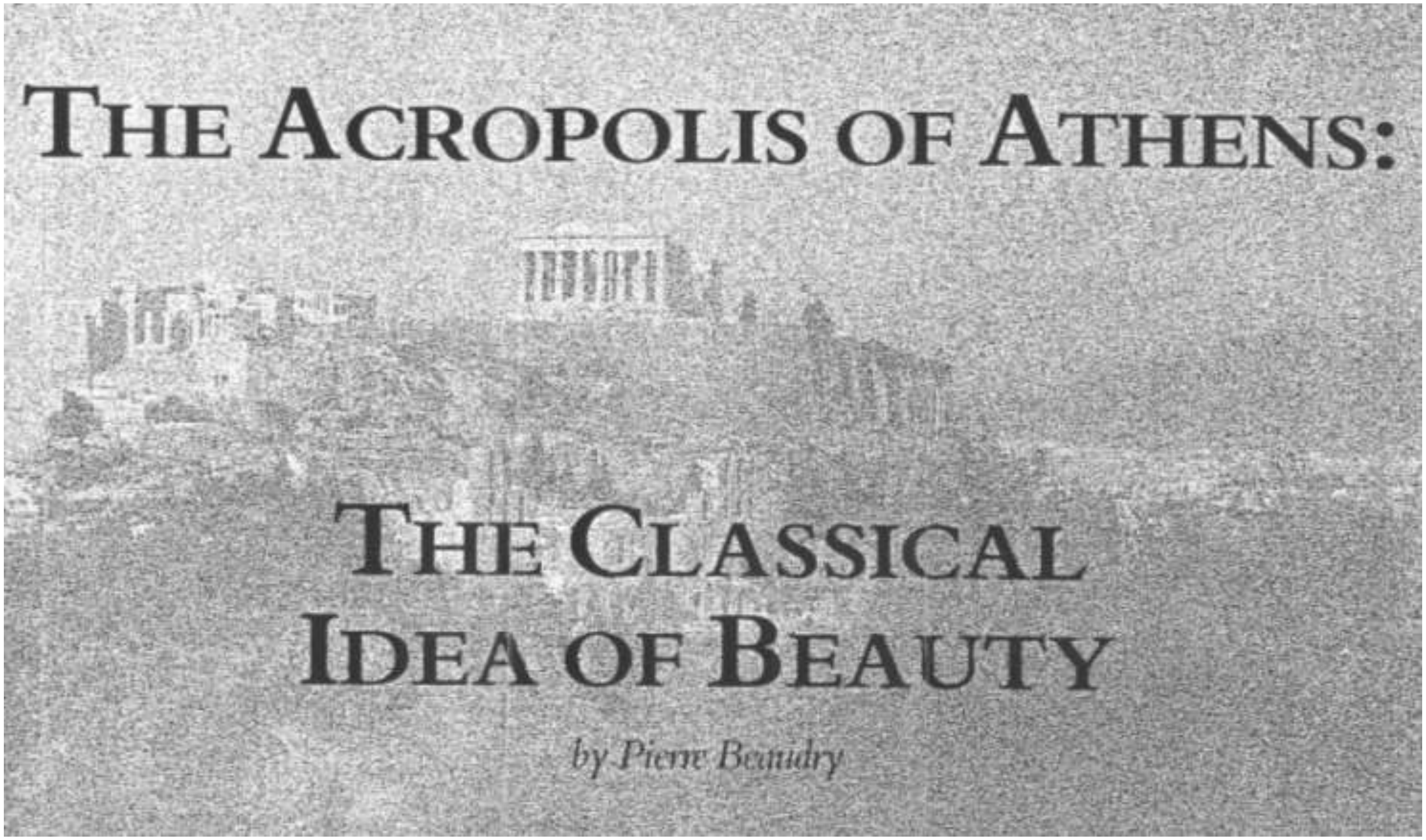
Reproduced from The New Federalist, June 24, 1988.



"Take the exemplary case of the Acropolis. Studies show that the Acropolis is the result of the unfolding of a single, coherent plan, always subsumed by the Classical Greek notion of Golden-Section-pivoted beauty in plastic art. In effect, the resulting construction has the quality of a single, if "polyphonic" act of composition." (Lyndon LaRouche, ***The Substance of Morality, Part I***, The Schiller Institute, reprinted from Fidelio, Vol. VII, No. 4 Winter 1998.)

The Acropolis in Athens (447-405 B.C.), as we know it today, was designed and built under the Pericles republic (450-429 B.C.) by two outstanding architects, Iktinos and Mnesikles, representing the high point of classical Greek architecture.

Although there exist today no written treatises, no plans, no design models, but simply ruins—one can nonetheless reconstruct the entire Acropolis mainly by rediscovering the principle of harmonic ordering which still lies dormant in that masterpiece, after more than 2,500 years have passed.

The method of archaeological reconstruction I have used here will show that the composition of the Acropolis has nothing to do with the mishmash of trial and error and ad hoc solutions attributed to its architects by most of the literature on the subject.

There was only one school of architecture in ancient Greece; it came from the tradition of Thales of Miletos, the first Greek philosopher known to us, and from Solon of Athens, the father of the Attic republic, who brought back from the school of Ammon in Egypt during the late seventh and early sixth century B.C., the science of synthetic constructive geometry for city building.

Their method of composition was based rigorously on the principle of least action (isoperimetry) and proportionality of circular action, whereby each element in the construction reflects the whole, such that each part is a complex arrangement which generates other parts by means of the process by which it itself was generated.

It is demonstrable from the works of Iktinos and Mnesikles, the two principal architects of the city of Athens, that they had mastered a most exciting method of constructive geometry, for which conical projection of the Golden Section and self-similar spiral action formed the basis for the elaboration of the Parthenon (447-432 B.C.), the Propylaia (437-432 B.C.), and the Erechtheion (409-405 B.C.) on Mount Acropolis.

The Principle of Harmonic Ordering

Guided by a powerful desire for truth and beauty, the Athenian architects sought to express a fundamental principle which is common to living processes, music, city building, and the thinking process itself.

This principle, otherwise known as the principle of aesthetics, is best defined by Plato in ciple of aesthetics, is best defined by Plato in his dialogue, the *Timaeus*. "It is impossible to put two things beautifully together without a third. There must be a bond between them which shall bring them together. The best of bonds is that which fuses into one both itself and the things which it binds together, and proportion is that which is by nature best suited to accomplish this. Wherever there are any three numbers or surfaces or volumes that have such a mean that the first is to the mean what the mean is to the last, and the last is to the mean what the mean is to the first, then when the mean becomes first and last, and when both the first and last become mean alternately, they all come to be the same—and when they become the same as one aother, they all become one."

Thus beauty emerges from this underlying harmonic ordering as what Saint Augustine and Nicolaus of Cusa in later centuries defined as the harmony of the Christian Trinity: Unity—Equality—Connection.

Such an ordering is an expression of the curvature of the least-action principle, the only possible curvature of natural law for the physical universe. Any other arrangement is unlawful and therefore results in ugliness. This principle of harmonic ordering informs the school of Ammon in ancient Egypt, Thales of Miletos, Solon of Athens, Mnesikles and Iktinos, Plato, Saint Augustine, Nicolaus of Cusa, Leonardo da Vinci, Kepler, Guass, Riemann, and the school of Lyndon LaRouche today.

