(A pedagogical experiment in universal history) PART V

THE EGYPTIAN SCIENCE OF SHADOW RECKONING AND THE DOUBLING OF THE CUBE. BY CONIC FUNCTION

by Pierre Beaudry
(Class of constructive geometry for the Philippines LYM. 9/02/2006.)

1. HOW DO YOU CONSTRUCT LOGARITHMIC PROPROTIONALITY BETWEEN THE SOLAR SYSTEM AND THE HUMAN MIND?

I wish to start today's class with by responding briefly to Lyn's challenge for the {LYM SCIENCE PROGRAM} of August 28, 2006, especially the first step in which he related to the discoveries of Kepler and Leibniz. Lyn wrote:

"{***a.) The concept of the ontological infinitesimal of the Leibniz calculus, as derived from Kepler's notion of the action of planetary orbit as defined by the ontologically infinitesimal reflection of universal gravitation. This is the basis for the Leibniz discovery of the only competent version of the origin of the calculus***.}"

I think it is crucial that all of you should be concentrating on that program more and more every day. As for myself, I won't be able to answer all of your questions about that, but the way I propose to look at the "ontological infinitesimal" question of the Leibniz calculus is with the conical construction of logarithms as Lyn initiated that in Chapter III of his {***So, You Wish to Know all about Economics***}, which I have already e-mailed to you.

I think the best way to begin this is with Plato and his higher hypothesis of the principle of proportionality between human vision and the universal ordering of the heavens. This is the most useful connection to establish between {***Sphaerics***} and the human power of reason. In this respect, I shall relate for you what Plato said in his {***Timaeus***} and what Kepler said on the same subject in his {***New Astronomy***} with regards to Leibniz. I find it is an interesting way to address the three-body problem. Plato said:

"{ ***…God created and bestowed vision upon us so that we, contemplating the orbits of our intelligence in the heavens, might put them to use by applying them to the orbits of our reason, which are related to them…***" (Plato, {***The Timaeus***}, 47b.)

Now, connect this thought with the following Kepler statement about the harmonic proportionality of the solar system in relationship to human reason.

"{***For as the Sun in its revolution about its own axis moves all the planets by the emanation which it sends out from itself, so also the mind, as the philosophers tell us, understanding itself and all that is in itself, stimulates the use of reason, and by spreading and unfolding its simplicity, causes all things to be understood. And so closely are the motions of the planets around the Sun and the processes of reasoning linked and tied to each other, that if the Earth our home, did not measure out its annual circuit in the midst of other spheres, changing place for place, position for position, human reasoning would never struggle to the absolutely true distances of the planets, and to other things which depend on them, and would never establish astronomy***.}" (Kepler, {***New Astronomy***})

In Part III of his {***New Astronomy***}, Kepler investigated several hypotheses with respect to the motion of the Sun. These investigations are some of the most daring reflections ever made about the astrophysical principle of gravitation underlying the solar system as a whole, and which reflect Kepler's concerns about finding new ways to investigate the question of equal spaces measured out in equal times. I bring to your attention especially the following gravitational proportionality between the Sun, Mercury, and Saturn. I stress this because this question of proportionality will go a long way toward explaining what sort of tempering effect must be emanating from our solitary fast-spinning sun in order to determine a planet's periodic time within the minimum-maximum ratio of a conic function? Kepler established this internal proportional tension using Mercury as the minimum and Saturn as the maximum. Kepler wrote:

" {***Further, we see that the individual planets are not carried along with equal swiftness at every distance from the Sun, nor is the speed of all of them at their various distances equal. For Saturn takes 30 years, Jupiter 12, Mars 23 months, earth 12, Venus, eight and one half, and Mercury three. Nevertheless, it follows from what has been said that every orb of power emanating from the sun (in the space embraced by the lowest, Mercury, as well as that embraced by the highest, Saturn) is twisted around with a whirl equal to that which spins the solar body, with an equal period***.}" (Kepler, {***New Astronomy***}, p. 388.)

Consider, then, the following relationship between the sun and all of its planets and examine how the sun's rotation caused a similar proportion to be affected according the changing circumstances of each planet and their satellites. Thus, what is the significance of the proportionality between the semidiameter of the sun and the orb of each and all of the planets that it generated, millions of years ago, to form what Pythagoras and Kepler called the harmony of the spheres. Kepler wrote the following stunning {***Geistesmassen***} of proportionality for the solar system:

"{***From this it is considered that the rotation of the solar body anticipates considerably the periodic times of all of the planets: therefore it must rotate in its space at least once in a third of a year***.

However, in my {Mysterium Cosmographicum} I pointed out that there is about the same ratio between the semidiameters of the sun's body and the orb of Mercury as

there is between the body of the earth and the orb of the moon. Hence, you may plausibly conclude that the period of the orb of Mercury would have the same ratio to the period of the body of the sun as the period of the orb of the moon has to the period of the body of the earth. And the semidiameter of the orb of the moon is sixty times the semidiameter of the body of the earth, while the period of the orb of the moon (or the month) is a little less than thirty times the period of the body of the earth (or day), and thus the ratio of the distances is double the ratio of the periodic times. Therefore, if the doubled ratio also holds for the sun and Mercury, since the diameter of the sun's body is about one sixtieth of the diameter of Mercury's orb, the time of rotation of the solar globe will be one thirtieth of 88 days, which is the period of Mercury's orb. Hence, it is likely that the sun rotates in about three days.}" (Kepler, Op. Cit., p. 389.)

Clearly, as the daily rotation of the earth comes from the rotation of the sun, in the same manner the rotation of the moon comes from the rotation of the earth; and so it is for all of the other planets and moons. Could this be the basis of the calculus that Kepler was asking future mathematicians to investigate as a reflection of the principle of universal gravitation, and which would express the anti-entropy universal principle of the solar system as a whole? There is, definitely, a relationship between this process of proportionality and the Leibniz discovery of his calculus, especially as it became expressed by the natural logarithms of the arithmetic-geometric process of generating the catenary curve from a logarithmic curve. This is what I want to investigate with you today, because this logarithmic function also relates to the Archytas doubling of the cube, as I will show you later with the arithmetic-geometric mean.

To start with, just recall what Archytas had said about imagining going to the end of the universe and pocking a stick outside of its limits. What are you poking at? Nothing. You are poking at a bad Aristotelian sense perception infinite outside of the universe, a bad infinite appended to an imperfect dependency on sense perception that the oligarchies have played up as an image of eternity in the afterlife for the benefit of the poor uneducated and bestialized human beings yearning for it; because this is where they make believe perfection resides. Thus, they say to you: "The sooner you leave this imperfect world, though wars, famine, and disease, the better mankind will be in the perfection of the other life." This is how the Aristotelian view of the infinite became the underlying assumption for permanent religious warfare as in the crusades and in the Thirty Years War.

On the other hand, if you consider that the human access to the infinite is not in the large, but in the small, and that such an infinite is expressed by internal singularities of the infinitesimally small, as Leibniz showed in his calculus for the catenary/logarithmic curves, or with his delta, then universal principles, such as Kepler's principle of gravitation, can be captured, from the inside, within small angular measures of change by the harmonic proportionality between a maximum and a minimum. This can be provided within the continuous manifold of a conic function.

