

(A pedagogical experiment in universal history) PART VII (last class)

**THE EGYPTIAN SCIENCE OF SHADOW RECKONING
AND THE DOUBLING OF THE CUBE.
BY CONIC FUNCTION**

by Pierre Beaudry
(Class of constructive geometry for the Philippines LYM. 10/14/06.)

1. THE LEONARDO PRINCIPLE OF ORGANIZING : {THE LAST SUPPER}

"{Linear perspective...is calculated mathematically, [but] aerial perspective...can only be grasped by the sentiment. By comparing these two sciences, where one is sensual, the other ideal, the methodical course of one will help penetrate the mysteries of the other... Aerial perspective is the art of generating ideas by means of the senses, of acting on the soul by the organ of vision. It is in this way that it acquires its importance that it competes with poetry; that it can, like poetry, enlighten the mind, warm the heart, excite and nourish higher emotions. We shall emphasize the contribution that it can bring to morality and to government; and how, in the hands of the skillful legislator, it will be a powerful means of instilling horror of slavery, and love of the fatherland, and will lead man to virtue.}"

-Lazare Carnot,
from the "Drawing" section of the Public Works
curriculum, Ecole Polytechnique, 1794.

In this last class, I would like to propose a hypothesis which addresses the interface of two domains that Lyn developed in his report **{On the Subject of Education}**, EIR, December 17, 1999. The hypothesis deals with the lawful ordering of universal principles as they apply to the domains of science and of Classical artistic composition.

Lyn wrote an important series of educational proposals to be adopted for our school system as well as for any nation around the globe. One specific insight was with reference to Wilhelm Furtwangler and Leonardo da Vinci. Lyn wrote: « **{The greatest orchestra conductor of the Twentieth Century, Wilhelm Furtwangler, described his method of conducting, as « performing between the notes. » Leonard da Vinci, centuries earlier, identified the principle of composition, in painting and plastic art generally, to the same effect. The issue is the same I raised above, in summarizing the**

significance of the principle of « least time ». What we define as distinct sense-impressions, may each really exist as sense-impressions, but one must not make the mistake of « connecting those dots » in a simply deductive way. This warning, against deductive modes for purporting to « connect the dots » applies as forcefully to art as it does to a mathematical form of physical science. } (Lyndon LaRouche, Op. Cit., p. 29.)

The point that Lyn is making is that the dots to be connected should not be connected between what looks like or sounds like different sense perceptions. The points to be connected are between non-linear points of different domains and pertain to how universal physical principles get connected. So, you cannot find these types of connections by imitating nature. Imitation, or curve fitting, is a fallacy of composition, as Plato argued the case in *{The Republic}* against the poets who use imitation, *{mimesis}*, in their art. And, at any rate, nature and art always have more to give that geometry can handle. As it is the case for dynamic physical geometry in science, in opposition to synthetic geometry, mechanical modeling, or benchmarking. Therefore, Classical artistic composition is essentially in dynamical opposition to imitation or copying of natural sounds, or natural forms. Classical artistic composition, as Leonardo understood it, is intended for the elevation of the soul, as described by Carnot. Let me remind you what Leonardo wrote to that effect in his notebooks:

"Represent your figures in such action as may be fitted to express what purpose is in their minds...A picture, or rather the figures therein, should be represented in such a way that the spectator may easily recognize the purpose in the minds by their attitudes...The hands and arms in all their actions must display the intention of the mind that moves them..." (*{The Notebooks of Leonardo Da Vinci}*, Oxford University Press, 1952. p.185 and 222)

So, the intention of painting is to make visible what is not seen; just like music makes audible what is not heard. From that vantage point, think of the principle of composition of Leonardo as being an early form of a well-tempered musical Lydian modality applied to the domain of plastic art, and especially to *{The Last Supper}*. With the subject matter of the opening sublime moment of the passion of Christ, Leonardo applied to this fresco-painting the discovery of the very idea of the Italian Renaissance, that is, what Nicholas of Cusa had been teaching in the social application of his Isoperimetric Principle, and which consisted in replicating the reflexive principle of *{tempering Pythagorean commas}*. For example, see the complexity of what Leonardo called his method of *{light and shadow propagation}*. He wrote:

"{Every body is surrounded by a limiting surface. Every surface is full of infinite points. Every point makes a ray. The ray is made up of infinite separating lines. In each point of any line, there intersect lines proceeding from the points on the surface of bodies, and they form pyramids. At the apex of each pyramid there intersect lines proceeding from the whole, and from the parts of the bodies, so that from this apex one can see the whole and the parts. The air that is between bodies is full of the intersections formed by the radiating images of these bodies.