Figure 1

The Golden Section

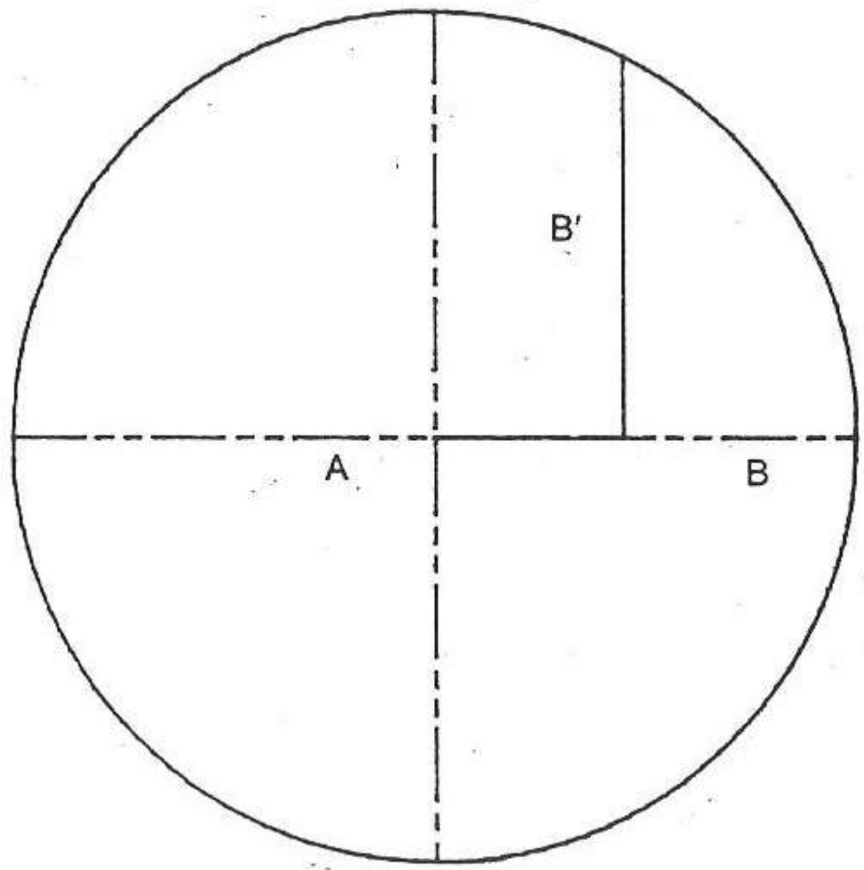
For the Athenians, a measure is a "mean of means," and not simply a length in and of itself. Inscribed in a circle, the harmonic relationship between a first A, a mean B′, and a last B, may relate alternately to the length, the breadth, and the height of a building. Their ratios are such that $A/B' = B'/B$.

This figure shows the ratios to be in golden proportions because $B + B' = A$.

To construct this, take any length B′ and make it grow by half of itself at a right angle to itself.

Circumscribe it, and you have generated a perfect Golden Section.

Figure 1.



$$\frac{A}{B'} = \frac{B'}{B} = \text{Golden Section}$$

Because $B + B' = A$

Figure 2

Musical Intervals

The Athenians constructed their temples by replicating the living process and the process of composition of musical intervals.

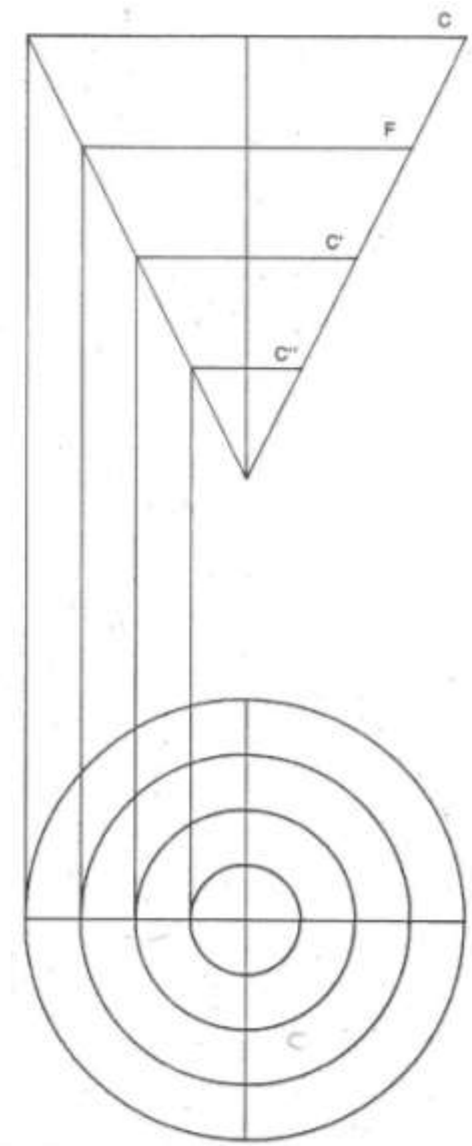
For anything that grows to become the double of itself, it must first divide itself by half and grow by half of itself, and then by a third of itself again. The Athenians gave musical terms to this growth process:

1) The diapente (fifth) is one and one-half, or $3/2 = F/C'$.

2) The diatessaron (fourth) is one and one-third, or $4/3 = C/F$.

3) The diapason (octave) is $2/1 = C/C'$.

It is fitting that the Athenians used musical proportions for the design and construction of the buildings of the Acropolis, since Athena, the goddess of this sacred precinct, is the goddess of reason, presiding over the arts, poetry, and music.



Diapente $\frac{3}{2} = \frac{F}{C}$

Diatessaron $\frac{4}{3} = \frac{C}{F}$

Diapason $\frac{2}{1} = \frac{C}{C'}$

Figure 3

Plan of the Acropolis

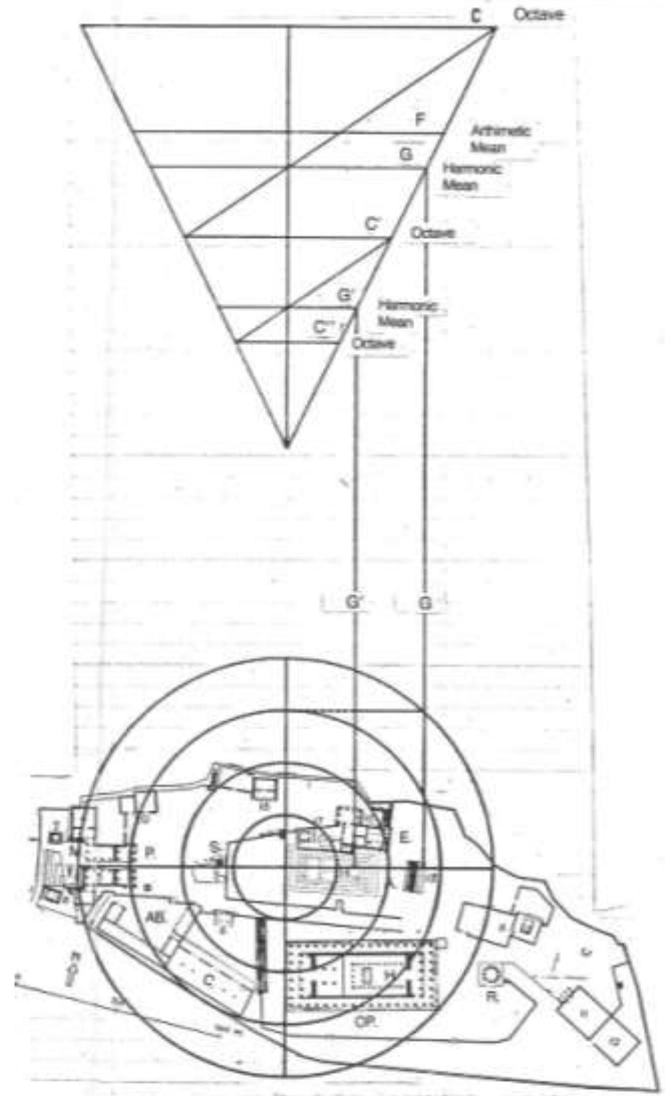
When you divide the same circle by three and by four simultaneously, the value of the diapente/diatessaron ratio (3/2 over 4/3) generates the Golden Section.