From that vantage point, follow the Apollonius method for the construction of the cone and you will see why Lyn indicated that Eratosthenes was excited about the

Archytas construction of doubling the cube. And that is because he had discovered how to define lawfulness by construction from the inside of the system. Also, consider that the {***self-similar conical spiral function***}, that we are now going to begin working with, reflects the same kind of process; that is, the physical-geometric definition of work as Lyn had defined it in {***So, You Wish to Lear All About Economics***.}

Now, take **Figure 1.** as an elementary form of constructing a conical projection of the solar system from the inside, and apply the following Kepler insight about locating the Sun on the axis of the cone. As we shall see, this is precisely the way to construct a metaphorical synthetic conic function for both the musical system and the solar system. Now, with this Keplerian noospheric proportionality in mind, let's complete the construction of the cone that Lyn started to elaborate in {**So, You Wish…p. 50-51.**}

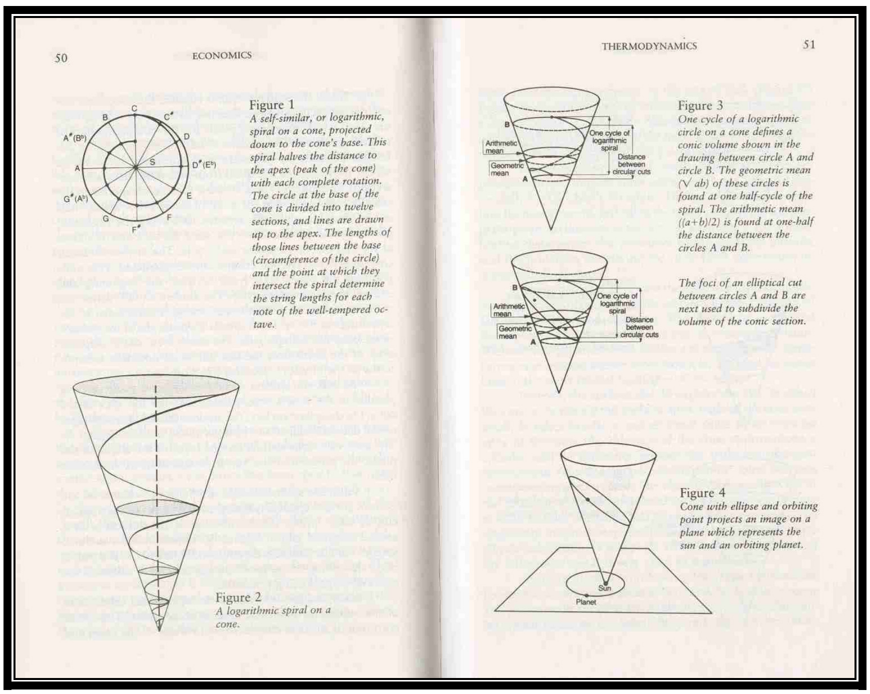
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Figure 1
A self-similar, or logarithmic, spiral on a cone, projected down to the cone's base. This spiral halves the distance to the apex (peak of the cone) with each complete rotation. The circle at the base of the cone is divided into twelve sections, and lines are drawn up to the apex. The lengths of those lines between the base (circumference of the circle) and the point at which they intersect the spiral determine the string lengths for each note of the well-tempered octave.

Figure 2
A logarithmic spiral on a cone.

Figure 3
One cycle of a logarithmic circle on a cone defines a conic volume shown in the drawing between circle A and circle B. The geometric mean (√ ab) of these circles is found at one half-cycle of the spiral. The arithmetic mean ((a+b)/2) is found at one-half the distance between the circles A and B.

The foci of an elliptical cut between circles A and B are next used to subdivide the volume of the conic section.

Figure 4
Cone with ellipse and orbiting point projects an image on a plane which represents the sun and an orbiting planet.

Figures 1. [The LaRouche projection of the Continuous and Discrete Manifolds.]

How do you construct a cone such that it is an appropriate metaphor for a growing economic process such as exemplified by the solar system, or by the well-tempered musical system? How do you regard the cone as being relevant for both the scientific and the Classical artistic domains? Such a conic construction must bear directly on the living process of the universe as a whole, such as an expression of the least action of a spiral action reflecting both anti-entropic living processes and thinking processes, the Biosphere and the Noosphere? Lyn has already indicated how this is relevant for determining the golden section pertaining to living processes and to music. How can we relate it to the Leibniz calculus with respect to Kepler?

A. HOW TO CONSTRUCT THE LOGARITHMIC CONE.

Ho do you construct the self-similar spiral? I will now show you how to construct that spiral by means of logarithms, in much the same way that Leibniz constructed the catenary curve with logarithms. I would like you to begin this construction right now and follow the Leibniz method for discovering an unknown curve; therefore, I will submit this to you in the form of the following theorem:

{Given the Kepler proportionality of planetary orbits within the solar system as a whole, find the logarithmic spiral that generated them.}

To do that, you must first discover how logarithmic conic sections are generated, with a straight edge alone, and without a mathematical formula. In constructing this cone, you must first establish the apex angle, which is defined by the amplitude of a self-similar logarithmic spiral action. A model representation of such a conic apex angle projection for the real solar system, for example, would be so wide and so extended that it would practically be impossible to use as a pedagogical device. See the Keplerian idea of a planetary orbit on page 51 of Lyn's {**So, You Wish to Learn All About Economics},** Figure 4. However, since all logarithmic self-similar spiral actions are proportional, regardless of their apex angles, we can accommodate ourselves with a smaller angle and construct the solar system as well as the well-tempered musical system with a different apex projection angle without problems.

Take a sheet of drawing paper and construct the axis of your cone by drawing a perpendicular axis from the base of the cone to the apex. Think that this axis is going through the sun at the apex, as if it {***moves all the planets by the emanation which it sends out from itself***.}"Conceive of your cone as being upside down on your drawing paper, following the {***Sphaerics***} tradition of the stereographic projection of Hipparchus. For our purpose here, we shall make the cone 18 centimeters high, and 18 centimeters wide at the base. Next, draw the two sides of the upside down cone, from the base to the apex. Complete the conic cross-section by dividing the cone by half, and then, by dividing the portion from the middle of the cone to the apex, by half again. [See **Figure 2**.] Lastly, generate two elliptical cuts representing the minimum and maximum range over the two octaves. This elliptical function is the key to the construction of all of the logarithms of the twenty-four intervals for two octaves of the well-tempered system. Thus, you can now begin to construct the logarithmic spiral spanning two octaves of the conical musical system; say the octave of C-128 to C-256 and the octave of C-256 to C-512. This is how Archytas also constructed his cone for doubling the cube. I sent you the illustrations last night but we will be discussing them after having constructed this cone.

B. THE DODECAHEDRAL CONIC-FUNCTION

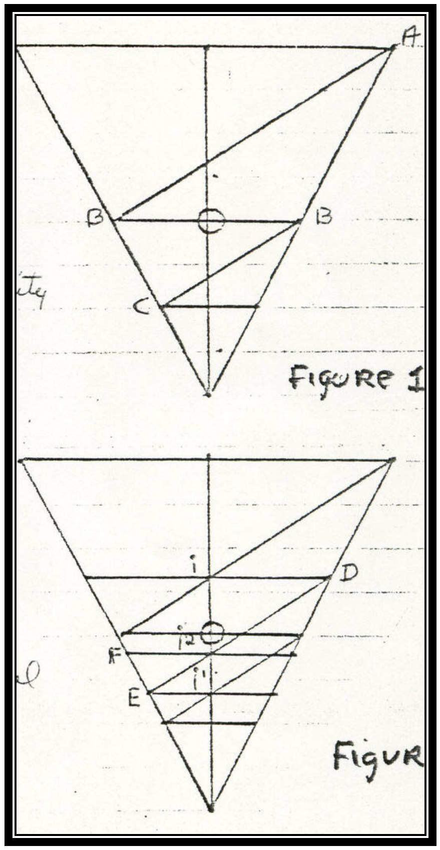

Figure 2. [The Logarithmic conic function.]

Here is a logarithmic conic function that you can construct with a straight edge and compass alone. The construction is based on establishing the boundary conditions for the DODECAHEDRAL CYCLE OF TWELVE PENTAGONS, that is the cycle of 12 and 5, which is expressed by the logarithmic cycle of fifths. Bear in mind that during the construction of this cycle, you will have to account for the Pythagorean commas, and that the tempering of any dissonance will have to be corrected by hand.