The images of the figures and their colors are transferred from one to the other by a pyramid. Each body fills the surrounding air with its infinite images by means of these rays. The image of each point is in the whole and in each part of the line caused by this point. Each point of the one object is, by analogy, capable of uniting the whole base of the other. Each body becomes the base of innumerable and infinite pyramids. One and the same base serves as the cause of innumerable and infinite pyramids turned in various directions, and of various degrees of length. The point of each pyramid has in itself the whole image of its base. The centerline of each pyramid is full of an infinite number of points of pyramids. One pyramid passes through the other without confusion... }"

Now, Leonardo used the very same method and applied it to shadows as if he was defining an artistic composition as a form of the Leibnizian calculus with respect to infinitesimal differentiations of secondary derived light and shadow reflections from primary lights and primary shadows. He described the method in the following manner as he made plans to write seven books about this subject of non-linear perspective:

"{Shadow is the obstruction of light. Shadows appear to me to be of supreme importance in perspective, because without them, opaque and solid bodies will be ill defined; that which is contained within its outlines and the outlines themselves will be ill understood unless it is shown against a background of a different tone. Therefore, I state as my first proposition concerning shadows that every opaque body is surrounded and its whole surface enveloped in shadow and light. And to this I shall devote the first book.

Moreover, these shadows are of varying degrees of darkness because they have been abandoned by a varying quantity of luminous rays; and these I call primary shadows because they are the first shadows to form a covering to the bodies concerned. And to this I shall devote the second book.

From these primary shadows there issue certain dark rays, which are diffused through the air and vary in intensity according to the density of the primary shadows from which they are derived; and consequently I shall call these shadows derived shadows, because they have their origin in other shadows. And of this I shall make the third book.

Moreover these derived shadows in striking upon anything create as many different effects as there are different places where they strike; and of this I will make the fourth book.

And since where the derived shadow strikes, it is always surrounded by the striking of the luminous rays, it leaps back with these in a reflex stream towards its source and mingles with and becomes changed into it, altering thereby somewhat of its nature; and to this I shall devote the fifth book.

In addition to this, I will make a sixth book to contain an investigation of the many different varieties of the rebound of the reflected rays, which modify the primary shadow by as many different colors as there are different points from whence these luminous reflected rays proceed.

Furthermore, I will make the seventh book treat of the various distances that may exist between the point where each reflected ray strikes and the point whence it proceeds, and of the various different shades of color which it acquires in striking against opaque bodies." (*The Notebooks of Leonardo Da Vinci*), Oxford University Press, 1952. p. 129-130)

This principle of drawing and painting was, in essence, the same method that was taught to me by Viator Savignac at the Bourget Classical College of Rigaud, near Montreal, where I attended my first years of classical studies, at the age of 13. Savignac used to explain to us the method of Leonardo by saying: "*If you wish to paint the forest, never paint the trees, just paint the shadows.*" This is how he would describe the Leonardo method of artistic composition that some people have described as *{sfumato}*, that is, the "smoky" quality of softness that enveloped all of his subjects as if were dealing with a least action principle of universal change. Leibniz himself addressed this very same question of least action of light propagation two hundred years later in a letter to Huygens, in which he stated his implicit agreement with Leonardo. He said: "*The whole question lies in the manner with which you have yourself considered that each point is itself radiating, and how you have composed a general wave for all of these auxiliary waves.*" (Leibniz, Letter, June 12-22, 1694. Note also that Leibniz had access to Leonardo's work through the *{Huygens Codex}* that Huygens' father owned.

So, in first approximation, the scene of *{The Last Supper}* moving in front of you is like a polyphonic chorus composition of twelve voices expressing the relevant emotional colorations of the beginning of the passion of Christ through the least action principle of light and shadow propagation in the form of the Lydian modality of intervals of well-tempered dissonances, or commas, in the way that Lyn defined them for Classical thorough-composition. For example, listen to a good rendition of the *{Miserere}* portion of the Dresden *{Missa Solemnis}* by Beethoven, and think of the Leonardo fresco in the same way. Think of it also as if you were organizing and singing on a street deployment. These same emotional colorations should be coming out of you and impact the public in the same way that Leonardo described his reflexive process of light, color, and shadow. That is the intention of Classical art: *{change people for the better}*. That is the purpose of *{The Last Supper}*. Now, let's look at it from the standpoint of musical composition as well as from the standpoint of the scientific method of Lyn.



Figure 1. {The Last Supper}.

Leonardo was among those who created the singing discipline of *{bel canto}*, whereby the principle of register axiomatic shifts of the six adult human voices, tuned in accordance with the idea of the Isoperimetric Principle of Cusa, became the basis for all future musical, or plastic forms of Classical artistic composition; much in the same way that Kepler, and Gauss after him, had discovered the significance of the principle of gravitation determining the asteroid belt « interval singularity » of our solar system with the proportionality principle of the arithmetic-geometric mean. The key thing that Leonardo discovered was the significance of such anomalies that overlapped both the domains of Classical artistic composition and science.