By projecting from a cone, the diapente/diatessaron function locates precisely the Propylaia (1), the Parthenon (2), and the Erechtheion (3) on the Acropolis.

The musical references are: C, the octave; F, the arithmetic mean; G, the harmonic mean; C′, the octave; G′, the harmonic mean; and C″, the octave.

The harmonic mean octave of G-G′ locates the Golden Section and the position of the Parthenon floor plan, which is itself a double golden rectangle.

This is the harmonic ordering underlying the Platonic solids that Kepler used to elaborate the harmony of the spheres. The diapente/diatessaron function is also similar to Gauss' arithmetic/geometric mean function. It is a harmonic/arithmetic mean function, a mean of two means, that is, a double mean function.



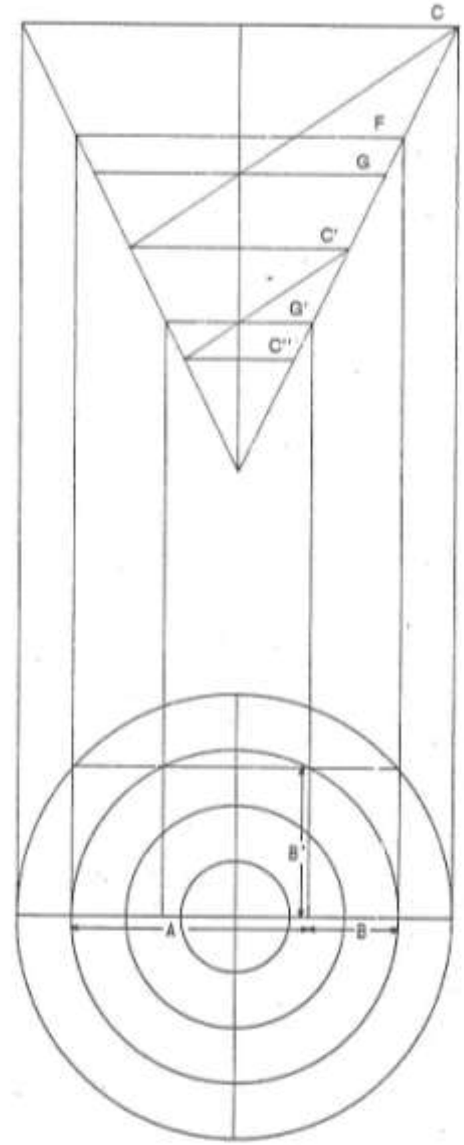
Athens, Acropolis II and III. Plan (Dinsmoor).

A. Archaic Athena Temple
AB. Artemis Brauronia Precinct
B. Beule Gate
C. Chalcotheca
E. Erechtheion
H. Hecatompedos Naos (Parthenon)
M. Monument of Agrippa
N. Nike Temple
OP. Older Parthenon
P. Propylaia
R. Roma and Augustus Temple
S. Statue of Athena Promachos

Figure 4

The Parthenon

The great temple to Athena, the Parthenon, built by Iktinos, is based on a spiral progression which can be expressed in living processes by the Fibonacci series. It is constructed as a chain of ratios 1/2, 2/3, 3/5, 5/8, 8/13, 13/21 . . . , where each element is derived successively from the preceding one. When the convergence upon the Golden Section reaches the ratios 8/13 and 13/21 by successively replacing the first by the mean and the mean by the last, then the first becomes 13 (the height), the mean becomes 29 (the breadth), and the last becomes 65 (the length). For this reason, the triglyph-metope relationship above the architrave defines the intercolumnization in the same golden rectangle proportion as the front elevation is to the entire floor plan (stylobate) of the Parthenon.



$\frac{A}{B'} = \frac{B'}{B}$ = Golden Section

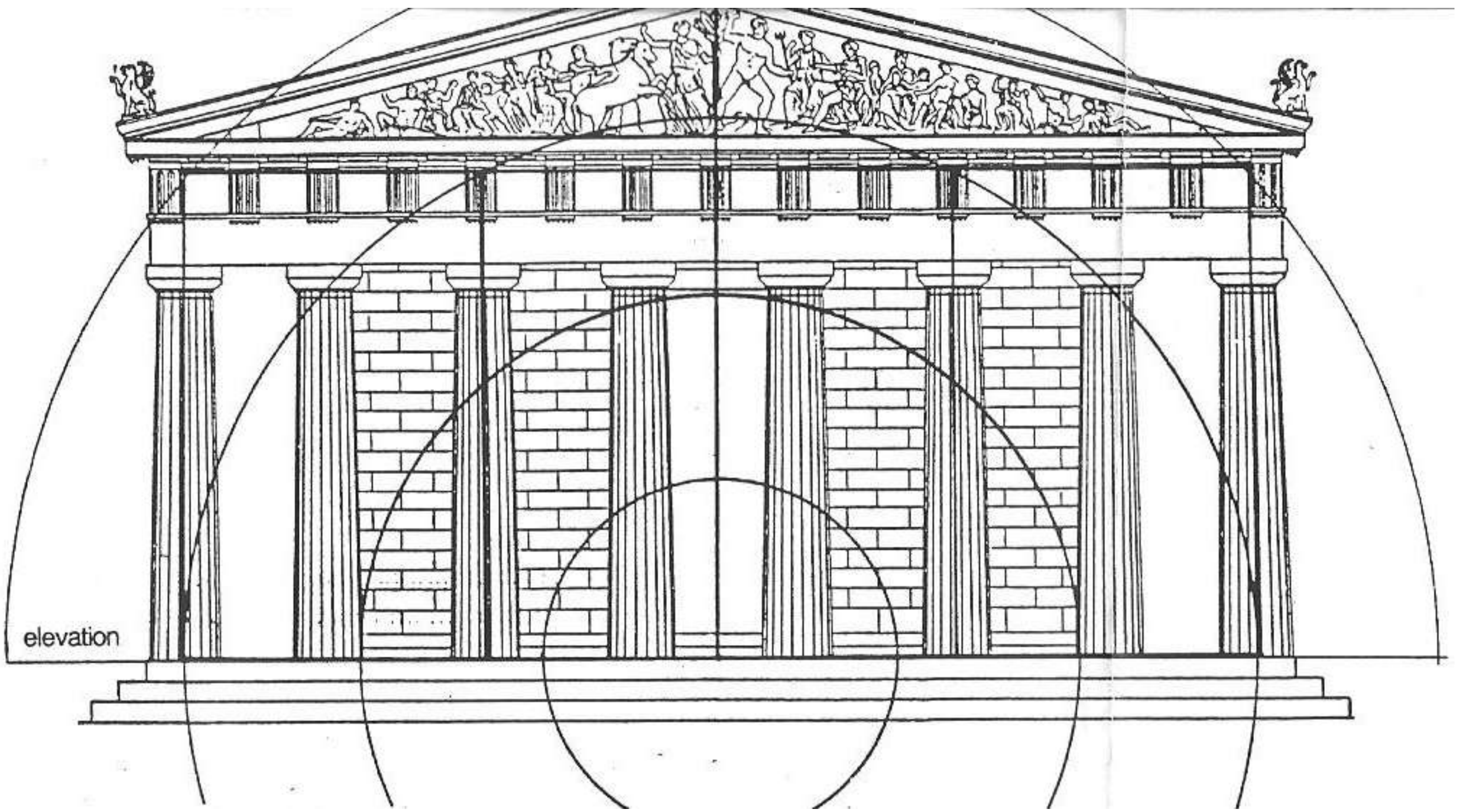


FIGURE 4 THE PARTHENON golden rectangle.