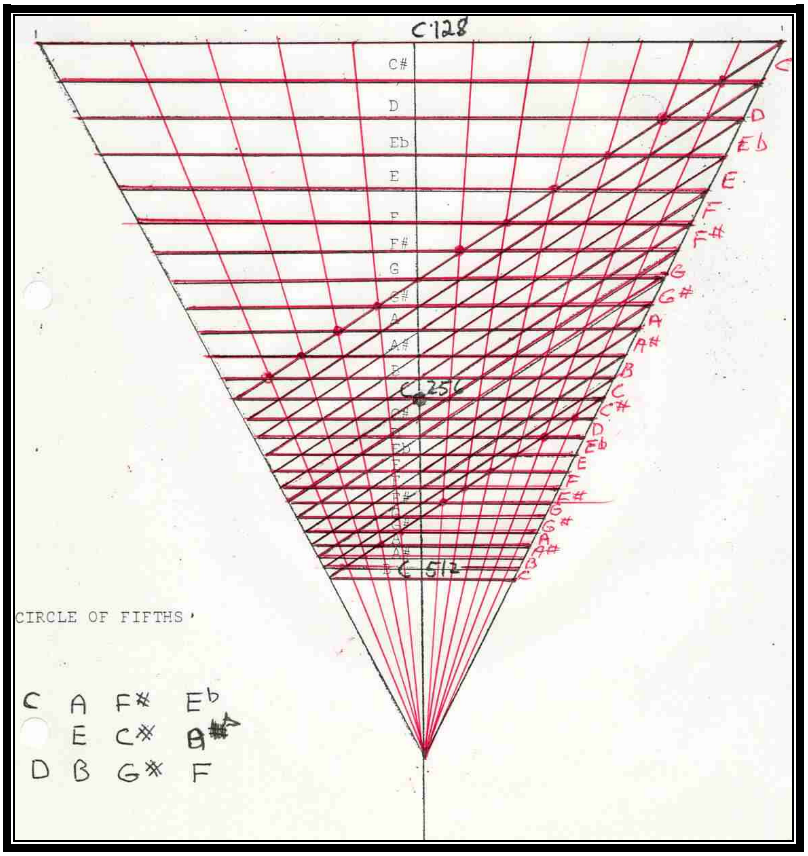

Figure 3. [The Dodecahedral Cycle of Fifths.]

The purpose of this exercise is to think of this upside-down cone as a thought object, a {***Geistesmassen***}, or a {***stereo-idea***} corresponding to what Lyn called a Riemannian continuous manifold, which must be related to a discrete manifold represented by the horizontal plane view of the conic projection. You must view the two different domains simultaneously, as if you were seeing a single stereographic whole representing the principle of {***invariance***} of the complex domain relating the continuous manifold and the discrete manifold.

Thus, any object of the complex domain should be viewed as being simultaneously projected onto the vertical section of the cone and on the horizontal section of the plane, all at once. This is determined by the boundary conditions of the invariant connection between the two manifolds. In other words, in order to see the horizontal planes in your mind, you must project, from this flat profile elevation of the cone, the flat horizontal view corresponding to it, as if you were looking from the upper part of the solid cone base and down into the center of its apex, as in **Figure 4.** In order to avoid an overloading of lines, you won't need to draw all of the circles right away. The three circles bounding the two octaves will be sufficient for the time being.

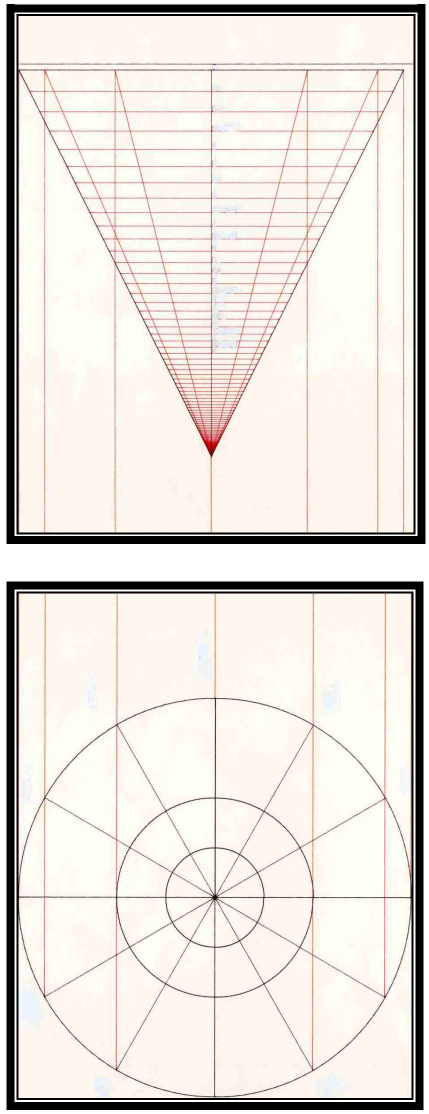

Figure 4. [Logarithmic Conical projection from the continuous Manifold onto the Discrete Manifold.]

However, all of the 12 rays originating from the apex of the cone must be connected between the two manifolds. Ordinary white paper should be used, not grid paper. The best way to do such orthographic projections is with a small drawing board, a T-square, and a pair of scalene triangles 90/60/30 degrees, as I described them in the Egyptian and Archytas models for doubling the cube.

We will first construct this continuous manifold as it must look like in your mind, and that is, as a Kepler expressed the proportionality of planetary orbits in the text above in order to establish the correct position of the self-similar logarithmic spiral that generates it. Later we will connect the spiral of the continuous manifold with the spiral of the discrete manifold, as if you viewed it from the top. The exercise will end with drawing the horizontal view of its projection below the vertical projection. The two plane projections of the continuous and discrete manifolds must therefore be connected, from apex to apex, orthographically and stereographically to show the principle of {***invariance***} between the two projection. After you have done that, e-mail me your results. You won't need any equations or any mathematics. No numbers are required, just the identification of the twelve intervals of the musical system.

Once you have grasped the geometrical principle of invariance between the process of projection of the characteristics and their anomalies as shadow-images in the discrete manifold, then, you will have a sense of the anti-entropic process that Kepler was referring to with his proportionality between Mercury and Saturn. The next step, as Lyn stated is to go to the higher domain: "{***In second approximation, higher-order invariances identify those changes in the continuous manifold which are carried over into the discrete manifold as transformations in invariants of the discrete manifold. {Relativistic transformations in the metrical properties of action} in the discrete manifold belong to this second, higher-order class of projective invariances. A {unique experiment} has as its subject-matter such a higher-order transformation in metrical characteristics of principles of action in a discrete manifold. Riemann's 1859 treatise on shock-wave generation is a model of principles of {unique experiment}***.} (Op. Cit., p. 57.) And, I venture to add that the Platonic determination of the Dodecahedron generating the five Platonic solids, the discovery of the astrolabe by Hipparchus, and the Gauss 1799 dissertation are also typical examples of such a {***unique experiments***}.

So next, after you have completed the boundary determinations elements of the discrete manifold from the projection of the continuous manifold alone, then identify the twelve intervals of the well-tempered system and confirm the function of the DODECAHEDRAL CYCLE OF FIFTHS. Locate at the extremity of the twelve radii of the discrete manifold, the series of twelve intervals of the well-tempered system as it appears in {***So, You Wish…***p. 50). You can define that cycle of fifths by self-similar spiral action alone. Start counting by intervals of 5, or intervals of 7, beginning at C located on the rim of the maximum circle at 12 o'clock, and proceed to rotate clockwise from C to G. The series of fifths that you will generate will take you back to C after having gone through all of the fifths as in the following series:

C, G, D, A, E, B, F#, C#, Ab, Eb, Bb, F.