The dissonances of the human voices, for example, which Lyn identified in the anomalies, or paradoxes of *{Jesu, Meine Freude}*, and which are expressed in the *{St. John Passion}*, and the *{St. Mathew Passion}*, later developed by Bach, were undoubtedly inspired from the same well spring source of genius as the visible dissonances of *{The Last Supper}* by Leonardo. So, the question that we shall now investigate is : what is the great dissonant event of *{The Last Supper}*?

The great dissonant moment to be captured in Leonardo's work is a great anomaly, which represents the inversion of an existential freakout, that is, the irony where one has to turn one's mortality into immortality. Look at the body language of each and all of the apostles and you can't fail to realize that you are summoned by Leonardo to inquire about what is going on in their minds. The brush of Leonardo is not depicting self-evident visual forms in themselves, but reflexive shadows of their thinking processes. He is making the invisible visible. He is showing you what cannot be seen. A careful study of the underlying method of both the musical composition of Bach and the plastic art composition of Leonardo reveals the presence and the treatment of the same principle of *{least time}* of Huygens, Fermat, Leibniz, and Bernoulli which is generated by means of the well-tempered Lydian interval modality. Now, listen to what Lyn is saying and apply it to the fresco:

« {The *developmental principle characteristic of a Classical musical composition, is a nested set of ironies, which converge upon a single, pervasive metaphor. Each of these contrapuntal ironies, has the quality of a necessary dissonance to be resolved. One must see the dissonance in this case not as some arbitrary dissonance, but as reflecting the same principle of irony underlying the Classical method of sculpture associated with Scopas and Praxiteles, and Leonardo's {The Last Supper}. It is not dissonance in the sense of falseness, but dissonance in the sense of a true ontological paradox. Just as a validatable discovery of a universal physical principle resolves the valid dissonance we see as an ontological paradox, so a great Classical musical composition defines a subsuming musical –ontological paradox, whose solution is the identity of that composition taken as an individual whole. }*» (Op. Cit., p. 33) Any questions about that?

2. LEONARDO PAINTING BETWEEN THE NOTES

The reproduction of {*The Last Supper*} that I e-mailed you is not the original but a reconstructed version of the original painting by Leonardo. When you have a chance, you should get a large reproduction of the original for your office and be mindful of the deterioration of the work by unfortunate circumstances. Now, the reproduction I sent you is not a copy of the original Leonardo, but a reconstructed version by a copyist, which has eliminated the intrusive doorframe of the original and is showing the underneath of the table. I am using it because it identifies certain details that the deteriorated original does not show. From left to right, centered around the figure of Christ, there are four groups of three apostles each: the first group, on the left, is represented by {*Bartholomew, James the Minor, and Andrew*}, the second group is {*Judas, Peter, and John*}, the third group, on the right of Christ, is {*Thomas, James the Major, and Philip*}, and the fourth group, at the extreme right, includes {*Matthew, Thaddeus, and Simon*}. Note how Bartholomew, on the extreme left, is rising on his toes, while Simon, on the extreme right, is digging in his heels. That is a very telling anomaly between the two extremes. Now, find the multiple continuous mean proportionals between those two extremes and you will find the underlying unity of the composition.

Look at this ensemble as the framework for a Classical composition of polyphonic counterpoint in which the unity of effect of the whole composition is reflected into Jesus Christ, at the center of the perspective, but which is also replicated by each individual voice reflecting differently the same thought object, the same {*Geistessmassen*}, that Jesus Christ, as the subsuming central figure, internalizes by reflecting all of the voices as the unifying principle of the composition as a whole. Note the paradox, which is expressed by dissonances between the four groups of three and Jesus. Note that all of the receding lines of linear perspective all converge on the head of Christ and have the purpose of establishing a principle of continuity/discontinuity between the painting and the dining room of the monastery, thus, creating an irony between the two. The monastery rule is silence during meals.

The question of the unity of composition of *{The Last Supper}* is really the very first thing to identify, if one is to understand the organizing principle of the painting at all, that is: how do you explain the explosion of emotions that Leonardo has painted in this dramatic scene? *{The issue, here, is really: how do you relive the principle that masters and unifies all of the different emotions that Leonardo has portrayed in this apparent explosive shock-wave reaction? In a paradoxical way, the answer to this question can only come from understanding why Christ is apparently so calm, while the apostles are so apparently agitated, why Bartholomew is so jumpy while Simon is in such a state of denial. }* That is the central anomaly of the whole scene, which begs the question. What must have happened that produce such an apparent opposite paradoxical effect between Christ and his disciples? Somebody must have said something to cause such a commotion. What was it? If you know the answer don't say it. Take a few moments to see if someone can discover it without looking at the back of the book. If you don't start with asking these questions you cannot internalize the significance of these agitations, there can be no understanding at all about the discovery of principle that Leonardo had composed within *{The Last Supper}*, nor can there be any understanding about the effects of a universal physical principle upon history as a whole.