Figure 5

Propylaia and Erechtheion

The west elevation of the Propylaia, the entrance of the Acropolis, is a very beautiful composition mixing into one all of the previously mentioned ratios: the octave 2/1, the fifth 3/2, the fourth 4/3, and the Golden Section. The Propylaia is a mixture of both the Doric and Ionic orders.

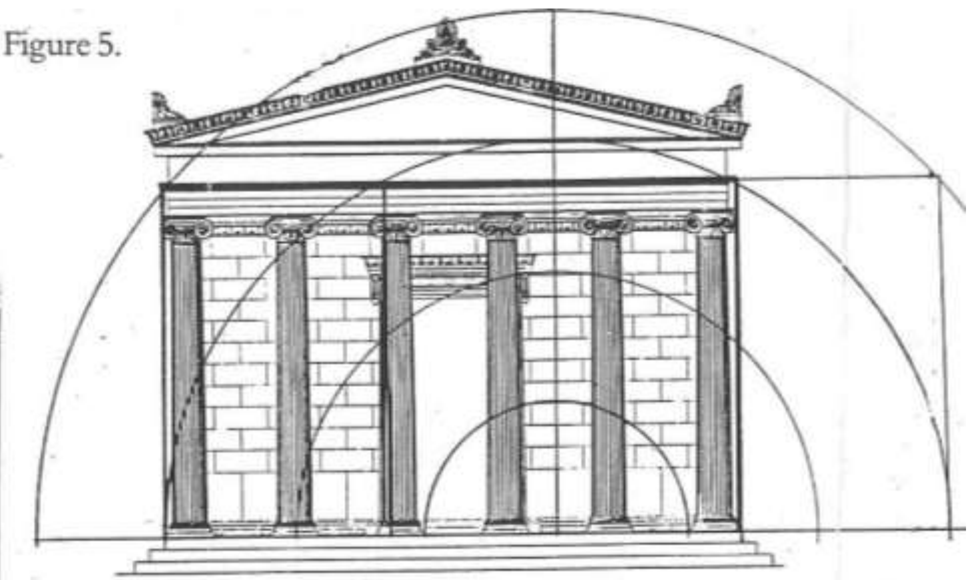
Other ratios can also be found, since the division of the maximum circle by three and four ultimately divides its radius into 12 equal parts: the basis for the 12-inch ruler!

Mnesikles' initial design for the Propylaia was to be entirely symmetrical; however, he was forced to modify his plan under the political pressure of the fundamentalist cult of the temple of Athena Nike.

The Erechtheion is undoubtedly the most beautiful gem of the entire Acropolis. The east elevation, including the architrave, forms a perfect golden rectangle. The only temple exclusively in the Ionic order, it is dominated by the ratios of 2/1 (double square floor plan) and the Golden Section.

The volutes of the Ionic capitals of the Erechtheion and the Propylaia, as well as the curvature of the Doric capitals of the Propylaia and the Parthenon, show that the properties of spirals and of conic sections (including the hyperbola) were already well known during the 5th century B.C.

Figure 5.



Erechtheion east portico

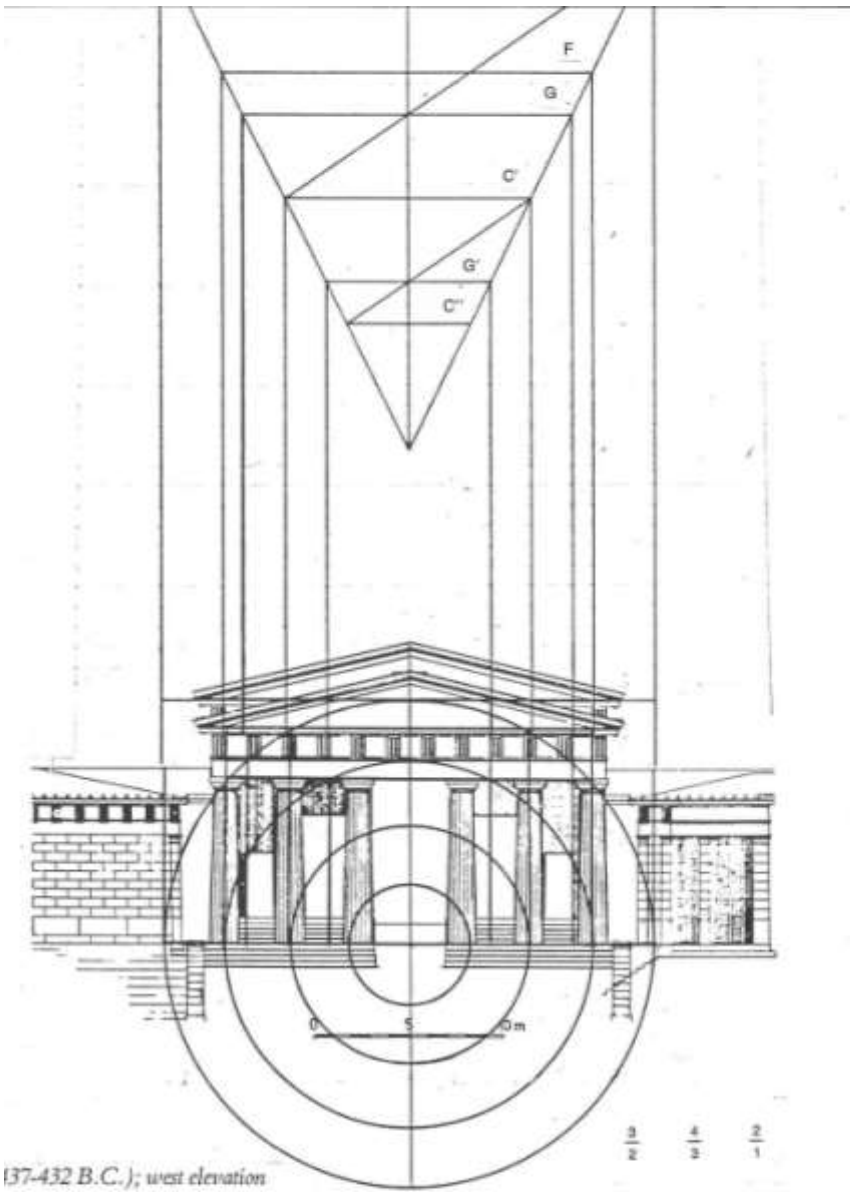


FIGURE 5 PROPYLAIA

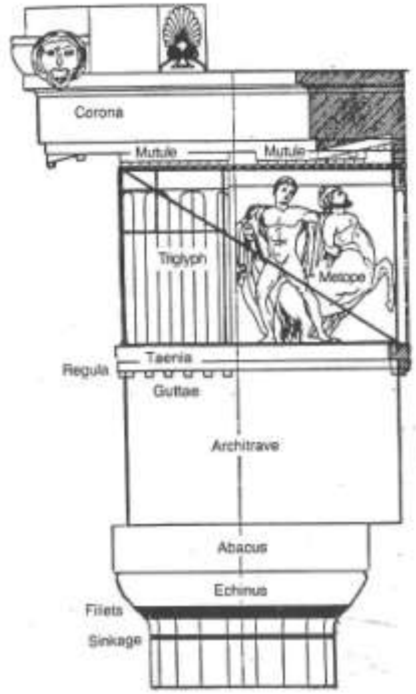


Figure 6

Leonardo da Vinci: The Proportions of the Human Body

Leonardo da Vinci's schema for the proportions of the human body illustrates the origin of the three Greek orders—Ionian, Doric, and Corinthian—as taken from the ratios of the human body.