Think of this construction as an expression of the anti-entropic process of self-similar spiral action behind this whole process. This is the same principle that Kepler used to relate the proportionality between Mercury and Saturn, with respect to the sun and that Leibniz developed with the logarithmic dynamic for the catenary/logarithmic curves. It is for the same reason that Lyn steered the LYM to look into the uniqueness of the five Platonic solids, into Napier, and into the Pentagrama Myrificum of Gauss. These are all derivatives of the dodecahedron. Thus, the ordering of twelve logarithmic intervals of action for each octave of the well-tempered musical system, or for the solar system, works in this fashion because the universal principle of gravitation is what generates and tempers the logarithmic spiral action in this form. The result is the selforganizing process of the division of half, and half of the half, as reflected in the 256 series, which is also an underlying characteristic of the continuous manifold of the universe as a whole. However, it is the singularities of the Pythagorean commas, and of the register shifts that determines 256, not 256, which determines these singularities.

FIN 9/02/06

(A pedagogical experiment in universal history) PART VI

THE EGYPTIAN SCIENCE OF SHADOW RECKONING
AND THE DOUBLING OF THE CUBE.
BY CONICAL FUNCTION

by Pierre Beaudry
(Class of constructive geometry for the Philippines LYM. 9/16/2006.)

1. HOW TO "CONNECTS THE DOTS" BETWEEN THE CONTINUOUS AND THE DISCRETE MANIFOLDS.

How do you map change? As Lyn indicated in {***So, You Wish to Know All About Economics***}, visual space is the image projected on the screen of our sensory perception as if from the conical projection of a continuous manifold onto a discrete manifold. That change is the thought object, {***Geistesmassen***} that you are developing when the spiral action "connects the dots" between the continuous and discrete manifold. What you are connecting are not the dots of sense perception, but the dots of singularities and discontinuities. Take [***Figure 3***] and observe how your mind makes the change in that connection. The point is that you don't make a deductive copy of one to the other; the truth about the subject matter is the change between the dots of the two manifolds. So, even though you see this happening before your eyes, it is not your eyes but your mind that makes that connection between the shadows of the two manifolds. The crucial implication, as Lyn wrote, is that you your mind can grasp the {***inbetweenness***} of the change which transforms one manifold into the other:

"{***Like Riemann, we identify visible space as the {discrete manifold} and the higher space of self-similar conical-spiral constructions as the {continuous manifold}. We require that mathematics for physics be constructed entirely within the continuous manifold, and functions of the discrete manifold be accounted for mathematically as projections of images of the continuous manifold upon the visible (discrete) manifold. To this purpose, we require that the student employ the self-similar conical-spiral***

action to elaborate a synthetic geometry of continuous-manifold space in the same sense that circular action is employed to construct a synthetic geometry of visible space (the discrete manifold). All mathematics for physics must be derived and proven mathematically solely by the synthetic-geometric method of construction within a continuous manifold, and algebraic functions treated as nothing more than descriptions of synthetic-geometric functions of a continuous manifold.

"For us, as for Riemann, experimental physics centers upon those {unique experiments} which prove mathematical (geometrical) hypotheses pertaining to the continuous manifold by means of experimental observations made in terms of the projected images of the discrete manifold. This possibility depends upon a geometrical principle of topology, {invariance}. In first approximation, {invariance} identifies those characteristic features of the geometry of a continuous manifold which are "preserved" through the process of projection as characteristics of the images of the discrete manifold. In second approximation, higher-order invariances identify those changes in the continuous manifold which are carried over into the discrete manifold as transformations in invariants of the discrete manifold. {Relativistic transformations in the metrical properties of action} in the discrete manifold belong to this second, higher-order class of projective invariances. A {unique experiment} has as its subject-matter such a higher-order transformation in metrical characteristics of principles of action in a discrete manifold. Riemann's 1859 treatise on shock-wave generation is a model of the principles of {unique experiment}.}" (Op. Cit. p. 55-57)

Now, take **[Figure 3.]** and **[Figure 4.]** and make the connections between the twelve rays that are projected from the apex of the cone. All of the twelve logarithmic circles that you have painfully constructed in the last class, and which represent the range of action of the continuous manifold, intersect those twelve rays in the discrete manifold at points that mark the passing of the change between the two different manifolds. The transfer of those twelve points per octaves is but the shadow of the change in curvature. The anti-entropic self-similar logarithmic spiral action is therefore merely a first approximation of least action principle underlying the synthetic geometric construction that I submitted to you two weeks ago, and that was:

{Given the Kepler proportionality of planetary orbits within the solar system as a whole, find the logarithmic spiral that generated them.}

If you have any problems in locating the relevant change in topological invariance, which are preserved between the two manifolds, we should discuss them now, as a matter of course. As for the dual aspect of the logarithmic spiral, think of the interval between F and F# as a pedagogical device for illustrating the arithmetic-geometric mean interval of the logarithmic spiral when it reaches the ambiguous arithmetic/geometric half-way mark in its motion up and around the cone. As in the Leibniz catenary function, the "up" portion of the interval is arithmetical, while the "around" portion of the interval is geometrical. " It's not very precise, but consider that not form of geometry could ever be appropriate for expressing such living processes.

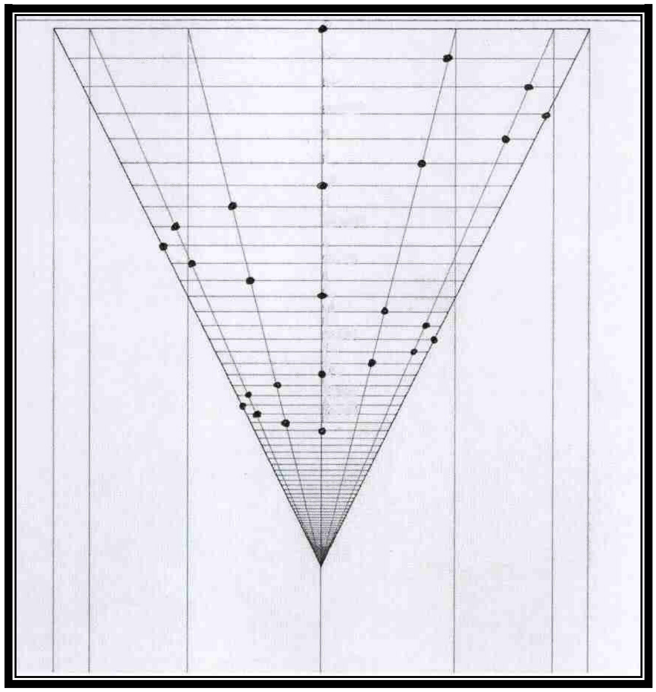

Figure 5. [Mapping the Logarithmic Spiral onto the Continuous Manifold.]

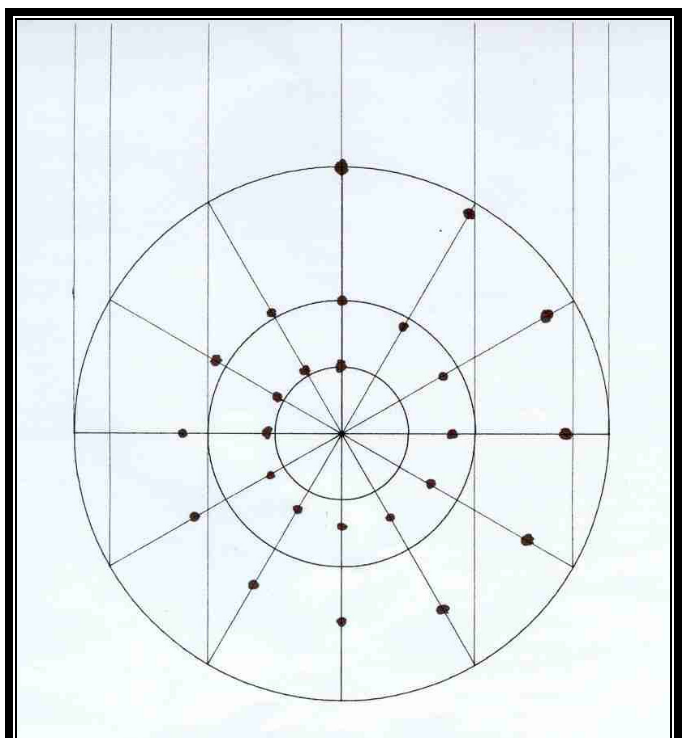

Figure 6. [Mapping the Logarithmic Spiral onto the Discrete Manifold.]