In his notes, Leonardo related to what Mathew recounted in his Epistles, that is, the drama that unfolded immediately after Christ uttered these very emotionally charged words before his apostles. Christ said:

«*{ 'Verily I say on to you, that one from among you shall betray me.' And struck by a profound consternation, each one of them asked him, 'Lord, is it I?' He replied: 'He who has shared this meal with me, shall betray me. The Son of man is leaving; as it has been written of him. But, woe betides he who has betrayed the Son of man! That man would wish he had never been born. }* » (Mathew 26; 20-22)

One must fully internalize these terrible truthful words in order to grasp the shock that these words must have produced within the assembly of the twelve. It is that very unraveling moment of mortality and immortality of the passion that the *{The Last Supper}* is depicting. What Leonardo is conveying to the audience, in this *{sublime event,}* is the experience that each of the twelve apostles lived at that moment, as the truth of those words hit them and as they were recoiling from it, as if they were between two contradictory states: *{It cannot be me...}* and *{...Lord, is it I}*. This is the perplexity which must accompany any great discovery of physical principle, and without which such a discovery would be a fraud. This irony is the dramatic life and death question of the political organizer of a Renaissance, the sublime and dramatic paradox that a Renaissance man like the members of this organization today must recreate consciously as the active principle of his or her daily activities. From that vantage point, *{The Last Supper}* is a mirror about us as a revolutionary movement! Any questions about that?

Now, from that non-linear perspective, look at the sub-groups having the function of acting on the whole from that higher dimensionality, where the passing from a lower manifold to a higher manifold represented the resolution of such dissonances into the unity of the composition represented by the sublime serenity of Christ. That is the true

intention of the aerial perspective in *{The Last Supper}*, which, as Carnot said, can only be grasped by the sentiment, that is, cognitively. This is to be conveyed as if a shock-wave were to have come from the dynamic yet peaceful center where Christ is sitting, with his arms peacefully extended and his eyes lowered in the serene acceptance of what is to become of his mortality, and had traveled simultaneously to the two ends of the long table, hitting every apostle differently, and resonating off of each of them, in a polyphony of dissonances; and from which all are recoiling back to Christ and to the spectators in the dining room. Like the drama of a Classical Shakespearean play, Leonardo's *{The Last Supper}* forces the spectator to relive this drama on the stage of his own imagination, and to come out of the experiment a better human being than he was before entering that experiment.

Thus, the unity of composition of *{The Last Supper}* represents, in reality, the shock wave principle of creativity itself behind the apparent world of the senses, that is, the ecumenical Christian principle of organizing axiomatic changes that have the power of shattering the fishbowl domain of socially accepted public opinion. What Leonardo painted was the relationship between the visible behavior of human beings and the invisible principles unifying their reactions in the real world. This means that it is not the particular visible individuals that count, nor their specific « fishbowls », but the cultural change of a world based on the ecumenical principle of *{agape}* as represented by Cusa's renaissance. Thus, Leonardo's *{The Last Supper}* reflects as many paradoxes that form Lydian dissonances, which have to be resolved, in a well-tempered form of *{painting between the notes}*. Such is also what Lyn had identified as the social function of Classical music, Classical art, and Classical drama. Any questions?

3. THE MEANS OF CHANGE: THE THREE LYDIAN SPIRALS OF MOZART

Now, how can synthetic geometrical conic functions represent such ironies in the well-tempered system? The principle of Lydian intervals used by Leonardo in *{The Last Supper}*, is the same as the one used by John Sebastian Bach in his composition of the art of the fugue and more specifically in his *{A Musical Offering}*. Lyn also showed that it was the same principle which was later developed by Mozart, in his piano sonata *{Fantasy K. 475}*. It can also be recognized in Beethoven's *{Sonata Quasi-Una Fantasia}*, otherwise known as the *{Moonlight Sonata}*, as well as in a significant number of Mendelssohn's piano sonatas. As Lyn demonstrated, all great Classical composers, such as Hayden, Mozart, Beethoven, Schubert, Shuman, Mendelssohn, and Brahms, have used the same principle of Lydian intervals of thorough-composition. (See Lyndon H. LaRouche, *{Politics as Art}*, in Fidelio, Spring 2001.) A simple exercise can give an idea of how to generate such Lydian intervals on a keyboard.

Do the following experiment on a keyboard. Start with dividing the octave of C-256 to C-512 and think of the octave as the interval of a logarithmic spiral action. Divide the action of the octave in half by going halfway up the scale to F#, and listen to the interval of action between C and F# played in succession, then simultaneously. It is quite dissonant. How can you temper that interval of dissonance? You can do it by dividing the

half action by half again, that is, by going back to Eb. Note how, when you play successively C, F#, and Eb, the dissonance disappears but only to reappear in a successive progression, where you go halfway the octave of Eb to A. Do not play chords, but successive intervals, and listen very closely to the differences in dissonances and their tempering. Next, after creating the dissonance of a half octave between Eb and A, temper it by a return to F#, and lastly, by going from F# to C-512, you will create another dissonance which can be tempered again, in the same fashion, by going back to A, and so forth. If you proceed like this with successive intervals of spiral actions, in a continuous flow of intervals of minor thirds, in an ascending manner, you can actually hear the Pythagorean commas defining the progression. Note that it is these commas, which determine the 256 series and not the 256 series, which determines the commas. Each one of these three spiral actions corresponds to a similar mean of change up and down the cone with different colorations and different shades.