The first ratio is 1/6, taken from the ratio of man's hips (side to side) to his height. The other, 1/10, taken from the ratio of his hips (front to back) to his height. By taking the arithmetic mean of the two ratios, you obtain the Ionian order ratio 1/8. Similarly, by taking the means of the Ionian with the larger and smaller ratios of man, you get, respectively, the Doric order 1/7 and the Corinthian order 1/9 (the Corinthian order is absent from the Acropolis). Note that Leonardo's man is not only divided into six segments, but also into four. By this double division, you will arrive again at the Golden Section by means of the diapente/diatessaron function. Leonardo's series converging on the Golden Section 1/3, 3/4, 4/7, 7/11, 11/18, 18/29 . . . is also applicable to the Acropolis.

Leonardo's construction is projected from Cusa's isoperimetric circle, expressing man as the unity of the maximum and the minimum in the universe.

Cusa's isoperimetric theorem can be described in the following way. If you inscribe an equilateral triangle in the maximum circle C, the circle inscribed in that triangle would be the minimum C′, and their ratio would be $2/1 = C/C'$.

By increasing the number of sides of polygons (square, pentagon, hexagon, etc.) of the same perimeter (isoperimetric) of this triangle, one can imagine that a repeated series of inscribed circles for such polygons would grow larger toward the maximum, while the series of circumscribed circles for the same polygons would grow smaller toward the minimum. The isoperimetric circle is the locus where the two series will meet, that is, where the maximum and the minimum coincide. The isoperimetric circle coincides with circle E, which locates the floor base (stylobate) of the Parthenon.

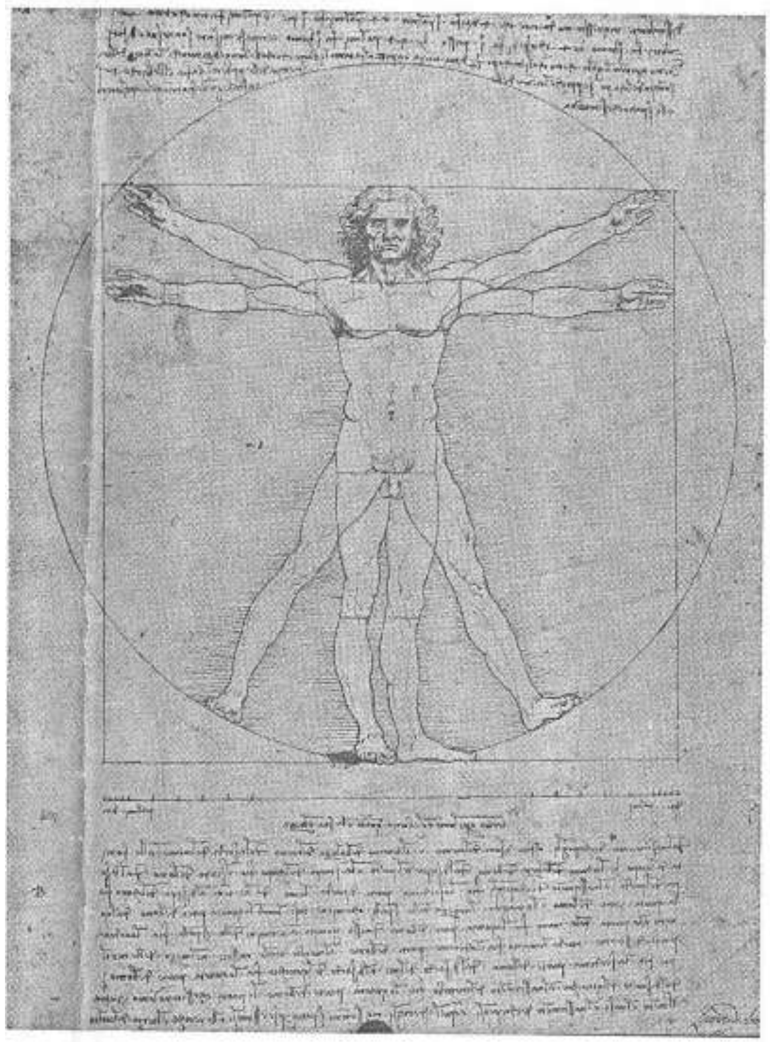


Figure 6.

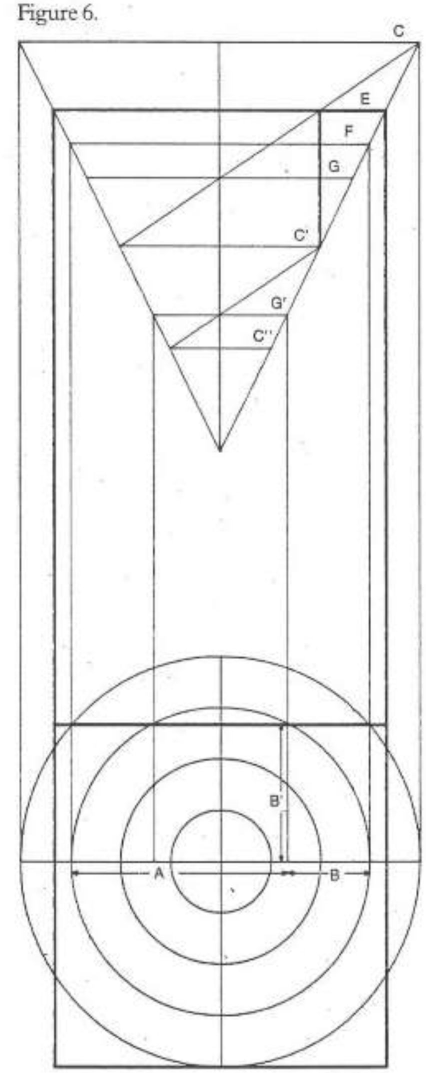


Figure 7

Continuum Theorem Of Classical Beauty

Any fundamental discovery in physical science, from before Plato's time until today, and through tomorrow, may be adduced from the implicit least action form of harmomic ordering, $A/B' = B'/B$. Beauty emerges "when a bond fuses into one both itself and the others it binds together."

By converting the ratios of lengths of simple circular action into ratios of intervals of an elliptical function, the ratio $(A - B)/(A' - B')$ becomes the value of a specific ellipse E. The iterative progression of Gauss' arithmetic/geometric mean function, including the singularity of the well-tempered F-sharp, generates a well-ordered harmonic series such as those of Fibonacci and Leonardo. 9 Ellipses

To construct this, consider that these ratios of intervals are the same for the circle as for the ellipse. By calculating the ratio of intervals instead of lengths, you can quantify the rela- [illegible] occur twice for each ellipse: one for the aphelion and perihelion (A and B), and the other for the arithmetic and geometric means (A' and B'). Each ellipse is a mean ellipse such that the interval between its arithmetic and geometric means becomes the interval of the aphelion and perihelion of a subsequent ellipse, while the interval of its aphelion and perihelion becomes the interval of the arithmetic and geometric means of a previous ellipse. (On the elliptical orbit of a planet, aphelion represents the position on the ellipse at which the planet is farthest from the Sun; perihelion, the point on the orbit at which it is closest to the Sun.)