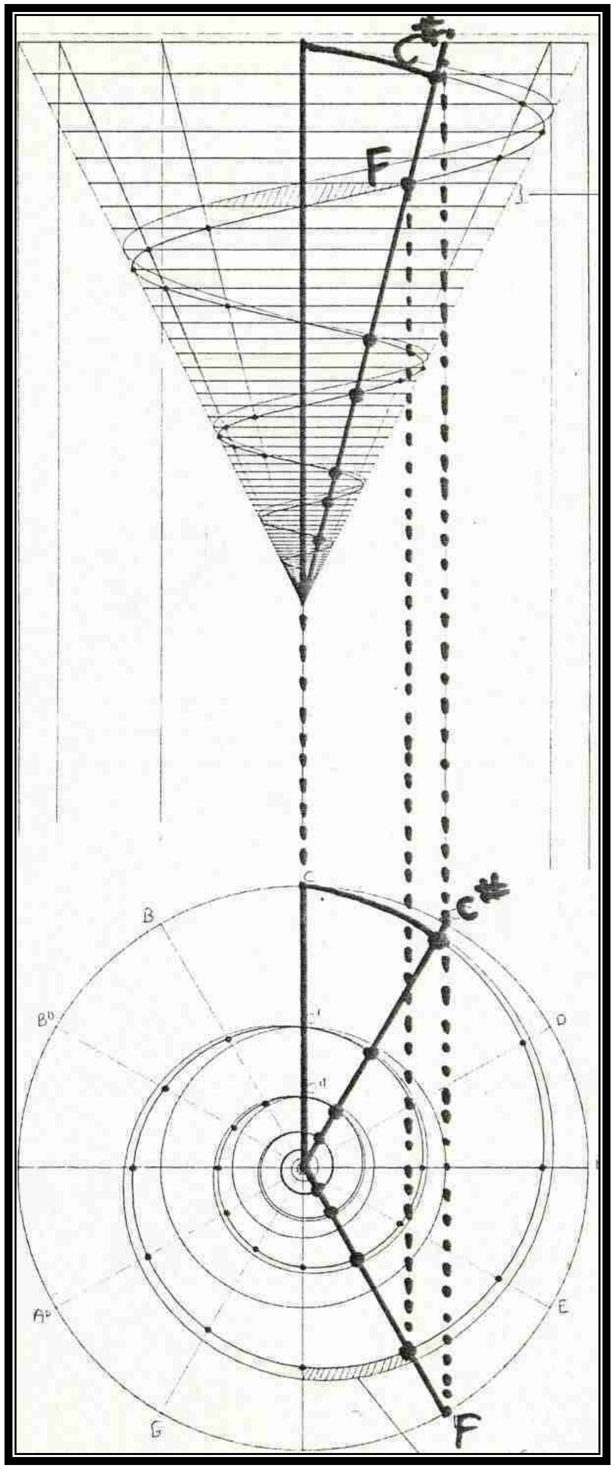

Figure 7. [Connecting the two Manifolds.]

Now, I want to add something that we did not discuss in class but that just popped up in the Morning Briefing of September 20, 2006. You will find, there, a lengthy expose of an exchange between the LYM in the US and Lyn on the question of {LYM upgrade.}

written up by Sky, I believe. It is a very important discussion and I hope this will be forwarded to you as soon as possible. In the discussion, there is a question which relates to "synthetic geometry as a means of conveying an idea" and which relates to Chapter 3 of {***So, you Wish to Learn All About Economics?***} that we have been working with for the past period. This is very important for what we are doing here. The questioner asked: {***[…] You've stated that this synthetic geometry forms the basis for understanding elliptical function, continuous and discrete manifold and many more things that you've laid out in that chapter. Should the student still do all this synthetic geometry to understand the idea you are conveying so as to form the basis to move on to Gauss, Dirichlet, and Riemann?[…]***}

Lyn responded as follows: {***[…]I am proceeding from an overview of an integrated process of development, in which each stage is generated from the irony created by the deceptive appearance of a successful conclusion reached by the preceding stage.***

It is not the image, which conveys the idea: it is recognizing that the image is deceptive in pretending to provide a solution, when its failure to accomplish that prompts the discovery of the needed next question. E. g. discovering the elliptical orbit, is not a solution; it begs the question, of what is generating the ellipse. E. g. discovering the elliptical orbit of Earth and Mars, begs the question of the principle of harmonic orderings among the planetary orbits.

"Synthetic geometry" is a phrase which begs the issue of physical, rather than formal geometry. This quest leads to the posing of the notion of a physical universe whose dimensions are not formal-geometric, but are universal physical principles. The way in which those principles are configured, in respect to one another, defines a physical geometry, as might be represented by a Riemannean tensor. This is the outcome, by Riemann, of Gauss' posing the of "hypergeometry."} (Morning Briefing, September 20, 2006)

Now, I want to make a comment on this statement by Lyn, so that there is no confusion about what the purpose of formal constructive geometry is. Note that Lyn did not say that the work in formal synthetic geometry should not be done, but that he is putting the emphasis on proceeding from a higher integrated process of overviewing the general creative process of the physical universe as a whole. What Lyn called {***deceptive appearance of a successful conclusion***} is what I have been calling the illusion of {***geometrical shadows***} as projected onto the wall of Plato's Cave.

On the one hand, the question of {***deceptive appearance of a successful conclusion***} involves a very special paradox that all of you should address and resolve. The deception, here, lies not in its construction, but in the fact that formal synthetic geometry appears to work {***successfully***} at providing an apparent solution. This is the trap of synthetic geometry that I have warned about several times in the class before. The point is that there is both "success", and "failure." For example, the great dodecahedron that Kepler discovered and built, is a perfectly beautiful example of a {***successful***} construction of synthetic geometry; but this geometrical object, in and of itself, is meaningless. It is merely a decoration and is but an illusion if it has the pretension of providing a solution to the physical geometry of change in the universe as a whole, including the so-called golden section. As Dr. Moon once told me: "{***The dodecahedron is simply a butterfly catcher. However, if you catch the right kind of butterflies with it, you might get a fusion reaction***.}"

On the other hand, if the dodecahedron serves judiciously as a shadow metaphor projected on the dimly lit wall of Plato's Cave, and that it is used for the purpose of conveying an approximation of the harmonic ordering among the planetary orbits of the solar system, then its formal geometry begs the question: What generated it? What kind of universal physical principle was able to organize space in such a dynamic manner as to produce the dodecahedron? This is how mathematical formulas get to be mere shadows of geometric constructions, and geometric constructions mere shadows of universal physical principles. This is why Lyn says that there is no such thing as a formal geometry of the universe, because the universe changes all the time from the dynamics of universal physical principles acting within it. That is the function of the Riemannean tensor behind the deceptive integral appearance of the dodecahedron.