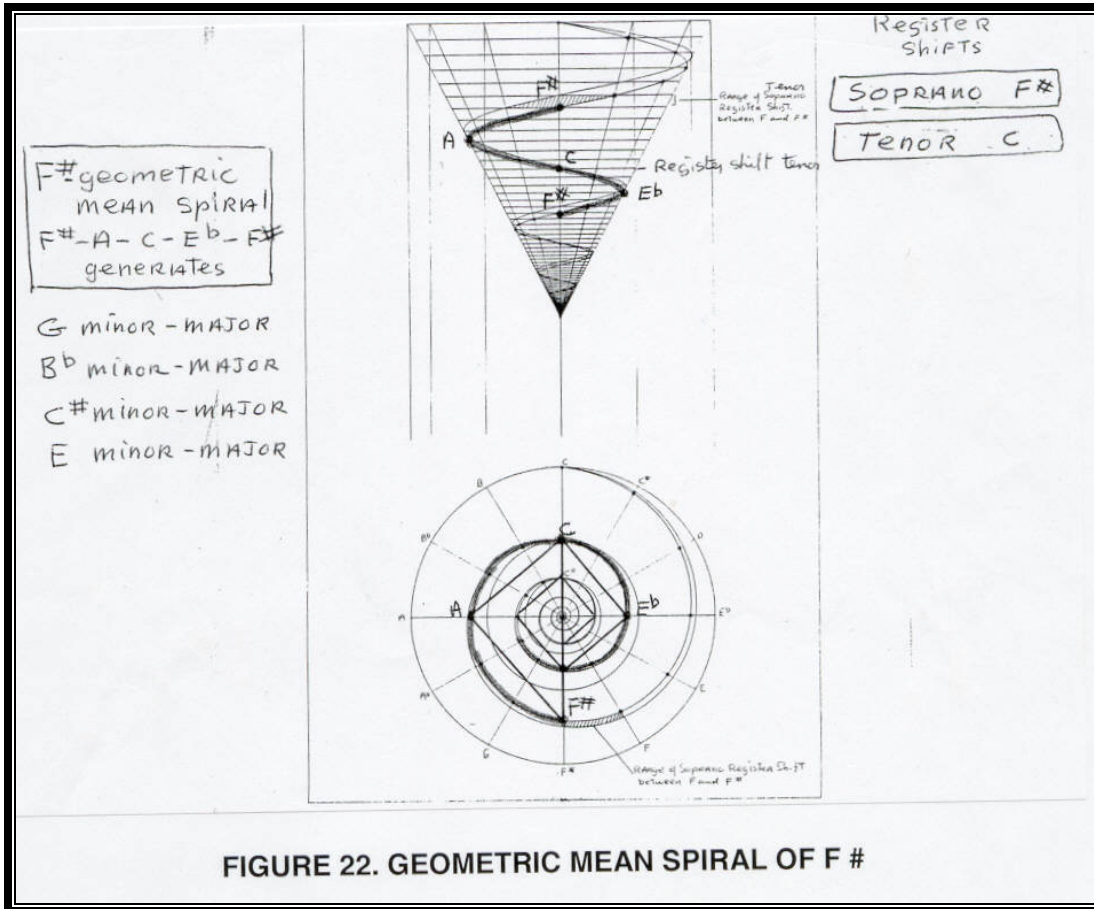


Figure 2. [Geometric Mean Spiral of F#.]