The harmonic ordering is such that in one direction the ellipses become rounder and culminate in a circle, while in the other direction they become more elongated, ending in an absolutely straight line!

By investigating the nature of these intervals, you will also discover the inverse of the arithmetic/geometric mean function; that is, if the progression toward the arithmetic/geometric mean is defined by the compounding of $A' = (A + B)/2$ and B' = the square root of (A times B), the reverse process shall be defined by compounding $A' = A +$ the square root of $(A^2 - B^2)$ and $B' = A -$ the square root of $(A^2 - B^2)$.

By virtue of this iterative progression, it becomes evident that the elliptical function of Gauss actually describes the harmonic ordering of Cusa's minimum/maximum principle.

Since $(A - B)/(A' - B') = E = (\sqrt{E'}) = (\text{twice } \sqrt{E''})$, then $E/E'^2 = E'^2/E''^4 = 1$.

By applying this to the structure of the Parthenon, the Golden Section itself becomes a derivative of a Gaussian elliptical function. From this standpoint, the first city man builds on Mars can be built on the same principle of musical harmonic ordering as the Acropolis in Athens.

Figure 7.

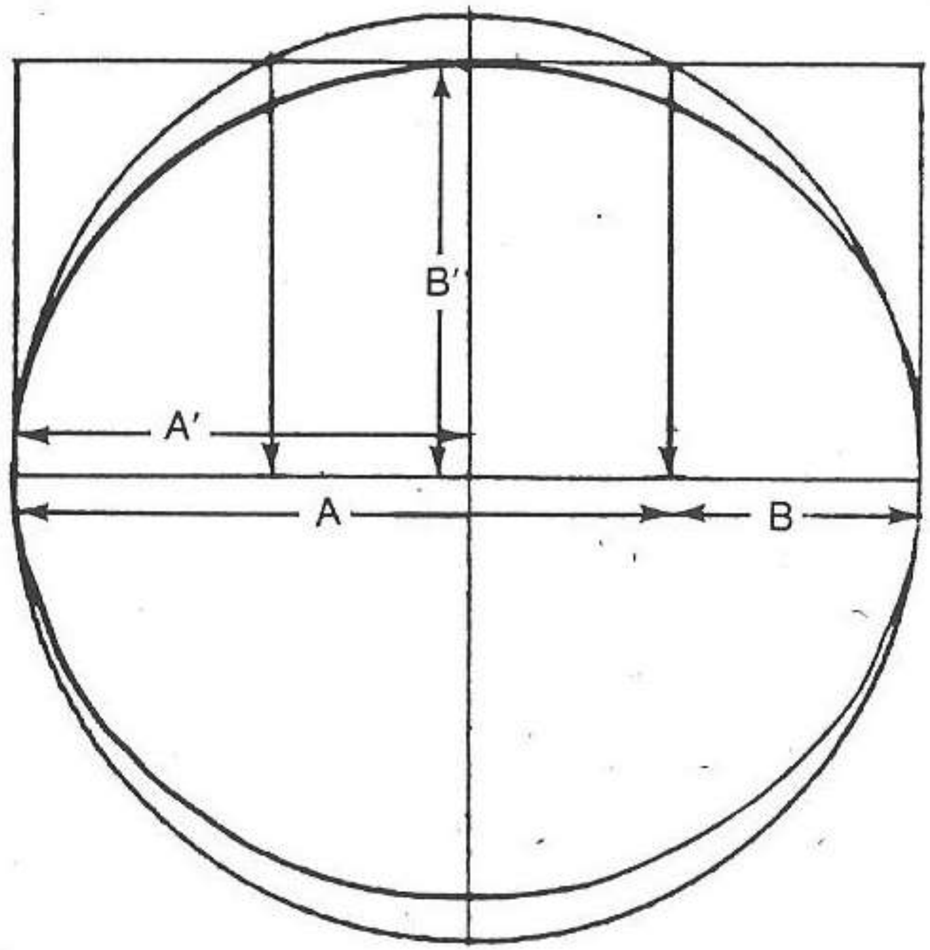


FIGURE 7 THE ELLIPTICAL FUNCTION OF GAUSS AS CUSA'S MINIMUM-MAXIMUM PRINCIPLE. (An illustration of the elliptic function transformation is missing in this report.)

Figure 8

From the Acropolis to the Well-Tempered System

The equal-tempered system and the well-tempered system are derived from same ordering principle that underlies the Acropolis. To construct this, with a straight edge alone, consider the harmonic location of the Parthenon, as described in Figure 3. The octave of G, or the harmonic mean octave, with respect to the C octave, is your starting point for the elaboration of the entirety of the 12 intervals of the equal-tempered system. Part of the key to understanding this is the process of formation of the diapente-fifth (division by half, half of the half). The same process generates Gauss' arthimetic/geometric mean function.

To construct this, go back to the conical part of Figure 3. Draw a ray projected from the apex of the cone and traveling on the surface of the cone through the singularity X, intersecting the lower boundary elliptical cut (minimum) of octave C'-C" with the harmonic mean circular cut of the G octave. This ray also projects upwards to the singularity X', intersecting the upper boundary elliptical cut (maximum) of octave C-C'. Trace a circular cut at intersection X' and draw an elliptical cut from X' to X. This locates a new octave which is the fifth of the fifth, the fifth of G, the harmonic mean of the harmonic mean octave.

Draw again a new circular cut at the focus of that last elliptical cut. The new singularity Y intersecting the upper boundary elliptical cut of octave C-C' may be projected by another ray from the apex of the cone intersecting also the lower boundary elliptical cut singularity Y' of octave C-C'. This will generate still another elliptical octave from Y to Y'. This is the fifth of the fifth of the fifth (the circle of fifths).

By continuing this process very accurately, the last elliptical cut will cross the axis of the cone at precisely the starting point, the midpoint of the cone. At that point, you have reached a maximum; that is, you have constructed two full octaves of 12 intervals each, no more, no less. Thus, you have proven by construction, using a straight edge alone, that there cannot be more than 12 tones in our musical universe. These are, in their order of discovery, C, G, D, A, E, B, F-sharp, C-sharp, A-flat, E-flat, B-flat, F, and back to C. Each circular cut is the geometric mean of each succeeding and preceding cuts in the logarithmic series. Project all circular cuts onto 12 equal parts, dividing a circle in the plane. All the radii will intersect the geometric spiral passing through each of the 12 divisions (Figure 8a).

To transform this equal-tempered system into a well-tempered system, use a geometric (logarithmic) spiral and an arithmetic (Archimedean) spiral (Figure 8b). The harmonic ordering of the geometric spiral is determined by the geometric mean square root of AB, while the harmonic ordering of the arithmetic spiral is generated by the arithmetic mean $(A+B)/2$. The two spirals generated clockwise and counterclockwise, respectively, will intersect one another precisely at the octave and at the well-tempered F-sharp, the arithmetic/geometric mean corresponding to the register shift of the soprano voice. Note that the arithmetic/geometric mean coincides with the elliptical cut, and the two spirals.