However, if the paradoxical synthetic geometrical construction of the dodecahedron gives you the illusion of success and puts you into a state of perplexity, as it should, the same synthetic geometrical construction gives you a cognitive power that nothing else, except the experience of musical polyphony, will give you. The constructive proof is in the pudding, as they say, to be used as a means of "hammering your personality" by proving both the success of its illusion and the illusion of its success. As Lyn put it on the subject of Pope Benedict XVI: {***Accordingly, we must approach the management of the affairs of the universe in a manner which is governed by a careful blending of certainties and humilities.***}"

So, my question, which is a question that you should be asking yourself, is how can the Archytas model for doubling the cube help us understand LaRouche's economics? Well, I would say that, on the one hand, it reflects a very sensuous grasp of the anti-entropic {***principle of power***} of going from a lower geometry to a higher geometry, and on the other hand, it reflects a first approximation of attempting to establish the boundary condition of an {***internally self-bounded universe***.} So, these are the two main ideas to consider, from the top down, with respect to {***So, You Wish to Know All About Economics?***} So, as we have seen, the model itself is a very powerful means of showing the LaRouche-Riemann functional relationship between the {***Continuous manifold***} and the {***discrete manifold***}, as Lyn developed it in that book. This is expressed in the three forms of circular actions that are embodied in the Archytas construction and that is: 1) The generation of the {***Torus-Cylinder curv***e}. 2) The generation of the {***Cone-Cylinder curve***}. 3) The generation of the {***Cone-Torus curves***}. As Lyn indicated earlier, the focus should be on the significance of the {***conical function***}, which is the key to the Archytas construction.

2. FROM KEPLER TO NAPIER TO LEIBNIZ.

In 1616, there appeared in London a very unique publication by a Scotsman by the name of John Napier (Nepair), called {***A DESCRIPTION OF THE ADMIRABLE TABLE OF LOGARITHMES***}, which had been translated from the Latin by Edward Wright, who had also developed an instrumental table and a conical projection to help find the proportional parts of any spherical logarithms, including those that were not included in his table. Ironically, the book had been dedicated to the Company of Merchants of London, which was trading with the East Indies, and whose interest in money blinded them from forever enjoying the true wealth that laid hidden in Napier's discovery.

This book was actually a compilation of the notes from Napier who, as a Mariner, was able to reconstruct, from his own experience of sea voyages, the lost "Art of Navigation" of {***Sphaerics***} that was initiated by ancient astronavigators, and in whose memory, such "noble knowledge" of trigonometry and proportionality had been, somehow, brought to life again. {***The Admirable Table of Logarithmes***} was explicitly written in memory of such ancient {***Sphaerics***} astronomers as Hipparchus, and Apollonius, and was intended for modern astronavigators as opposed to Venetian astrospeculators.

Two experiments are worth replicating with respect to the crucial work of Napier. One is the Pentagrama Myrificum, as Gauss called it, and the other is the application of proportional logarithms to {***Sphaerics***}. I want to introduce briefly these two experiments at this point because they are extremely useful exercises for understanding the function of boundary conditions between the discrete manifold and the continuous manifold.

As Napier showed, the method of construction of logarithms is aimed at finding any small interval of action within a function that is continually proportional. The numbers of these logarithms, representing the shadows of their angular distances, are in such a proportion that, given three numbers, {***the doubling of the second minus the first produces the third***.} It is important to note here that these numbers, as such, have no significance in and for themselves and do not reflect linear distances between two points. They are the mere indicators, indices, or shadows of intervals of action, which reflect the real world behind them, and relate to angular intervals of action on the surface of the sphere. So, given any two numbers representing such intervals of action, you should be able to find any number of mean proportionals between them as small as you may wish them to be.

For example, let me show you the geometrical "calculus" construction of Napier, in the small, as designed for him by Edward Wright in the form of a Triangular Table for finding the different proportional parts of logarithms. This projection is taken directly from the idea of {***Sphaerics***}, and is very similar to the conical projection that Thales, Hipparchus, and Apollonius have used in their constructions of conics, and that Napier reconstructed for the purpose of his logarithms. Of the three different methods that Napier developed for deriving such logarithms, the Golden Rule, the Logarithm Table, and the conical function of the Triangular Table shown in **Figure 8**, this latter method is the simplest and most cognitively elegant of the three. It is better because it shows the actual geometrical construction of the purely mathematical Golden Rule. Let's use the following construction as an example.

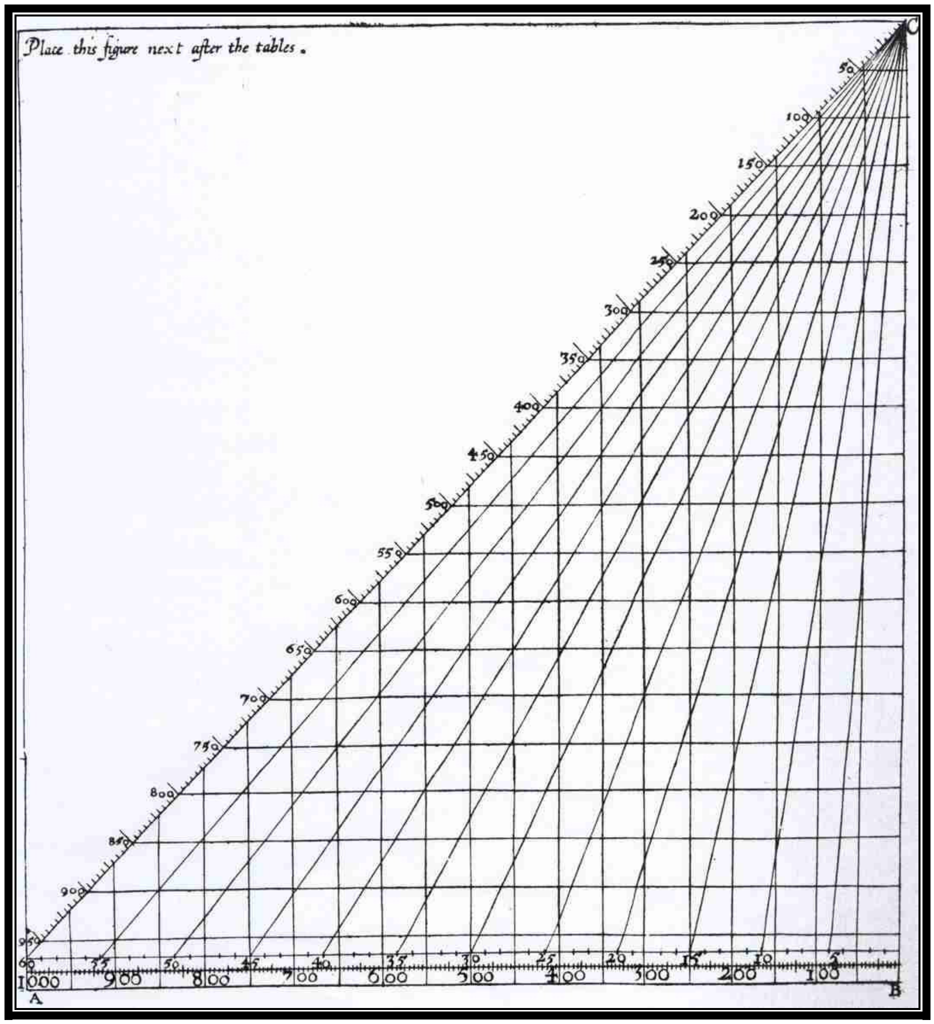

Figure 8. [The Napier Triangular Table of Logarithms.]

At the end of the Napier book, Henry Briggs wrote a short explanation of this method whereby if you required to find a logarithm, say 141766, which is the logarithm of half the spherical angle of 120 degrees 24' 49", you must translate that logarithm into the precise angular measure, which is 60 degrees, 12' 24 ½". However, the problem is that this logarithm is not found in the Napier Table of Logarithms. On the other hand, since it is located somewhere between two known logarithms that were given in the Table, 141834 and 141667, which are the differentials of 12 and 13 minutes located between 60 and 61 degrees, then we must find the small interval of 24 ½ seconds between 12 and 13 minutes.

