the Earth. This is as the infinite motion of simultaneity of eternity. How do you make that visible in a way that is a true representation and not a sophistical fallacy of composition? If you find the answer to that question, you have found the solution to the paradox of the astrolabe.

In their study of shadows, the Egyptians noticed the discrepancy between the projection of light from a sphere, and the flat shadow that was cast onto a plane, as in a

Sundial. That relationship between the sphere of the heavens and a plane on the earth became the first solid geometry paradox of ancient Egypt. The paradox was: how do you project a curved surface onto a plane surface? How can a sphere be represented onto a plane? How can the relationship between the two be normalized? That problem had been a very puzzling and perplexing paradox from the very beginning of astronavigation and it is very difficult to determine exactly when it was actually resolved. I will later submit a hypothesis that the solution may have been found when Hipparchus created the astrolabe during the second century BC, and is said to have discovered the {*Precession of the Equinoxes*}. It may have been discovered before that, but, so far, I have not met with much satisfactory evidence.

Indeed, if you project a light from the center of a transparent sphere, all of the curved lines on the surface of the sphere will appear as straight lines when projected onto a plane ceiling! However, if someone tried to see the difference between the sphere and the plane by squishing half of a hollowed out orange or grapefruit onto a plane, you would rapidly discover that you have a very messy situation on your hands, because it simply doesn't work. The two surfaces are incommensurable. Ahhh!!!

Here we are, again, with the problem of incommensurability between two geometric species. How did the Egyptians and the Greeks discover a way to measure curvedness and straightness at the same time? The Egyptians invented the conic section that came to be known as the compass and the conic angle divider. They realized that any measurement with the conic section of a compass enabled them to solve the paradox between curvedness and flatness. Indeed, angular measurements with a compass are the same on a curved surface as they are on a flat surface. So, what appeared to be impossible became possible with the use of the compass. This is the reason why the Classical Greeks established that everything they could construct in geometry had to be done with a compass and a straight edge alone. So, it is important to reestablish that tradition of the conic function. Therefore, the question is, how does an angular projection make possible what appears to be impossible? How could a conic projection solve the paradox of the sphere and the plane?

3. THE DOUBLING OF THE CUBE BY ARCHYTAS

The first thing you must avoid when you are first introduced to the Archytas doubling of the cube is the trap of fumbling all over the cone, the torus, and the cylinder, as if they were things in and of themselves. They are not. They are visual traps. What you must do, immediately, is to look behind the visible domain and reach out for the principle of what Lyn has always identified as multiply connected circular action. The reason you want to concentrate on intervals of circular action is because they always express the principle of least action in some form of proportionate way. For example, the difference between the doubling of the square and the doubling of the cube is proportional to the difference in the circular action that is required between determining one mean between two extremes and two means between two extremes. This is the way that proportionality of different actions brings closure to the physical boundary conditions of a change in power in the physical universe. Therefore, the doubling of the cube will require a doubly connected circular action, and the way to discover that is by way of a stereographic conic function of projective geometry. The idea to be grasped, here, in this whole construction, is to generate the solution by both an orthographic and stereographic projection of the conic function as indicated by Lyn. How do you do that? Let me describe this by constructing a workable Archytas model that you can use in your deployments.

Start with the same white cardboard material as you did before, with the Egyptian model, and establish a baseboard 23 cm by 45 cm. Draw the appropriate circle and conic triangle lines exactly as in the previous Egyptian model. The crucial difference, here, between the two models is that Archytas transformed the double conical projection of the Egyptian Sphere into a Cylinder on which he generated two bold curves by means of a Cone and a Torus. So, instead of drawing the great circle of a sphere, start by drawing the base of a cylinder with an 18 cm diameter. Determine all of the same parameters as in the previous Egyptian construction as the {*necessary predecessor*} to the Archytas construction to draw the shadow-line AMD'.

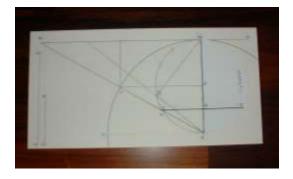


Figure 2. [Construct the baseboard of the Archytas construction.]

Construct two parts of a half cylinder whose height is 9 cm. Draw on each part the letters identifying the different points of the base board, that is, A, B, M, C, then line BB' and BJGH. Next, construct the two bold curves of the Torus-Cylinder and the Cone-Cylinder and establish point P as the intersection singularity of four degrees between the two double curves. Think of the intersection of those two bold curves as the traces of two circular actions generating a single unity of effect that is stereographic in character. Then, think of the Torus-Cylinder Action separately from the Cone-Cylinder Action. They are both constructible only by circular angular rotation. The reason you want to start with the construction of the Cylinder is that this is the only one of the three solids that is not moving, and it is the only solid on the surface of which the two bold curves can be traced.

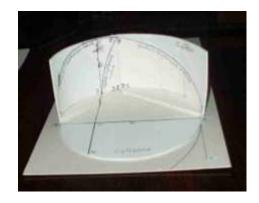


Figure 3. [Construct the Torus-Cylinder curve on a half-Cylinder.]

THE TORUS-CYLINDER ACTION



Figure 4. [Construct the Torus half circle and two animation tracers.]

Construct the half circle of the Torus as you did the Cylinder base, with a diameter of 18 cm. The two circles have the same diameter. Starting from an initial position at AC, rotate the Torus half-circle one full circumference around the fixed pivot point at A. Trace a quarter circumference on your baseboard and note that the Torus half-circle motion intersects the Cylinder at every point of its wall. Construct the stereographic image of that Torus-Cylinder curve. The motion of the Torus is leading toward a point of singularity, point P, which is the only point that intersects simultaneously the three surfaces of the Cone, the Torus, and the Cylinder! There is no other point, anywhere in that whole complex construction which intersects the three surfaces all at once. Everywhere else, the surfaces of the three solids only meet two by two. So, the question is, can you find in the Torus-Cylinder Action any reason to stop at point P? The answer is no.