If you generate both spirals clockwise, you can locate easily that the halfpoints of their rotations define the interval of a half tone between F and F-sharp, the interval of the register shift for the soprano voice.

All projections were computer-generated by Mark Fairchild. The slight difference in the vertical and horizontal radii is caused by a mechanical defect of reproduction.

Figure 8a.

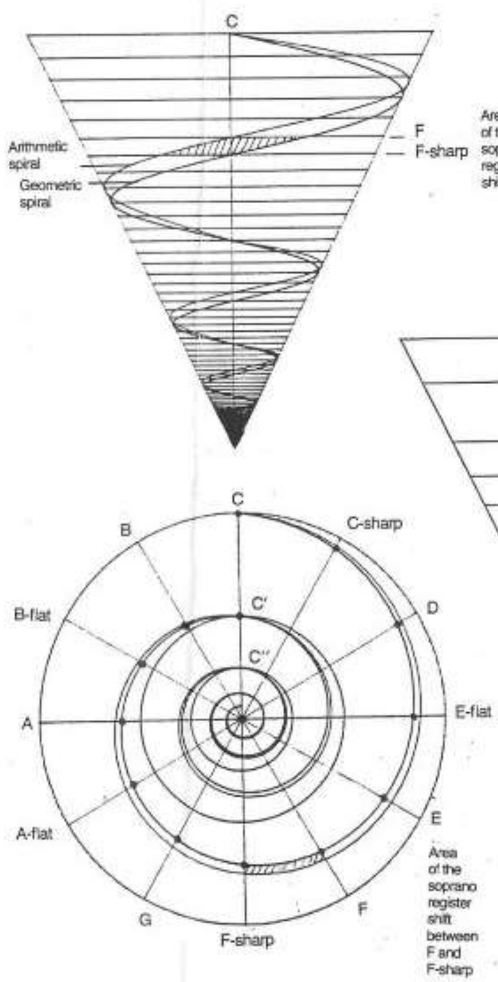
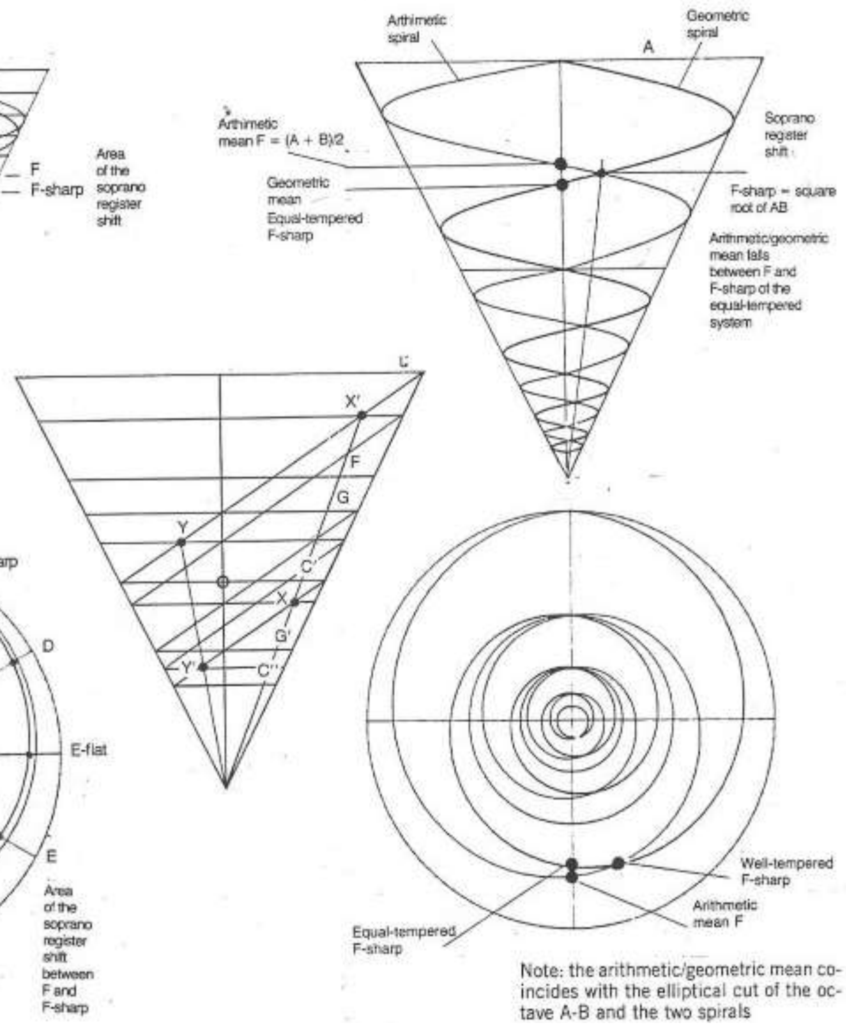


Figure 8b.



(Correction: The arithmetic mean G and the harmonic mean F have been unfortunately inversed in all of the projections.)

The Continuum Theorem of Classical Beauty

by Pierre Beaudry

This note serves as a correction to the concluding section of last week's *American Almanac* article, titled *The Acropolis of Athens: The Classical Idea of Beauty*. I must apologize to the readers for having presented an incomplete illustration referred to as Figure 8. Figures 8a and 8b shown here complete that illustration.

In order to understand the implications of harmonic ordering developed by Plato, Cusa, Leonardo, and Kepler from the standpoint of elliptical functions of the Gauss-Riemannian domain, one must realize that the physical universe is not only characterized by negentropic singularities, but is also characterized by a continuous underlying harmonic ordering. There may be bumps in our understanding of the universe, but there are no holes in the universe.

The arithmetic-geometric mean progression of this Gaussian elliptical function is characterized by a very unique harmonic ordering which can be called a "hypergeometric fifth." This is a dual progression, which divides simultaneously, an arithmetic spiral action and a geometric spiral action by half, and again divides by half the combined remaining intervals. This determines a certain growth process expressed by the elliptical function.

Think of the transformation of this family of ellipses as expressing a precise sequence of pulses harmonically propagating at a faster and faster rate, and increasing the strength of a continuous wave. Each elliptical sequence catches up with the next at an accelerated rate, and converges upon a unique limit value, creating at the limit, the singularity of a shock

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Figure 8a.