As Napier puts it, in the language that will later be much better expressed by Leibniz in his calculus, one could easily find the exact logarithm, as small as one would wish, if one were to take the difference between the two Table logarithms, which is 167, and take the difference between two Tabular Arcs of one minute, which is 60 seconds. These two "tabular differences" must be related to a third difference, that Napier called the "occurring difference", which is taken from the difference between the first of the two Table logarithms, which is 141834, and the sought for angular value of 141766. And that third interval of difference is 68. Thus, the three intervals of differences **167, 60, and 68** will help us discover the fourth proportional part that is exceeding 60 degrees and 12 minutes by the very small amount of 24 ½ seconds. How can you construct that in the continuous manifold of the conic function?

The purely mathematical way of finding this small interval of difference, Napier called his rule of proportion, or the Golden Rule, which consists in multiplying the third 68 by the second 60 and dividing the product 4080 by the first 167, which will be almost 24 ½. However, this mathematical expression is merely the shadow representation of a geometrical process of conical proportionality. For example, use the same intervals of action **167, 60, and 68**, and take the two which are intervals of logarithms, that is **167 and 68** and locate them on the hypotenuse of the Triangular Table [**Figure 8**.] If you drop a perpendicular from the location of the first value of **68** to the extension of a horizontal line drawn from the second value of **167**, the point where this little "delta" triangle meets a diagonal line coming from the apex of the triangle can be projected down to the base of the triangle, which locates the proportional value of 24 ½ seconds. However, this encounters the same difficulty as the one, which occurred, in our last class, when you attempted to find the octave of D or the octave of C# in the projection of the well-tempered system. It requires very precise tempering.

However, since the concurrence of the horizontal and vertical projections of the first two intervals of the Napier projection is not easily discernable with precision, the problem can be resolved by increasing their values proportionately by 5. Thus, if you locate the interval between ***340 and 835*** and apply the same projective method, you shall find the fourth proportional interval value to be precisely 24 ½ located at the base of the triangle. Bear in mind that this Napier Triangular Table is not exactly the same as our conical function, but the principle of proportionality behind its projection is exactly the same. The beauty of Napier's discovery has been expressed very beautifully by two poems published with his book.

"{***It was at hand, and yet it was unseen,***
Invisible, and yet was clear to wit.
As it could wish, or as it could have been
In Art or Nature; yet Art mist of it […]
For who with ease hath done what none ere could
Is most like God in workes of rarest skills
This argues He can do what ere he would
In Art with ease, if he hath but a Will. […]}"

John Davies of Hereford

"Arts, in themselves, have such divine Perfection,
As Human reason cannot always see;
Yet God all good, to man gives such direction
At hidden things sometimes discovered be:
What many men and ages could not find,
Is, at the last, by some one brought to mind […]}"

Ri Leuer

THE NAPIER PENTAGRAM

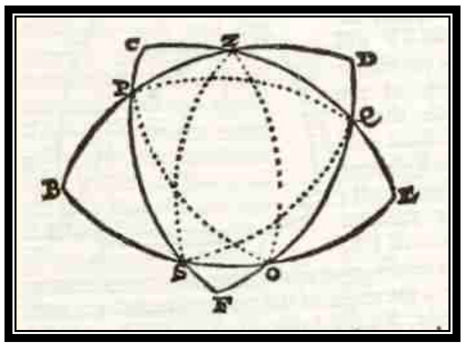

Figure 9. [Napier's Pentagram.]

{***Given the discrete manifold of Napier's pentagram, find the continuous manifold that projected it.***}

If one can get a sense of the infinite in the small, by means of the Leibniz infinitesimal, by the Gaussian arithmetic-geometric mean, or by the Isoperimetric Principle of Cusa, then, how do you get a sense of the unity of the whole? The unity of the whole, which the dodecahedron represents with respect to the regular Platonic solids, for example, gives you such a sense of integration, and so does the dodecahedral conical projection of the well-tempered musical system that we just built. The Pentagram construction of Napier represents the same result of an integration function displayed on as an image on the screen of your sense perception. How can you construct the spherical continuous manifold projecting this shadow image on the discrete manifold? You can do this experiment with five hoops.

The underlying principle behind the Napier construction of **Figure 5** is simply this: {***the integral uniformity of circular action by five great circles around the surface of a sphere is such that they form a pentagram of right angle triangles all around the surface of the sphere; such that, the first great circle must cut the second, the second must cut the third, the third must cut the fourth, as the fourth must cut the fifth, in the***

same proportion that, the fifth cuts the first at right angle*.}** This is the simplest form of geometric closure in the domain of {Sphaerics***}, and which gives you an insight into the singularity of the dodecahedral function with respect to the harmonic ordering of the universe as a whole. However, none of the other angles of the Napier Pentagram are required to be equal. Recall that the Egyptian model for doubling the cube was also a double conical right angle projection. Any Questions?

3. THE CONICAL FUNCTION OF THE ARCHYTAS CONTINUOUS MANIFOLD

By now, it should be clear that the Archytas construction is an ancient format of the continuous manifold. And, by now, you should also have begun to grapple with the construction of the Cone, the Cone-Cylinder curve, and the conical section intersecting the Torus in the Archytas doubling of the cube. This last intersection is occurring both inside of the Cone and inside of the Torus, and not on the surface at all. However, this is where the {***Stereo-idea***} thought object of the conical function becomes visible to your mind as the central feature of the Archytas doubling of the cube. The necessity of determining the lawfulness of the universe from the inside, with constructible universal principles, as opposed to external deductions, statistics, or authority from “peer review committees”, is extremely important because without this subjective element of human reason, the universe could never become intelligible.

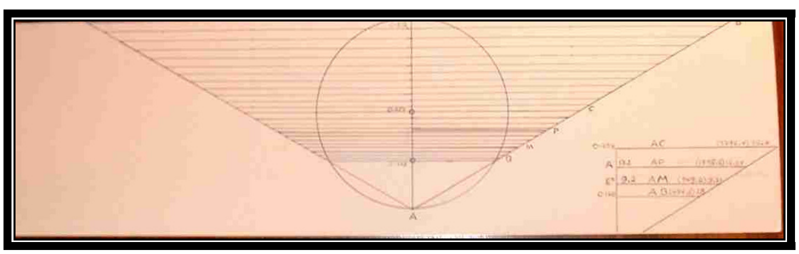

Figure 10. [Meridian Conical Section of the Archytas Cone.]

Archytas determined the apex angle of his model to be 120 degrees because it was predetermined by the two extremes, AB and AC, which had to be in a proportion of two to one, and therefore, had to follow a hexagonal projection across the base of the Cylinder. [**Figure 6.**] That is the only possible conical projection for the Archytas model. As a result, his meridian conical section had to be made up of two scalene triangles, with their shortest side connected, back-to-back, against the axis of the cone. We must, therefore, follow the same requirement, when building our own Archytas model, and establish that the axis is AC with the apex at A, the base at C, and the apothem at AD. You should proceed to construct the outline of this cone and include the identification of two relevant musical octaves also in proportion of two to one.

I recall, as we did before, that if you rotate the conical section ACD around the axis AC by an angle of 45 degrees, the 38-degree angular intersection of the rotated apothem will establish the two mean proportionals, AM and AP, and determine point P as the intersection between the Cone, the Torus, and the Cylinder. The important feature of this conical function is that it determines the paradoxical axiomatic transformation straddling two different powers between the two different domains of the plane surface and of the solid. In other words, this is not merely a position of change measurable by a simple linear distance; this is a non-linear change of angular position expressing a change of power of the human mind in being able to change the universe. This axiomatic angular transformation is a reflection of the universal physical principle that causes the axiomatic change between the doubling of the square and the doubling of the cube; that is to say, where an increase in power over the universe occurs between the {***flatland***} domain of square roots of Aristotle, Euclid, and Euler, and the {***stereo though object***} domain of cubic roots of Plato, Gauss, and Riemann.