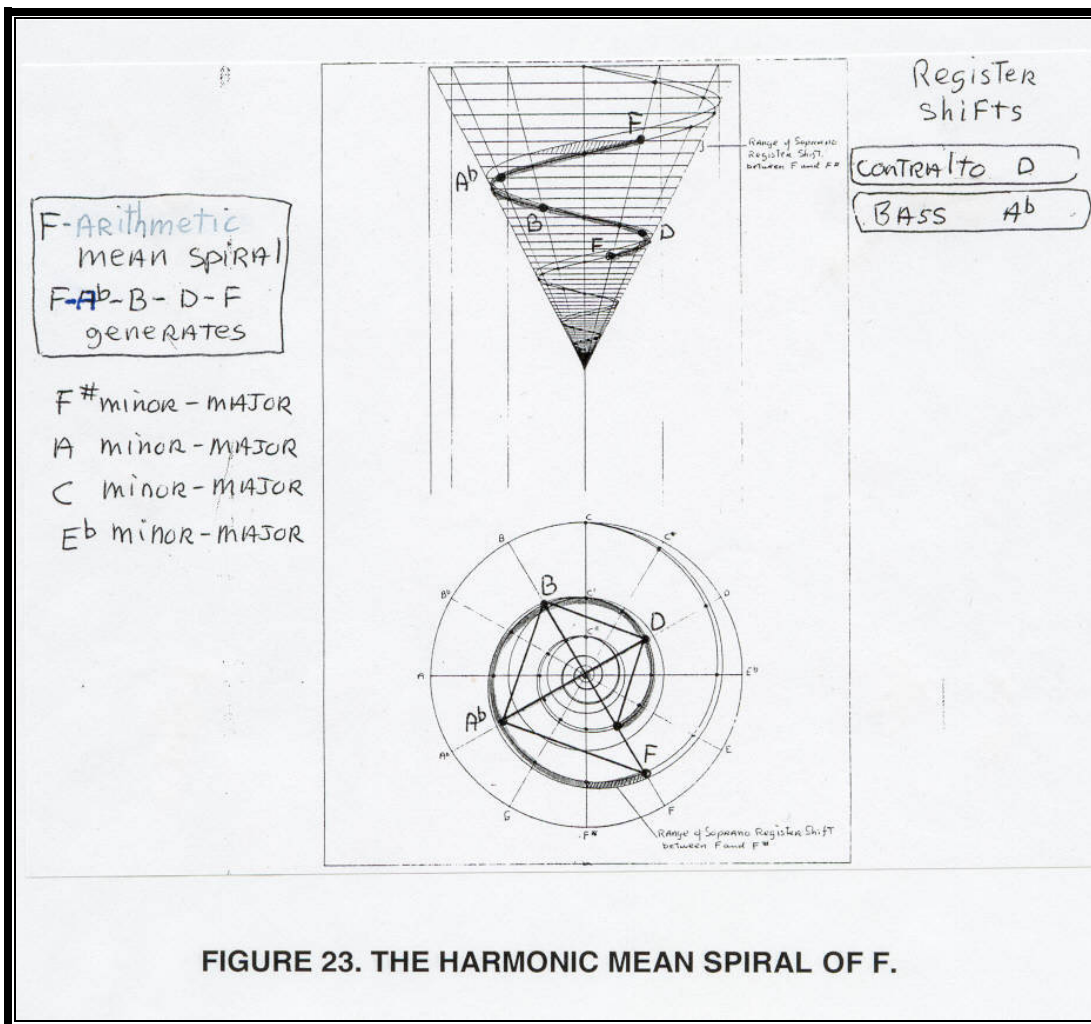


FIGURE 23. THE HARMONIC MEAN SPIRAL OF F.

Figure 3. [Harmonic Mean Spiral of F.]