A = aphelion

B = perihelion

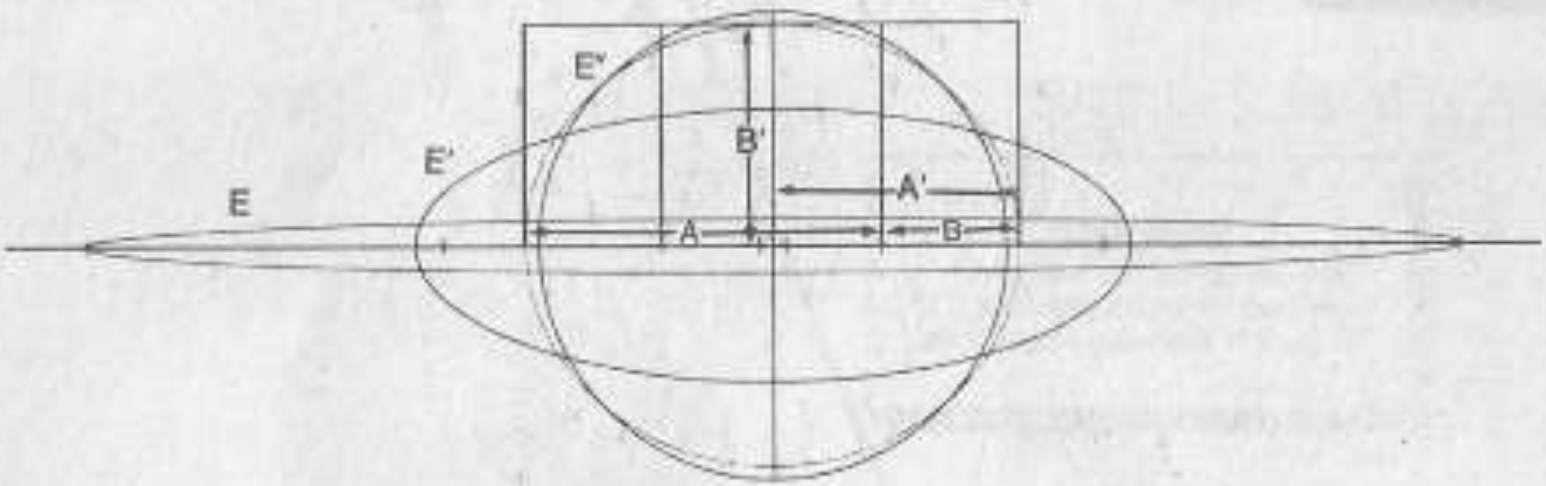
A′ = arithmetic mean

B′ = geometric mean

$(A/B') = (B'/B) =$ Golden Section

When $(A - B)/(A' - B') = E$

then $(E/\sqrt{E'}) = (\sqrt{E'})/(\sqrt[2]{E''}) = 1$



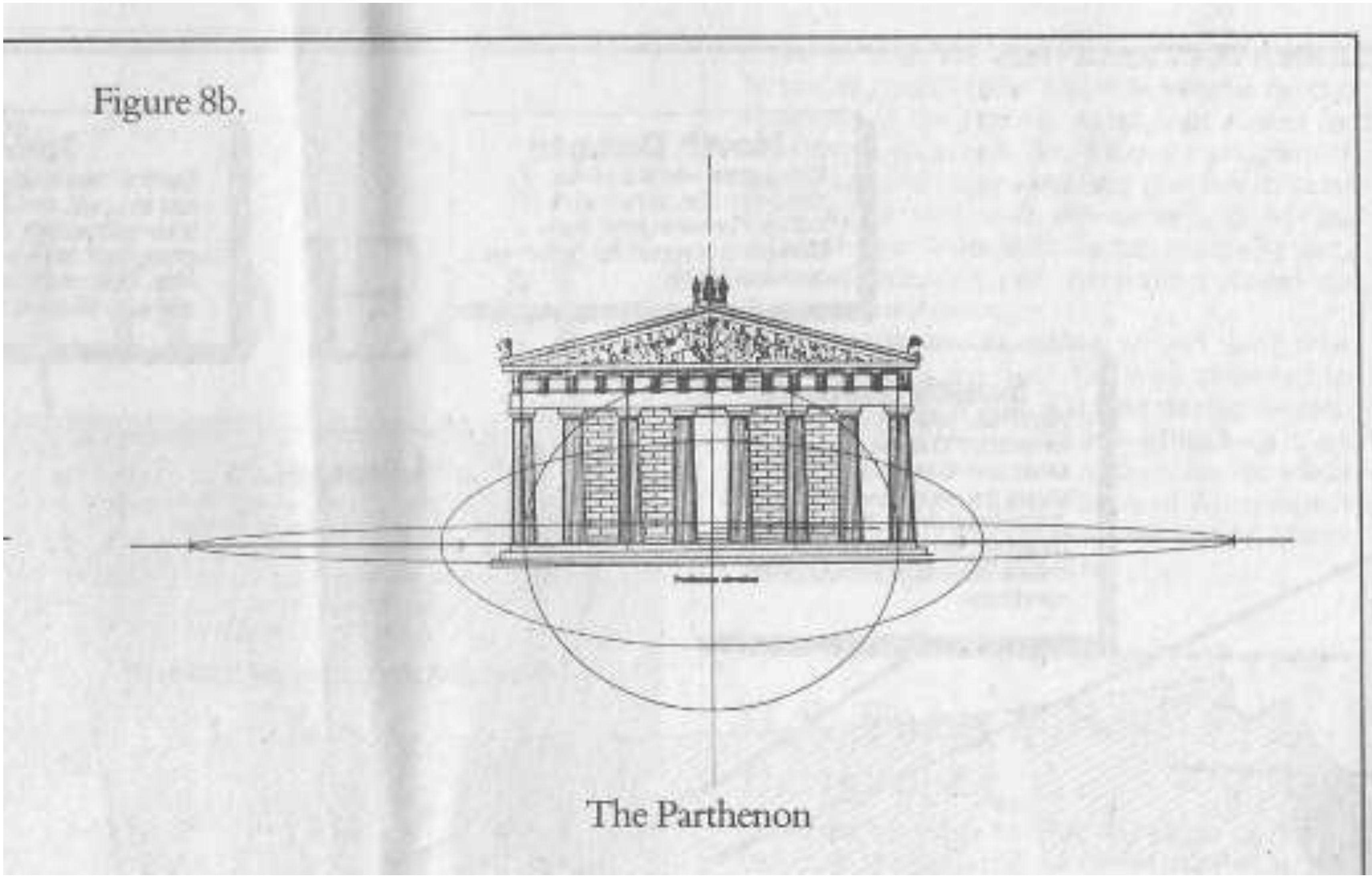


Figure 8b.
The Parthenon

front, the arithmetic-geometric mean circle.

The rate of increase of the intervals between ellipses taken pairwise $E \rightarrow E'^2 \rightarrow E''^4 \rightarrow \ldots$ is simply the inverse of the Gaussian arithmetic/geometric progression. To compute the progression toward the circle, you must compound the arithmetic mean $A' = (A+B)/2$ and the geometric mean $B' = \sqrt{(A \times B)}$. For the inverse of the progression, you must compound $A' = A + \sqrt{(A^2 - B^2)}$ and $B' = A - \sqrt{(A^2 - B^2)}$, which will lead you to a straight line.

By investigating the nature of such intervals of action, you will discover that they are harmonically ordered to such a degree that the original form of simple circular action harmonics $(A/B') = (B'/B)$ is integrated into the elliptical function and is simultaneously subjected to a nonlinear transformation in the form of $(A-B)/(A'-B')$, a ratio of ratios carried through between pairs of successive ellipses as successive singularities, defining a well-ordered negentropic curvature of physical space-time.

Recall here, Plato's idea of harmonic proportionality from his dialogue *Timaeus*, "when the mean becomes first and last and when both the first and last become mean alternately, they all come to be the same to one another . . . and when they become the same to each other, they all become one."

Thus, when $(A-B)/(A'-B') = E$ then the convergence on the arithmetic-geometric mean relates all ellipses harmonically such that $E/\sqrt{E'} = \sqrt{E'}/(\sqrt[4]{E''}) = 1$.

Figure 8b shows the "Golden Mean ellipse" mapped onto the front elevation of the Parthenon. This mapping is merely heuristic, since the Greeks did not know elliptical functions.

The two foci of the "Golden Mean ellipse" fall at the center of the third columns, while the perihelion and aphelion locate the centers of the corner columns. Note that the arithmetic-geometric mean circle of Figure 8a should fall precisely on the inside corner columns and on top of the frieze.

FIN.

P.S.