When you rotate the Torus half-circle, perpendicular to the base circle of the Cylinder, you see in your mind the trace of a bold curve, as it has been called, on the surface of the Cylinder, and as Bruce and Jonathan have demonstrated in their own pedagogical. The curve has been called "bold" because it is daring, because it is the coastline that rises "bold" between the two domains of Euclidean Flatland and the domain of Pythagorean Sphaerics. This bold curve is a double Torus-Cylinder curve; meaning that it traces the shadow-contact of the two surfaces as the motion of the Torus half-circle constantly intersects the fixed surface of the Cylinder in its angular rotation. As you follow that trace on the Cylinder, imagine that the same trace is moving slowly on the circumference of the Torus half-circle, from C toward P. Now, consider this curving action as an {*axiomatic change indicator*}, for it has no other meaning than to trace the shadow leaving point C and moving toward the singularity of point P as if it were tracing the pathway of the axiomatic change between the doubling of the square and the doubling of the cube.

You can easily establish this curve by having the Torus half-circle stop anywhere you choose along the base of the Cylinder and trace two other different positions on two animation tracers that will reflect the positions of two different points on the Torus-Cylinder curve. (See **Figure 4**.) Bear in mind that what is special about that point P is that it is the point where the Torus and the Cylinder meet the {*conic function*}. Ahhh!!! So, there you have it. There was a reason for the Torus-Cylinder Action to stop at P, and that reason is to be found in the cone. So, there must be a second {*bold curve*}, generated by the Cone-Cylinder Action of the {*conic function*}, and which will contain the reason why the Torus-Cylinder curve must stop at P. This is where the Egyptian missing link comes in. How can we construct that?

It is essential to discover this point by a construction process and not simply assert its existence by intuition. The reason why it is necessary to go through the constructive proof of a discovery is because a so-called intuitive proof is a front for a false underlying assumption. This is the reason why Bernoulli, for example, wrote a letter to Newton, urging him to send him his method of construction for the catenary curve, because Newton was flaunting his solution without showing what was behind it.

Of course, Newton never sent him his method of construction for the catenary because he had copied the answer from the back of Leibniz's book. So, you see, Newton was hiding behind his "intuitive proof" because he had not found it by construction and he wanted to use that cover as a means of exercising authority and power over people. That is the underlying assumption, which is hiding behind an intuitive proof, and that is why you must always provide a constructive proof in everything that you do.

THE CONIC-CYLINDER ACTION



Figure 5. [Construct the scalene conic section and the Cone-Cylinder curve on a half-Cylinder.]

Lastly, let us construct the bold curve of the {*conic function*}. Rotate the scalene conic section through the entire half-Cylinder. That 90-degree rotation generates a quarter of a cone whose axis is hinged at AC in the plane. Now, as you elevate the tip of the scalene-conic section around the axis AC, the quarter conic rotation traces a curve along the entire surface of the half-Cylinder up to a maximum point at D'. Cut the Cylinder along that Cone-Cylinder curve. This last step shows why the whole process of the double circular action stops at point P on the Cylinder. It is because this angular elevation is a mixture of 45 and 38 degrees, that is, {*the angular difference between doubling the square and doubling the cube!*} That was the crucial singularity to be discovered. That is the singularity where the crucial discontinuity of a change of power between the plane and the solid becomes intelligible to your mind's eye.

As a result, point P becomes a {*quadratic singularity point*} intersecting four surface contacts: two between the Torus and the Cylinder surfaces and two between the Cone and the Cylinder surfaces. In fact, this is the only {*quadratic singularity point*} in the entire Archytas model. This point of discontinuity could also be likened to a thermodynamic phase-space transformation between solid, liquid, and gas. As Lyn often demonstrated, and as the current crisis-point in history also shows, a point of high density of singularities represents the turning point of a physical axiomatic change. So, the significance of point P is that it acts as a catastrophic shock-effect point, or as a turning point of opportunity, at any rate, as a change of power, a Riemannian change of geometry. That is what point P is all about. It is an axiom busting point of four degrees, something like a four-degree osculation that Leibniz talked about in his {*Acta Eruditorum*} papers.

Thus, point P determines the summit of the orthographic shadow-line PM, along the Cylinder wall, which establishes the two cubic roots, AM and AP, corresponding to the sides of two cubes that respectively double and quadruple the initial cube whose side is AB. It is also interesting to note that these two cubic roots also have a certain correspondence to the Lydian musical conic function, which divides the octave by half and half of the half in the logarithmic spiral action of the well-tempered system. Most emphatically, however, this passing from the domain of the square roots and the cubic roots leads directly to the crucial point made by Gauss in his 1799 polemic against d'Alembert, Euler, and Lagrange and their "fictions" of imaginary roots. The Archytas construction obviously provides Gauss with the constructive proof that there is an axiomatic difference between shadow and "*{merely a shadow of a shadow.}*"

Thus, Archytas established the two mean proportionals that were required to be found between two extremes in a ratio of 2/1. Such is the {*quadratic proportionality*} of the conic section where **AB** : **AM** :: **AP** :: **AP** : **AC**. That is the pivot of the {*quadratic conic function*} that Lyn identified as the key that unlocks the Archytas theorem, and which brings it in congruence with the Pyramid of Egypt, where {*the height of the Great Pyramid is to its apothem as two mean proportionals are to the doubling of the cube.}*

FIN PART II, July 22, 2006.