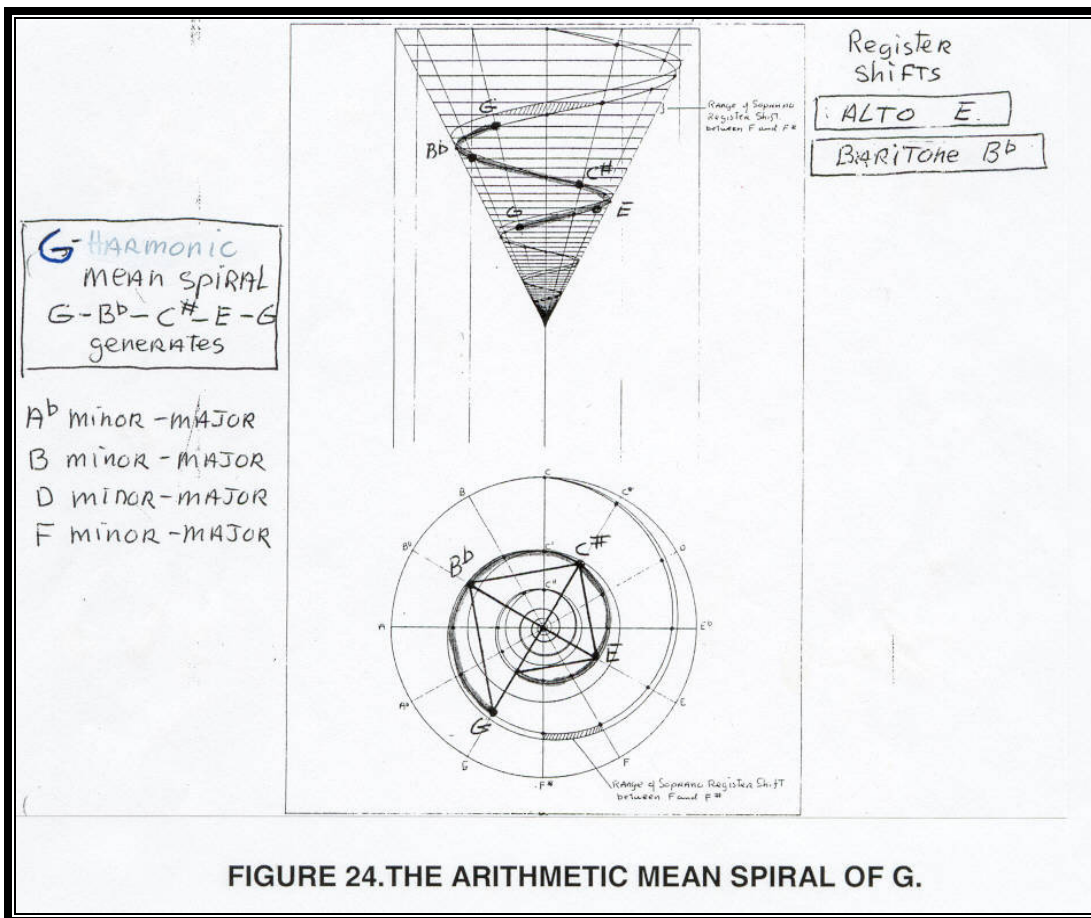


FIGURE 24. THE ARITHMETIC MEAN SPIRAL OF G.

Figure 4. [Arithmetic Mean Spiral of G.]

In the least inadequate domain of what Lyn had identified as the continuous and discrete manifolds of synthetic geometric, my hypothesis for the arithmetic/geometric spiral action (double spiral) represents the least action form of generating singularities and dissonant discontinuities. Consider that all three double Lydian spirals, representing the action of each of the six vocal human voices, taken two by two, represent a least inappropriate heuristic device reflecting the production of axiomatic register shifts, or axiomatic passages, from a lower geometry to a higher geometry. There are only three such Lydian spiral actions for the 24 minor and major keys of the well-tempered system, and all three are developed extensively in the Mozart {*Fantasy K. 475.*}

With these three spiral progressions only, you will cover all of the twelve Lydian intervals of the well-tempered musical system. Take **Figure 2**, for example, and look at the four key changes for Soprano and for Tenor. My hypothesis is that the F# geometric mean Lydian spiral for soprano and tenor voices (with C and F# register shifts) generates key changes in G, Bb, C#, and E minor and major; the F harmonic mean Lydian spiral of contralto and bass voices (with D and Ab register shifts) generates key changes in F#, A, C, Eb minor and major; and the G arithmetic mean Lydian spiral of alto and baritone voices (with E and Bb register shifts) generates key changes in Ab, B, D, and F major and

minor. I would very much welcome comments on the part of voice teachers on this hypothesis.

Then, in each case, of the six human voices, invert the motion and play the same intervals on a keyboard, in a descending manner. Each descending process of spiral action will be different. Such geometric spiraling motions, ascending and descending will generate all slightly minor-dissonant intervals, which, when projected from the continuous manifold onto the discrete manifold, represent a partitioning of octaves into half, and half of the half. These dissonances all appear to be of equal division of the ascending and descending scale progressions in the discrete manifold, but they are not equal. They are all associated with Lydian intervals which, when organized in the same key, or when leading to a naturally ordered set of changes of four different keys for each of the three progressions, they bring closure to the composition like an ensemble of arithmetic-geometric mean functions. Think of Mozart's {*Fantasy K. 475*} as such a function of functions.

The image contains two musical score excerpts with handwritten annotations. The top excerpt, labeled 'Adagio', shows the first four measures of Mozart's Fantasy in C, K. 475. It features a piano part in the lower register and a vocal line in the upper register. Handwritten annotations include 'Tenor Soprano' and 'BARITONE ALTO' in boxes, indicating voice ranges. Dynamics 'f' and 'p' are marked, and a 'pp' marking is present under the vocal line. The bottom excerpt, labeled 'Primo tempo', shows measures 161-164. It also features piano and vocal parts. Handwritten annotations include 'First double spiral' and 'Third double spiral' in boxes, indicating specific musical structures. Dynamics 'f' and 'p' are marked, and a 'pp' marking is present under the vocal line. The entire figure is enclosed in a black border.

Adagio

Tenor Soprano

BARITONE ALTO

f p

pp

Tenor

BARITONE

FIRST 4 MEASURES OF MOZART'S FANTASY IN C- 475
FIGURE 25.

161 Primo tempo

First double spiral

Third double spiral

f p

pp

Tenor

MEASURES 161 – 164 OF MOZART'S FANTASY SONATA - 475.
FIGURE 26.

Figure 5. [Mozart Fantasy Sonata K. 475]

It seems that a musical *{fantasy}* was ironically the form that was initially chosen by Mozart to express such a reality principle. There are a few other pieces that resemble this one written by Bach and Beethoven. However, the Mozart piece is unique and it does not resemble anything else that he, himself, wrote. From that standpoint, it is a unique axiomatic piece for artistic composition, and it stands alone like Leonardo's *{The Last Supper}*. It is worth studying it and learning it thoroughly. And, I will tell you a secret: you can even learn that extraordinary peace without even knowing how to fluently read the score, that is to say, you can play it just by reconstructing it with these three spiral actions. When ordered in accordance with the register shifts of, say, the three adult male voices, creating tensions between them, the development of those three ascending-descending series of minor third progressions, into four keys each, will inevitably lead you to understand how to organize a thorough-composition of resolved dissonances.