Moreover, it must follow from what precedes that since the side of the original cube, AB, is one quarter of the apothem length AD, then AC' is half of the apothem length. If you were to make circular cuts across the cone at these two different points, B and C', you would have divided the cone into two octaves, as illustrated in **Figure 6**. Now, take the logarithmic cone that you have constructed last week, and map the logarithms one on one onto the Archytas conical section. Because they have the same height, the two cones will reflect everywhere the same logarithmic intervals of circular cross-sections.

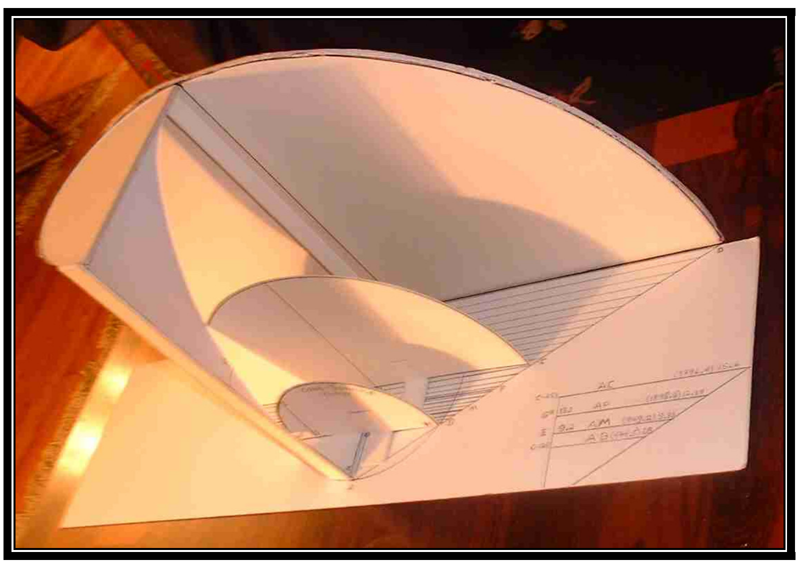

Figure 11. [The two Circular Octaves of the Cone and the Logarithmic Division of the Cylinder.]

Lastly, locate the two mean proportionals AM and AP within the range of the conical section representing the first octave, that is, between C-128 and C-256. These are reflections of a special form of {***cross-proportional arithmetic-geometric means***} relating the values of the two cultural and scientific domains of doubling of the cube and of the musical Lydian interval. Here, you will discover a wonderful congruence between the inside of the cone and the boundary condition of its apothem on the surface of the cone.

If you calculate the arithmetic-geometric mean between the two extreme values of the apothem, AC = 18 and AB = 9, the result is the value of 13.110837, which corresponds to the value of the Lydian interval of the well-tempered system. On the other hand, if you calculate the arithmetic-geometric mean between the two extreme values of the corresponding internal octaves, 15.6 and 7.8, which correspond to the radii of the two octave circular cuts, the result is the logarithm value of 11.3628, which is very close to the value of the first proportional mean, AM. Thus, from the standpoint of the arithmetic-geometric mean, the values for the doubling of the cube and for the Lydian interval are interchangeable.

Consequently, the two mean proportionals AM and AP are located close to the two Lydian intervals of minor thirds, namely that of A and Eb with respect to C and F#. This correspondence is not so unusual since the conical function of the well-tempered musical system is also established within the continuous manifold by a complex arithmetic-geometric mean function of dividing the cone by a complex half, and half of the half. So, it becomes clear that it is this conical complex function, which determines the musical series of 256, and not the 256 series, which determines the complex function.

Furthermore, this construction shows that the Archytas doubling of the cube is directly proportional to the well-tempered musical system, and to the solar system as a whole. Therefore, the conical function of the Archytas construction represents an astro-physical-musical pivot, which overlaps the cultural domains of Classical artistic composition, Classical Art, the harmony of the Spheres of Keplerian astrophysics, and the scientific domain of the Gaussian complex plane. However, before going into the cultural domain, I would like to add one last aspect of the Archytas harmonic division process, which is represented by the intersection between the conical section and the torus section.

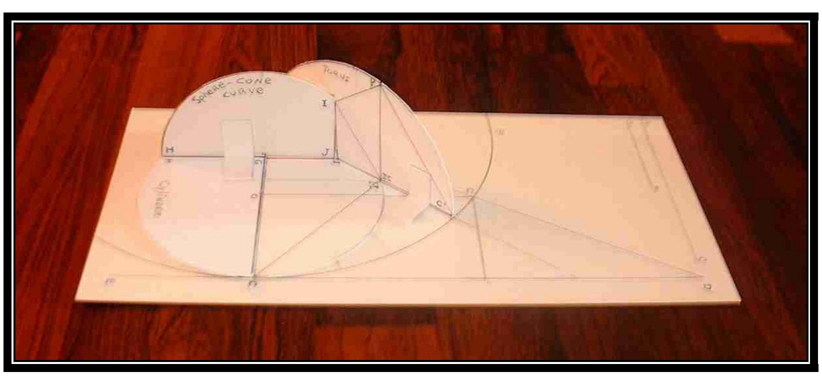

Figure 12. [The Intersection of the Conical Section with the Torus Section.]

The Archytas circular conical section BJGH, that we had traced initially on the Archytas base board, See [**Figure 12**.] is in the position of the octave corresponding to C-128 in the well-tempered system, as we defined it above, with C-256 in the middle, and C-512 at the base of the cone [**Figure 11**.]. Segment AJ of the torus half-circle intersecting the Conical section at point J corresponds to the side of a new cube whose area is ½ the area of the cube whose side is AB. This represents the inverse solution to the Archytas doubling of the cube. Therefore, the Archytas construction implies the geometrical construction of a logarithmic spiral form of action whose physical significance is to represent self-similar anti-entropic growth of living processes corresponding to the progression of the 256 series.

Thus far, from this construction, the mean proportionality remains the same as before, because the angular singularity of 45/38 degrees remains the same, but the two mean proportionals have changed because the two extremes are different. The two mean proportionals are now AB and AM between the two extremes AJ and AP, which are in a ratio of 2/1. So, the double mean proportionality has now become:

AJ : AB :: AB : AM :: AM : AP.

This establishes the constructive proof that the Archytas model not only doubles the cube, but also, quite legitimately, halves the volume of any given cube, as well. Furthermore, it establishes that the conical section BJGH is also the almucantar circle of a celestial sphere whose diameter is AOC. This is merely a shadow of the previous Egyptian model.

In conclusion, let me stress that this Archytas conical function is a very special {***cross proportional singularity***} of the arithmetic-geometric mean which intersects the Classical artistic and the scientific domains, such that this Archytas construction, originating from ancient Egyptian {***Sphaerics***}, takes us full circle into the domain of elliptic functions. It demonstrates how, through however unevenly and dimly perceived the shadows of its projection may be on the wall of Plato's cave, the external beacon of light casting the shadow of the Great Pyramid of Egypt down to us, today, is a reflection of the most powerful historical singularity of creative knowledge that the human mind was capable of producing more than 5,000 years ago. In so doing, such an ancient Egyptian thinker as Imhotep had set the stage of history and had defined the measuring instrument by which the battle for liberating the human mind would be fought for all centuries to come. However, don't look at this shadow as a pre-existing map that we have to follow linearly, but as a changing map of the universe that we have to change with our own creative discoveries of principle. Among the rubbles of civilizations past and future, this original Archytas construction, born out of the shadow of the Great Pyramid of Egypt, shall stand as a testimony to the endurance of the human mind's quest for truth and for its unceasing commitment to recognize its own optimistic spirit in reaching out for the development of future generations. Thank you for your attention and your generous perplexity. Any questions?

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