What is unique about these Lydian intervals is that they represent a quality, which permits you to change: that is, change moods, change coloration, change shades, with the different change of keys within the system itself, and ultimately change lawfully the system itself. Thus, what seems at first to be simply a mistaken dissonance becomes a willful axiomatic change to a higher domain. Then, you can bring the whole system beyond the limit to which the Lydian system of Mozart can be brought to, and then, you get into the domain of the late quartets of Beethoven. I know this is too much of a cursory geometrical overview, which should require much more development and musical examples, but any one of you who plays and studies the Mozart *{Fantasy K. 475}* will discover these forms of action as boundary conditions of the composition.

CONCLUSION.

So, this brings us to the end of this cycle of classes. However, we have not come to the end of a closed cycle. We are not as we were at the beginning, and I hope we have become better human beings than we were at the start. So, I think the best way to end this class is with the letter of Eratosthenes * to King Ptolemy that I have just discovered and which, I think, is what Lyn referenced in the Morning Briefing of June 6th 2006, and that I quoted in my first class on the question of the *{conical function}* underlying the doubling of the cube. Although we don't have any more time to discuss this letter, I thought it would be important for you to have it as an added insight.

With respect to the construction and application of models, the letter of Eratosthenes ends with a reference to the fact that Plato had proscribed the use of a mechanical form of geometry as a method of finding two mean proportionals for doubling the cube, and that he insisted that the dynamics of physical reality be the only appropriate means of finding them. Since purely mechanical forms of geometry tend to focus on sense perception results as opposed to non-visible causes underlying the physical processes, Plato chose to exclude mechanics from dynamics. This will be an important point to remember when you tackle Kepler's *{New Astronomy}*. If you have any question about it, send me an e-mail.

Similarly, the intention of this class was never to develop the doubling of the cube, as such, mechanically or otherwise, but rather to develop the power of the human mind to change the universe by means of carefully chosen shadows, which are cast on the dimly lit wall of Plato's Cave, and which open a window on physical universal principles. The function of such shadows, as Lazare Carnot put it in his class on Public Works, is “{*generating ideas by means of the senses, of acting on the soul by the organ of vision.*}” Thus, as insignificant as they may appear at first glance, shadows must be chosen and used wisely for the intention of leading to the non-visible underlying universal physical principle that projected them. The significance of the shadow angle of the Ecliptic in our Egyptian model for doubling the cube is a good reminder of how to discover a crack leading to the underlying action of such a universal physical principle.

So now, like Virgil, I must leave you at the threshold of a higher dimensionality that Delante and other LYM members will now take you to, as Beatrice did by accompanying Dante to the higher spheres. Saludos amigos, and don't forget Lyn's exhortation to {*believe nothing that for which you cannot give, yourself, a constructive proof.*} Thank you for your attention and for your precious collaboration.

FIN LAST CLASS, 10/14/06

*[Letter from Eratosthenes to Ptolemy.]

“{*Eratosthenes to King Ptolemy greeting.*”

There is a story that one of the old tragedians represented Minos as wishing to erect a tomb for Glaucus and as saying, when he heard that it was a hundred feet every way,

*Too small thy plan to bound a royal tomb.
Let it be double; yet of its fair form
Fail not, but haste to double every side.*

But he was clearly in error; for, when the sides are doubled, the (surface) area becomes four times as great, and the solid content eight times as great. Geometers also continued to investigate the question in what manner one might double a given solid while it remained in the same form. And a problem of this kind was called the doubling of the cube; for they started with a cube and sought to double it.

While then for a long time everyone was at a loss, Hippocrates of Chios was the first to observe that, if between two straight lines of which the greater is double of the less it were discovered how to find two mean proportionals in continued proportion, the cube would be doubled; and thus he turned the difficulty in the original problem into another difficulty no less than the former.

Afterwards, they say, some Delians attempting, in accordance with the oracle, to double one of the altars fell into the same difficulty. And they sent (for) and begged the geometers who were with Plato in the Academy to find for them the required solution. And while they set themselves energetically to work and sought to find two means between two given straight lines, Archytas of Tarentum is said to have discovered them by means of half-cylinders, and Eudoxus by means of so-called curved lines. It is, however, characteristic of them all that they indeed gave demonstrations, but were unable to make the actual construction or to reach the point of practical application, except to a small extent Menaechmus and that with difficulty.}" (T. L. Heath, {*APPOLONIUS OF PERGA, Treatise on Conic Sections*}, Barnes & Noble Inc., 1961, p. xviii.)

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